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General scalarization method of dynamic elastic fields in transversally isotropic media and its new applications[*]

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Обобщенный метод скаляризации динамических упругих полей в трансверсально-изотропных средах и его новые применения[***]

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Introduction. An efficient technique of tensor field scalarization is successfully used while investigating tensor elastic fields of displacements, stresses and deformations in the layered structures of different materials, including transversally isotropic composites. These fields can be expressed through the scalar potentials corresponding to the quasi-longitudinal, quasi-transverse, and transverse-only waves. Such scalarization is possible if the objects under consideration are tensors relating to the subgroup of general coordinate conversions, when the local affine basis has one invariant vector that coincides with the material symmetry axis of the material. At this, the known papers consider structures where this vector coincides with the normal to the boundary between layers. However, other cases of the mutual arrangement of the material symmetry axis of the material and the boundaries between layers are of interest on the practical side.

Materials and Methods. The work objective is further development of the scalarization method application in the boundary value problems of the dynamic elasticity theory for the cases of an arbitrary arrangement of the material symmetry axis relative to the boundary between layers. The present research and methodological apparatus are developed through the general technique of scalarization of the dynamic elastic fields of displacements, stresses and strains in the transversally isotropic media.

Research Results. New design ratios for the determination of the displacement fields, stresses and deformations in the transversally isotropic media are obtained for the cases of an arbitrary arrangement of the material symmetry axes of the layer materials with respect to the boundaries between layers.

Введение. При исследовании тензорных упругих полей перемещений, напряжений и деформаций в слоистых конструкциях из различных материалов, включая трансверсально-изотропные композиты, успешно применяется эффективный метод скаляризации тензорных полей. Данные поля могут быть выражены через скалярные потенциалы, соответствующие квазипродольным, квазипоперечным и чисто поперечным волнам. Такая скаляризация возможна, если рассматриваемые объекты являются тензорами относительно подгруппы общих преобразований координат, когда локальный аффинный базис имеет один инвариантный вектор, который совпадает с осью материальной симметрии материала. При этом в известных работах рассматриваются конструкции, где этот вектор совпадает с нормалью к границе между слоями. Однако, для практики представляют интерес и другие случаи взаимного расположения оси материальной симметрии материала и границы между слоями.

Целью является дальнейшее развитие применения метода скаляризации в граничных задачах динамической теории упругости на случаи произвольного расположения оси материальной симметрии по отношению к границе между слоями.

Методы исследования. Предлагаемый научно-методический аппарат разработан на основе использования обобщенного метода скаляризации динамических упругих полей перемещений, напряжений и деформаций в трансверсально-изотропных средах.

Результаты исследования. Получены новые расчетные соотношения для определения полей перемещений, напряжений и деформаций в трансверсально-изотропных средах на случаи произвольного расположения осей материальной симметрии материалов слоев по отношению к границам между слоями.

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Discussion and Conclusions. The present research and methodological apparatus are successfully used in determining the stress-strain state in the layered structures of transversally isotropic materials, and in analyzing the diagnosis results of the state of the plane-layered and layered cylindrical structures under operation.

Обсуждение и заключения. Предлагаемый научно-методический аппарат успешно использован при определении напряженно-деформированного состояния в слоистых конструкциях, выполненных из трансверсально-изотропных материалов, и при анализе результатов диагностики состояния плоскослоистых и слоистых цилиндрических конструкций, находящихся в эксплуатации.

Keywords: scalarization method, transversally isotropic medium, acoustic waves, composite materials.

Ключевые слова: метод скаляризации, трансверсально-изотропная среда, акустические волны, композиционные материалы.

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Introduction. When investigating the displacement, stress, and strain tensor elastic fields in layered structures of various materials, including transversely isotropic composites [1-3], an efficient method for scalarization of tensor fields proposed in [4] is successfully used. Then, the checked fields can be expressed through scalar potentials corresponding to quasi-longitudinal, quasi-transverse, and transverse-only waves, respectively. This scalarization is possible if the objects under consideration are tensors relative to the subgroup of general coordinate transformations, when the local affine basis has one invariant vector that coincides with the material symmetry axis of the material. Structures where this vector coincides with the normal to the boundary between layers are considered in [5]. However, on the practical side, other cases of the mutual arrangement of the material symmetry axis of the material and the boundaries between layers are of interest.

Materials and Methods. The work objective is the further development of the application of the scalarization method for the boundary problems of the dynamic elasticity theory to the cases of an arbitrary arrangement of the material symmetry axis with respect to the boundary between layers.

At first, in the coordinate system with an admissible reference point, there are components of the displacement and stress tensors relative to this frame. Then, knowing all the components of the tensor fields in the given reference coordinates, we can find the normal and tangential displacement and stress components corresponding to the area lying on the boundary by passing to the coordinate system associated with the boundary between layers. These components are used later to satisfy the boundary problem conditions [5].

To dig deeper into the foregoing, we will discuss it in the context of solving problems for the plane-layered structures.

Let us consider the case when the principal symmetry axis of a transversally isotropic material makes angle α with the normal of the plane boundary surface between the layers (Fig. 1). At this, we assume that the field is independent of the coordinate $\bar{y}$.

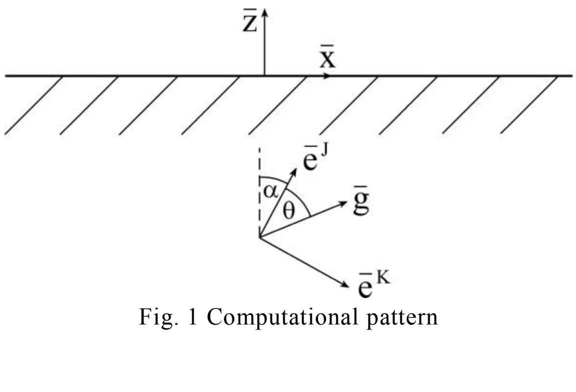

Fig. 1 Computational pattern

Design ratios

In accordance with [4]:

$$U_i = \left(\overset{(L)}{D}_1 \nabla_i + \delta_i^J \overset{(L)}{D}_2 \nabla_J \right)\phi + \left(\overset{(T)}{D}_1 \frac{1}{\overset{(T)}{g}} \nabla_i \nabla_J + \overset{(T)}{g} \delta_i^J \right) w + \sqrt{g}(\delta_i^K \nabla^N - \delta_i^N \nabla^K) v\,;$$

$$\begin{aligned}\sigma_{ij} &= (\overset{(L)}{d}_1 g_{ij} + \overset{(L)}{d}_2 \delta_i^J \delta_j^J + \overset{(L)}{d}_3 \delta^J_{(i} \nabla_{j)} \nabla_J + \overset{(L)}{d}_4 \nabla_i \nabla_j)\phi + \\ &+(\overset{(T)}{d}_1 g_{ij} \nabla_J + \overset{(T)}{d}_2 \delta_i^J \delta_j^J \nabla_J + \overset{(T)}{d}_3 \delta^J_{(i} \nabla_{j)} + \overset{(T)}{d}_4 \nabla_i \nabla_J \nabla_j) w + \\ &+2\sqrt{g}[a_2(\nabla_{(i}\delta^K_{j)}\nabla^N - \nabla_{(i}\delta^N_{j)}\nabla^K) + a_4(\delta^J_{(i}\delta^K_{j)}\nabla^N - \delta^J_{(i}\delta^N_{j)}\nabla^K)\nabla_J] v\,.\end{aligned} \tag{1}$$

Components of the tensor fields of displacements U_i and stresses σ_{ij} in the coordinate system associated with the anisotropy of the material x^J, x^K can be written as follows:

$$\begin{aligned} U_J &= i\,\overset{(L)}{\xi}(\overset{(L)}{D}_1 + \overset{(L)}{D}_2)\phi + \left(-\overset{(T)}{D}_1 \frac{\overset{(T)}{\xi}{}^2}{\overset{(T)}{g}} + \overset{(T)}{g}\right) w\,; \\ U_K &= i\,\overset{(L)}{\beta}\,\overset{(L)}{D}_1 \phi - \overset{(T)}{D}_1 \frac{\overset{(T)}{\xi}\,\overset{(T)}{\beta}}{\overset{(T)}{g}} w\,; \\ \sigma_{JJ} &= [\overset{(L)}{d}_1 + \overset{(L)}{d}_2 - (\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\overset{(L)}{\xi}{}^2]\phi + i\,\overset{(T)}{\xi}(\overset{(T)}{d}_1 + \overset{(T)}{d}_2 + \overset{(T)}{d}_3 - \overset{(T)}{d}_4 \overset{(T)}{\xi}{}^2) w\,; \\ \sigma_{JK} &= -\left(\frac{1}{2}\overset{(L)}{d}_3 \overset{(L)}{\xi}\,\overset{(L)}{\beta} + \overset{(L)}{d}_4 \overset{(L)}{\xi}\,\overset{(L)}{\beta}\right)\phi + i\,\overset{(T)}{\beta}\left(\frac{1}{2}\overset{(T)}{d}_3 - \overset{(T)}{d}_4 \overset{(T)}{\xi}{}^2\right) w\,; \\ \sigma_{KK} &= (\overset{(L)}{d}_1 + \overset{(L)}{d}_4 \overset{(L)}{\beta}{}^2)\phi + i\,\overset{(T)}{\xi}(\overset{(T)}{d}_1 - \overset{(T)}{d}_4 \overset{(T)}{\beta}{}^2) w\,. \end{aligned} \tag{2}$$

Here, the potentials of quasi-longitudinal ϕ and quasi-transverse w waves should satisfy the wave equation with the corresponding wave number g ($\overset{(L)}{g}$ or $\overset{(T)}{g}$), and they have the form:

$$\begin{aligned} \phi &= \phi_0 e^{i \overset{(L)}{\xi} x^J} e^{i \overset{(L)}{\beta} x^K}\,; \\ w &= w_0 e^{i \overset{(T)}{\xi} x^J} e^{i \overset{(T)}{\beta} x^K}\,, \end{aligned} \tag{3}$$

Where $\xi^2 + \beta^2 = g^2$;

$$(\Delta + g^2)\begin{matrix}\phi \\ w\end{matrix} = 0\,. \tag{4}$$

The coefficients D and d entering (2) are determined in [5] and are as follows:

$$\begin{aligned} \overset{(L)}{D}_1 &= \frac{\overset{(L)}{g}{}^2 (C_{13} + C_{44})}{\omega^2\rho - \overset{(L)}{h}{}^2 C_{44} - (\overset{(L)}{g}{}^2 - \overset{(L)}{h}{}^2)(C_{11} - C_{13} - C_{44})}\,; \\ \overset{(L)}{D}_2 &= \frac{\overset{(L)}{g}{}^2}{\overset{(L)}{h}{}^2} \frac{\omega^2\rho - \overset{(L)}{h}{}^2 (2C_{44} + C_{13}) - (\overset{(L)}{g}{}^2 - \overset{(L)}{h}{}^2) C_{11}}{\omega^2\rho - \overset{(L)}{h}{}^2 C_{44} - (\overset{(L)}{g}{}^2 - \overset{(L)}{h}{}^2)(C_{11} - C_{13} - C_{44})} = \frac{\overset{(L)}{g}{}^2}{\overset{(L)}{g}{}^2 - \overset{(L)}{h}{}^2} \overset{(L)}{D}_1\,; \\ \overset{(T)}{D}_1 &= \frac{-\overset{(T)}{g}{}^2 (C_{13} + C_{44})}{\omega^2\rho - \overset{(T)}{h}{}^2 (2C_{44} + C_{13}) - (\overset{(T)}{g}{}^2 - \overset{(T)}{h}{}^2) C_{11}}\,; \\ \overset{(L)}{d}_1 &= -\overset{(L)}{g}{}^2 a_1 - \overset{(L)}{h}{}^2 a_3 (\overset{(L)}{D}_1 + \overset{(L)}{D}_2)\,; \qquad \overset{(L)}{d}_2 = -\overset{(L)}{g}{}^2 a_3 - \overset{(L)}{h}{}^2 (2a_4 \overset{(L)}{D}_2 + a_5 \overset{(L)}{D}_2 + a_5 \overset{(L)}{D}_1)\,; \\ \overset{(L)}{d}_3 &= 2a_2 \overset{(L)}{D}_2 + 2a_4 (2\overset{(L)}{D}_1 + \overset{(L)}{D}_2)\,; \qquad \overset{(L)}{d}_4 = 2a_2 \overset{(L)}{D}_1\,; \\ \overset{(T)}{d}_1 &= a_1 \overset{(T)}{g}(1 - \overset{(T)}{D}_1) + a_3 \left(\overset{(T)}{g} - \frac{\overset{(T)}{h}{}^2}{\overset{(T)}{g}} \overset{(T)}{D}_1 \right); \end{aligned} \tag{5}$$

$$\overset{(T)}{d}_2 = a_3\overset{(T)}{g}(1-\overset{(T)}{D}_1) + 2a_4\overset{(T)}{g} + a_5\left(\overset{(T)}{g} - \frac{\overset{(T)}{h}{}^2}{\overset{(T)}{g}}\overset{(T)}{D}_1\right);$$

$$\overset{(T)}{d}_3 = 2a_2\overset{(T)}{g} + 2a_4\left(\overset{(T)}{g} - 2\overset{(T)}{D}_1\frac{\overset{(T)}{h}{}^2}{\overset{(T)}{g}}\right);\ \overset{(T)}{d}_4 = 2a_2\overset{(T)}{D}_1\frac{1}{\overset{(T)}{g}}; \tag{6}$$

$$a_1 = C_{11} - 2a_2;\quad a_2 = \frac{1}{2}\left(C_{11} - C_{12}\right);\quad a_3 = -\frac{1}{2}\left(C_{11}+C_{12}\right) + C_{13} + C_{44} - a_4;$$

$$a_4 = -\frac{1}{2}\left(C_{11} - C_{12}\right) + C_{44};\ a_5 = C_{11} + C_{33} - 2(C_{13} + 2C_{44}), \tag{7}$$

where C_{ij} are elasticity moduli of the material written down by the contracted index [6-7].

In the formulas (5) and (6) for these coefficients:

$$h = \frac{\cos\theta}{\cos(\alpha+\theta)}g_z;\qquad \beta = \frac{\sin\theta}{\cos(\alpha+\theta)}g_z;\qquad g = \frac{g_z}{\cos(\alpha+\theta)}.$$

The boundary conditions of the dynamic elasticity problems include the components of displacements $\bar{U}_z$, $\bar{U}_x$, and stresses $\bar{\sigma}_{zz}$, $\bar{\sigma}_{zx}$ written in the coordinate system related to the boundary (Fig. 1).

The reference coordinates $\bar{z}$, $\bar{x}$ and x^J, x^K, u, are interlinked through the relations[8]:

$$\bar{z} = -x^K\sin\alpha + x^J\cos\alpha;\ \bar{x} = x^K\cos\alpha + x^J\sin\alpha \tag{8}$$

or:

$$x^K = \bar{x}\cos\alpha - \bar{z}\sin\alpha;\ x^J = \bar{x}\sin\alpha + \bar{z}\cos\alpha. \tag{9}$$

Using relations (8) and (9), from the formulas [5]:

$$\bar{U}_i = \frac{\partial x^m}{\partial \bar{x}^i}U_m;\ \bar{\sigma}_{ij} = \frac{\partial x^m}{\partial \bar{x}^i}\frac{\partial x^p}{\partial \bar{x}^j}\sigma_{mp}, \tag{10}$$

where the coefficients with a bar tab refer to the coordinate system $\bar{z}$, $\bar{x}$ (let us call them "new" coordinates), and without a bar tab – to the coordinate system x^J, x^K ("old" coordinates).

We can write the components of the displacement $\bar{U}_z$, $\bar{U}_x$ and stress $\bar{\sigma}_{zz}$, $\bar{\sigma}_{zx}$ fields entering the boundary conditions through the components (2):

$$\bar{U}_z = \cos\alpha U_J - \sin\alpha U_K;$$

$$\bar{U}_x = \sin\alpha U_J + \cos\alpha U_K; \tag{11}$$

$$\bar{\sigma}_{zz} = \cos^2\alpha\sigma_{JJ} - \sin 2\alpha\sigma_{JK} + \sin^2\alpha\sigma_{KK};$$

$$\bar{\sigma}_{zx} = \frac{1}{2}\sin 2\alpha(\sigma_{JJ} - \sigma_{KK}) + (1 - 2\sin^2\alpha)\sigma_{JK}. \tag{12}$$

In the "new" coordinates, the potential functions (3) have the form:

$$\bar{\phi} = \phi_0 e^{i\overset{(L)}{\xi}(\bar{x}\sin\alpha + \bar{z}\cos\alpha)}e^{i\overset{(L)}{\beta}(\bar{x}\cos\alpha - \bar{z}\sin\alpha)};$$

$$\bar{w} = w_0 e^{i\overset{(T)}{\xi}(\bar{x}\sin\alpha + \bar{z}\cos\alpha)}e^{i\overset{(T)}{\beta}(\bar{x}\cos\alpha - \bar{z}\sin\alpha)},$$

and the displacements (11) are written as follows:

$$\bar{U}_z = \cos\alpha[i\overset{(L)}{\xi}(\overset{(L)}{D}_1 + \overset{(L)}{D}_2)\bar{\phi} + (-\overset{(T)}{D}_1\frac{\overset{(T)}{\xi}{}^2}{\overset{(T)}{g}} + \overset{(T)}{g})\bar{w}] - \sin\alpha[i\overset{(L)}{\xi}\overset{(L)}{D}_1\bar{\phi} + (-\overset{(T)}{D}_1\frac{\overset{(T)}{\xi}{}^2}{\overset{(T)}{g}} + \overset{(T)}{g})\bar{w}] \tag{13}$$

$$\bar{U}_x = \sin\alpha[i(\overset{(L)}{D}_1 + \overset{(L)}{D}_2)\bar{\phi} + (-\overset{(T)}{D}_1\frac{\overset{(T)}{\xi}{}^2}{\overset{(T)}{g}} + \overset{(T)}{g})\bar{w}] + \cos\alpha[i\overset{(L)}{\beta}\overset{(L)}{D}_1\bar{\phi} - \overset{(L)}{D}_1\frac{\overset{(T)}{\xi}\overset{(T)}{\beta}}{\overset{(T)}{g}}\bar{w}] \tag{14}$$

Here, the wave numbers ξ, β, g are determined with respect to the "old" frame (Fig. 2).

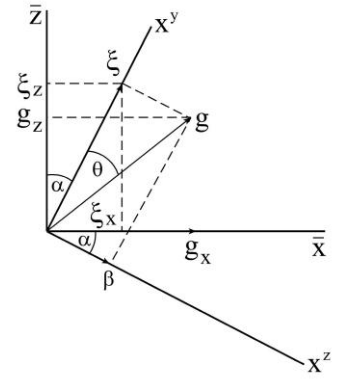

Fig. 2.Wave numbers “relations” diagram

The wave numbers ξ , β which are the projections of the vector $\bar{g}$ onto the “old” frame x^J , x^K , and the wave numbers g_x , g_z that are the projections of the vector $\bar{g}$ onto the “new frame” $\bar{z}$, $\bar{x}$ are interlinked through the following relations:

$$\xi = \frac{\cos\theta}{\sin(\alpha+\theta)} g_x = \frac{\cos\theta}{\cos(\alpha+\theta)} g_z ; \; \beta = \frac{\sin\theta}{\sin(\alpha+\theta)} g_x = \frac{\sin\theta}{\cos(\alpha+\theta)} g_z ;$$

$$g = \frac{g_x}{\sin(\alpha+\theta)} = \frac{g_z}{\cos(\alpha+\theta)} . \tag{15}$$

Substituting (15) into (14), we obtain expressions for displacements in the “new” coordinates:

$$\bar{U}_z = i\overset{(L)}{g}_z(\overset{(L)}{D}_1 + \frac{\cos\theta\cos\alpha}{\cos(\alpha+\theta)}\overset{(L)}{D}_2)\bar{\phi} + \overset{(T)}{g}_z(-\cos\theta\overset{(T)}{D}_1 + \frac{\cos\alpha}{\cos(\alpha+\theta)})\bar{w} ;$$

$$\bar{U}_x = i\overset{(L)}{g}_z[\frac{\cos\theta\sin\alpha}{\cos(\alpha+\theta)}(\overset{(L)}{D}_1 + \overset{(L)}{D}_2) + \frac{\sin\theta\cos\alpha}{\cos(\alpha+\theta)}\overset{(L)}{D}_1]\bar{\phi} + \overset{(T)}{g}_z[-\frac{\sin(\alpha+\theta)\cos\theta}{\cos(\alpha+\theta)}\overset{(T)}{D}_1 + \frac{\sin\alpha}{\cos(\alpha+\theta)}]\bar{w} \tag{16}$$

where, in accordance with (13) and (15):

$$\bar{\phi} = \phi_0 e^{i\overset{(L)}{g}_x\bar{x}} e^{i\overset{(L)}{g}_z\bar{z}} ; \; \bar{w} = w_0 e^{i\overset{(T)}{g}_x\bar{x}} e^{i\overset{(T)}{g}_z\bar{z}} . \tag{17}$$

Similarly, using (2), (14), and (15), from (12), we obtain the relations for the stress components entering the boundary problem conditions:

$$\bar{\sigma}_{zz} = \{\cos^2\alpha[\overset{(L)}{d}_1 + \overset{(L)}{d}_2 - (\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2] + \sin 2\alpha(\frac{1}{2}\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\frac{\cos\theta\sin\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2 + \sin^2\alpha(\overset{(L)}{d}_1 - \overset{(L)}{d}_4\frac{\sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2)\}\bar{\phi} +$$

$$+\{\cos^2\alpha\frac{\cos\theta}{\cos(\alpha+\theta)}i\overset{(T)}{g}_z(\overset{(T)}{d}_1 + \overset{(T)}{d}_2 + \overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) - \sin 2\alpha\frac{\sin\theta}{\cos(\alpha+\theta)}i\overset{(T)}{g}_z(\frac{1}{2}\overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) +$$

$$+\sin 2\alpha\frac{\cos\theta}{\cos(\alpha+\theta)}i\overset{(T)}{g}_z(\overset{(T)}{d}_1 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2)\}\bar{w} ;$$

$$\bar{\sigma}_{zx} = \{\frac{1}{2}\sin 2\alpha[\overset{(L)}{d}_2 - (\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2 + \overset{(L)}{d}_4\frac{\sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2] - (1-2\sin^2\alpha)(\frac{1}{2}\overset{(L)}{d}_3 + \overset{(L)}{d}_4\frac{\cos\theta\sin\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2\}\bar{\phi} +$$

$$+\{\frac{1}{2}\sin 2\alpha\frac{\cos\theta}{\cos(\alpha+\theta)}i(\overset{(T)}{d}_2 + \overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta - \sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) + (1-2\sin^2\alpha)\frac{\sin\theta}{\cos(\alpha+\theta)}i(\frac{1}{2}\overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2)\}\bar{w}; \tag{18}$$

Thus, the displacement and stress components entering the boundary conditions (the coordinate system $\bar{z}$, $\bar{x}$ is associated with the boundary) are determined by the relations (16) - (18) where the projections of the wave vector $\bar{g}$ in the coordinate systems $\bar{z}$, $\bar{x}$ and x^J , x^K are linked through the relations (15).

The matrices C describing the wave properties of the layers [9-10] used in [5] and being the key elements in the development of concrete solutions to the boundary problems in the layered structures in this case, when the axes $\bar{z}$, $\bar{x}$ form the angle α with the frame components $\bar{e}^J$, $\bar{e}^K$, have the form:

$$C = \begin{pmatrix} c_{(1)} & c_{(2)} \\ c_{(3)} & c_{(4)} \end{pmatrix} , \tag{19}$$

where

$$C = \begin{pmatrix} C_{(1)} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} & C_{(2)} = \begin{pmatrix} -c_{11} & c_{12} \\ c_{21} & -c_{22} \end{pmatrix} \\ C_{(3)} = \begin{pmatrix} c_{31} & c_{32} \\ c_{41} & c_{42} \end{pmatrix} & C_{(4)} = \begin{pmatrix} c_{31} & -c_{32} \\ -c_{41} & c_{42} \end{pmatrix} \end{pmatrix} \begin{pmatrix} e^{i\overset{(L)}{g}_z z} & 0 & 0 & 0 \\ 0 & e^{i\overset{(T)}{g}_z z} & 0 & 0 \\ 0 & 0 & e^{-i\overset{(L)}{g}_z z} & 0 \\ 0 & 0 & 0 & e^{-i\overset{(T)}{g}_z z} \end{pmatrix} e^{i g_x x};$$

$$g_x = \overset{(T)}{g}_x = \overset{(L)}{g}_x ; \tag{20}$$

$$C_{11} = i\overset{(L)}{g}_z(\overset{(L)}{D}_1 + \frac{\cos\theta\cos\alpha}{\cos(\alpha+\theta)}\overset{(L)}{D}_2) ;$$

$$C_{12} = \overset{(T)}{g}_z(-\cos\theta\overset{(T)}{D}_1 + \frac{\cos\alpha}{\cos(\alpha+\theta)}) ;$$

$$C_{21} = i\overset{(L)}{g}_z[\frac{\cos\theta\sin\alpha}{\cos(\alpha+\theta)}(\overset{(L)}{D}_1 + \overset{(L)}{D}_2) + \frac{\sin\theta\cos\alpha}{\cos(\alpha+\theta)}\overset{(L)}{D}_1] ;$$

$$C_{22} = \overset{(T)}{g}_z[-\frac{\sin(\alpha+\theta)\cos\theta}{\cos(\alpha+\theta)}\overset{(L)}{D}_1 + \frac{\sin\alpha}{\cos(\alpha+\theta)}] ;$$

$$C_{31} = \cos^2\alpha[\overset{(L)}{d}_1 + \overset{(L)}{d}_2 - (\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2] + \sin 2\alpha(\frac{1}{2}\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\frac{\cos\theta\sin\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2 + + \sin^2\alpha(\overset{(L)}{d}_1 - \overset{(L)}{d}_4\frac{\sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2)$$

$$C_{32} = \cos^2\alpha\frac{\cos\theta}{\cos(\alpha+\theta)}i\overset{(T)}{g}_z(\overset{(T)}{d}_1 + \overset{(T)}{d}_2 + \overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) - \sin 2\alpha\frac{\sin\theta}{\cos(\alpha+\theta)}i\overset{(T)}{g}_z(\frac{1}{2}\overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) +$$

$$+ \sin^2\alpha\frac{\cos\theta}{\cos(\alpha+\theta)}i\overset{(T)}{g}_z(\overset{(T)}{d}_1 - \overset{(T)}{d}_4\frac{\sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) ;$$

$$C_{41} = \frac{1}{2}\sin 2\alpha[\overset{(L)}{d}_2 - (\overset{(L)}{d}_3 + \overset{(L)}{d}_4)\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2 + \overset{(L)}{d}_4\frac{\sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2] - (1 - 2\sin^2\alpha)(\frac{1}{2}\overset{(L)}{d}_3 + \overset{(L)}{d}_4\frac{\cos\theta\sin\theta}{\cos^2(\alpha+\theta)}\overset{(L)}{g}{}_z^2 ;$$

$$C_{42} = \frac{1}{2}\sin 2\alpha\frac{\cos\theta}{\cos(\alpha+\theta)}i(\overset{(T)}{d}_2 + \overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta - \sin^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) + (1 - 2\sin^2\alpha)\frac{\sin\theta}{\cos(\alpha+\theta)}i(\frac{1}{2}\overset{(T)}{d}_3 - \overset{(T)}{d}_4\frac{\cos^2\theta}{\cos^2(\alpha+\theta)}\overset{(T)}{g}{}_z^2) . \tag{21}$$

Where $\alpha = 0$, the elements (21) of the matrix C coincide with the corresponding elements of this matrix for the case of coincidence of the normal to the boundary and the material symmetry axis direction that are represented by the formulas (3.82) in [5]. If $\alpha = \frac{\pi}{2}$, then we get the case when the symmetry axis of the material is tangent to the boundary surface, and the formulas (21) coincide with the expressions (3.89) - (3.91) in [5].

Knowing the expressions for matrices C, one can construct solutions to various problems using the research and methodological apparatus described in [5].

Conclusions. The present research and methodological apparatus were successfully used for determining the stress-strain state in the layered structures made of transversely isotropic materials, and for analyzing the diagnostics results of the state of flat-layered and layered cylindrical structures in operation.

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