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ИНФОРМАТИКА, ВЫЧИСЛИТЕЛЬНАЯ ТЕХНИКА И УПРАВЛЕНИЕ
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Estimation of proximity of controls synthesized on basis of maximum principle and ADAR method *

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Пример оценки близости управлений, синтезированных на основе принципа максимума и метода АКАР***

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Introduction. A special case of synthesizing the same electromechanical control system by the Pontryagin maximum principle and by the synergetic synthesis method is considered. The task was to solve the synthesis problem of the time optimal electromechanical position control system; herewith the travel resistance modulus linearly depended on the output coordinate of the system. This approach to the selection of the synthesis problem was because the synthesis of time optimal systems is one of the most widespread problems, and it is solved by increasing the efficiency of the existing control systems.

Materials and Methods. Synthesis of the time optimal linear control system based on the maximum principle is a widely accepted problem in the modern control theory. However, the procedure of synergistic synthesis does not have such formalization. This being the case, the paper suggests an approach that brings together these two methods, which, in our opinion, will increase the efficiency of the synergistic synthesis method through adding some features of the synthesis methodology for optimal systems.

Research Results. The paper formulates two key concepts. The first one is as follows: the application of the maximum principle for an object of the DC motor class when synthesizing the positioning algorithm under the conditions of linear loading functionally dependent on the engine rotation angle allows the time optimal system to be optimized. The second concept states that synthesis of a control system based on the synergistic approach enables to obtain a system close to optimal (quasioptimal), but after modifying the synergetic synthesis method itself. A hypothesis is formulated on the possible connection between the introduced (when implementing the procedure of state space extension in the

Введение. Рассмотрен частный случай синтеза одной и той же электромеханической системы управления методом максимума Понтрягина и методом синергетического синтеза. В качестве задачи была определена задача синтеза оптимальной по быстродействию электромеханической системы позиционирования, при этом момент сопротивления движению линейно зависел от выходной координаты системы. Этот подход к выбору задачи синтеза был обусловлен тем, что синтез оптимальных по быстродействию систем является одной из самых широко распространенных задач, которая решается при повышении эффективности действующих систем управления.

Материалы и методы. Синтез оптимальной по быстродействию линейной системы управления на основе принципа максимума — широко распространенная задача в современной теории управления. Однако процедура синергетического синтеза такой формализации не имеет. Исходя из этого, в статье предложен подход, сближающий эти два метода, который, по мнению авторов, позволит повысить эффективность метода синергетического синтеза, добавив в него некоторые особенности методологии синтеза оптимальных систем.

Результаты исследования. В работе сформулированы два основных научных положения. Первое - применение принципа максимума для объекта класса двигатель постоянного тока при синтезе алгоритма позиционирования в условиях линейной нагрузки, функционально зависящей от угла поворота двигателя, позволяет оптимизировать систему по быстродействию. Второе - синтез системы управления на основе синергетического подхода позволяет получить систему, близкую к оптимальной (квазиоптимальную), но уже после модификации самого метода синергетического синтеза. Сформулирована гипотеза о возможной связи между вводимыми, при реализации процедуры расширения пространства состояния в методе синергетического синтеза, постоянными времени с

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synergetic synthesis method) time constants with the optimal switching time of control defined in the maximum method.
Discussion and Conclusions. The synthesis through the maximum control technique and the ADAR method is performed. In virtue of the comparison of efficiency of these methods, a hypothesis is put forward on the possible compatibility of the studied methods.

определяемым в методе максимума оптимальным временем переключения управления.
Обсуждение и заключения. Выполнен синтез управления методом максимума и методом Аналитического Конструирования Агрегированных Регуляторов АКАР. На основании сравнения эффективности применения методов выдвигается гипотеза о возможной совместимости исследуемых методов.

Keywords: maximum principle, optimal control, operation speed, control algorithm, synergetic synthesis, ADAR method.

Ключевые слова: принцип максимума, оптимальное управление, быстродействие, алгоритм управления, синергетический синтез, метод АКАР.

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Introduction. In the 20th century, the engineering requirements, in particular, in space engineering, put forward a range of problems for which a new theory was developed – the theory of optimal control [1]. One of the main techniques for the synthesis of optimal control systems is the maximum principle developed by Soviet mathematician L. S. Pontryagin and his disciples in the fifties-sixties of the 20th century [2]. The application of this principle is based on the formalization of the synthesis problem with the transition to the form of the Mayer problem and the subsequent solution to systems of linear or, in some special cases, nonlinear differential equations [1]. It is worth noting that the need to solve systems of differential equations, and in the nonlinear case this is not always possible, is in many ways a limitation of both the maximum principle itself and the whole concept of synthesis of optimal control systems.

In recent decades, a new approach to the synthesis of control systems has been widely adopted. It relies on the synergetic concept of the analysis and synthesis of systems. The technique used, the author of which is A. A. Kolesnikov, is called the method of analytical design of aggregated regulators (ADAR) [4–5]. The proposed approach is based on the concept of synthesis of nonlinear feedbacks. They provide the asymptotic stability of the control system with respect to the required motion of the attractor in the state space [4]. This method differs from the methods of synthesis of optimal control systems in the absence of both the optimization criterion of the control system and the statement of the synthesized control optimality. From the point of view of the implementation of the synthesis procedure, the ADAR method has an undeniable advantage over the synthesis methods of optimal systems, which is expressed in the absence of restrictions on the nonlinearity of the system of differential equations [4-5].

From a practical standpoint, synthesis of the process or object management should be able to answer the question if there is another control that has the property of superiority over all others. Thus, the modern mass production constantly requires efficiency improvement to ensure market competition. One of the most popular ways to increase this efficiency is to optimize management processes. This approach is applicable both to the systems of automated assembly of equipment [6–7] and to the metal-cutting systems, in particular, to drilling control systems in metal-cutting machines [8–15]. The idea of combining these approaches to synthesis of the control systems has, from the authors' the point of view, an undoubted practical effect. In science terms, it is important to combine the ADAR method advantages, which are expressed in the possibility of considering synthesis of the complex nonlinear process dynamics in mechanical engineering [16–19], with a neat and definite formalization of the synthesis problem formulation and assessment of its achievement in the maximum principle [1–2] .

1. Synthesis of basic mathematical model and formulation of research problem

In the modern economy, the direction of the time optimal system synthesis, which enables to obtain a significant increase in the global system efficiency [7–8], has become a frequent practice. Thus, under solving the problems of automating the assembly processes of various equipment, the task of attaching different types of parts to each other [6–7] often arises. Here, the economic efficiency of the entire production process depends on the speed of this operation. The same situation is observed in metal cutting systems on metal-cutting machines [9–19], in which the faster the machining process is, the lower the costs of the entire production process. Based on the reasoning, under the assumption that it is necessary to synthesize a time optimal control system, which, considering the similarity of the processes of automated fastening of parts and drilling deep holes, can be illustrated by the following diagram (Fig. 1).

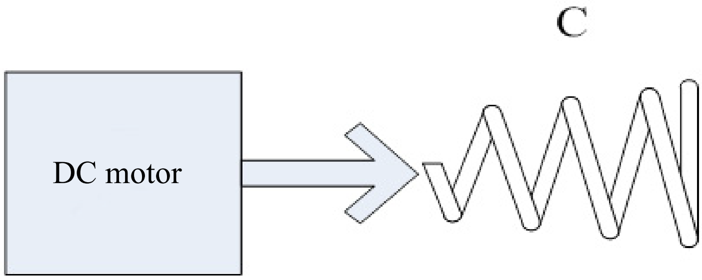

Fig. 1. Illustration diagram

Fig. 1 shows a DC motor performing either an equipment fastening operation or a feed in a drilling unit. In the first case, C is the combined load-deflection characteristic of the material that is compressed under twisting. In the second case, C characterizes a linear increase in the feed resistance under the accumulation of chips in the chip removal channels of the drill.

The electric drive, which ensures the conversion of electrical energy to mechanical energy of motion of the system actuators, is the basis for the support subsystem of the control system in both cases considered. Let us assume that in the present case, such a transformation is carried out by a DC motor with collector control, which is described through the following system of equations [20]:

$$\begin{gathered} U - c_e\omega = L\frac{di}{dt} + Ri \\ c_m i = J\frac{d\omega}{dt} + M_c \end{gathered},$$

where U is voltage applied to the engine manifold; i is current consumed by the motor; R, L are parameters of the electrical part of the engine; J is parameter characterizing the inertial properties of the engine rotor, the reduced inertial moment of all rotating masses; ω is engine rotor speed; M_c is external moment of resistance; c_m, c_e are mechanical and electric engine constants. In this case, the moment is presented as a function of the angle of rotation of the engine rotor, that is $M_c = C\alpha$. With this in mind (1), we rewrite as:

$$\begin{gathered} U - c_e\omega = L\frac{di}{dt} + Ri \\ c_m i = J\frac{d\omega}{dt} + C\alpha \\ \frac{d\alpha}{dt} = \omega \end{gathered}. \tag{2}$$

We express the current value in the second equation, apply the obtained value in the first equation, and get:

$$\begin{gathered} U\frac{1}{c_e} - \omega = \frac{LJ}{c_e c_m}\frac{d^2\omega}{dt^2} + \frac{RJ}{c_e c_m}\frac{d\omega}{dt} + \frac{LC}{c_e c_m}\omega + \frac{RC}{c_e c_m}\alpha \\ \frac{d\alpha}{dt} = \omega \end{gathered}. \tag{3}$$

We solve the first equation with respect to the highest derivative and rewrite it with the second one, and then we receive:

$$\begin{gathered} \frac{d\alpha}{dt} = \omega \\ \frac{d^2\omega}{dt^2} = -\frac{R}{L}\frac{d\omega}{dt} - \left(\frac{c_m c_e}{LJ} + \frac{C}{J}\right)\omega - \frac{RC}{LJ}\alpha + \frac{c_m}{LJ}U \end{gathered}. \tag{4}$$

Let us put $\alpha = x_1, \omega = x_2, \frac{d\omega}{dt} = x_3$; as constants, we introduce $\frac{R}{L} = a_{33}, \left(\frac{c_m c_e}{LJ} + \frac{C}{J}\right) = a_{32}, \frac{RC}{LJ} = a_{31}, \frac{c_m}{LJ} = b$. Then the system (4) takes the form:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= x_3 \\ \frac{dx_3}{dt} &= -a_{31}x_1 - a_{32}x_2 - a_{33}x_3 + bU \end{aligned} \quad . \tag{5}$$

or in matrix-vector form:

$$\dot{x} = Ax + Bu\,, \tag{6}$$

where $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_{31} & -a_{32} & -a_{33} \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 0 \\ b \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $u = \begin{pmatrix} 0 \\ 0 \\ U \end{pmatrix}$.

We take the DC motor whose parameters provide the following constant values: a_{31}=4.65, a_{32}=4.6, a_{33}=2, as a drive ensuring the system motion.

A matrix, with this in mind, will take the form: $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4,65 & -4,6 & -2 \end{pmatrix}$. From knowing *A* matrix, we find λ eigenvalues of *A* matrix: $\lambda_1 = -1.2666 + 0.0000i$, $\lambda_2 = -0.3667 + 1.8806i$, $\lambda_3 = -0.3667 - 1.8806i$.

As is clear from the obtained eigenvalues of the matrix, the control system is asymptotically stable according to Lyapunov [21]. Moreover, we can argue about the oscillatory nature of the processes proceeding in the system, since the eigenvalues contain not only negative real parts, but nonzero imaginary parts.

2. Synthesis of control by Pontryagin's maximum principle

First, we formulate the problem of the optimal control synthesis in the following form:

- using the maximum principle for the object described by the System (5), determine the optimal equation algorithm that ensures the transfer of the object from the initial state $x_1(0)=50$, $x_2(0)=0$, $x_3(0)=0$ to the final state $x_1(T)=0$, $x_2(T)=0$, $x_3(T)=0$ for *T* minimum time. Herewith, it is necessary to determine transition count, switching torque, and to construct curves *of* $u(t)$ control and $x_1(t)$, $x_2(t)$, $x_3(t)$ coordinates.

$|u| \le U_{max}$ restriction is imposed on the control action. The system parameters are as follows: $a_{31} = 1, a_{32} = 12, a_{33} = 20, b = 1$, U_{max}=440 V.

Solution: We write the Hamiltonian:

$$H = \sum_{i=1}^{2} \psi_i f_i\,, \tag{7}$$

where ψ_i and f_i are coordinates of $\vec{\psi}$ and $\vec{f}$ vectors. In addition to the system (5), we construct a system of equations for ψ_i auxiliary variables, where $i = 1, 2$, using the following relation:

$$\frac{d\psi_i}{dt} = -\sum_{j=1}^{2} \psi_i \frac{\partial f_i}{\partial x_i}, \quad i = 1, 2.$$

Or open :

$$\begin{aligned} \frac{d\psi_1}{dt} &= -\left[\frac{\partial f_1}{\partial x_1}\psi_1 + \frac{\partial f_2}{\partial x_1}\psi_2 + \frac{\partial f_3}{\partial x_1}\psi_3 \right] \\ \frac{d\psi_2}{dt} &= -\left[\frac{\partial f_1}{\partial x_2}\psi_1 + \frac{\partial f_2}{\partial x_2}\psi_2 + \frac{\partial f_3}{\partial x_2}\psi_3 \right] \\ \frac{d\psi_3}{dt} &= -\left[\frac{\partial f_1}{\partial x_3}\psi_1 + \frac{\partial f_2}{\partial x_3}\psi_2 + \frac{\partial f_3}{\partial x_3}\psi_3 \right] \end{aligned} \tag{8}$$

Considering (5), the equation system (8) takes the form:

$$\begin{cases} \dfrac{d\Psi_1}{dt} = 4,65\Psi_3 \\ \dfrac{d\Psi_2}{dt} = -\Psi_1 + 4,6\Psi_3 \\ \dfrac{d\Psi_3}{dt} = -\Psi_2 + 2\Psi_3 \end{cases} \tag{9}$$

$$\begin{cases} \dfrac{d\Psi_1}{dt} = 4{,}65\Psi_3 \\ \dfrac{d\Psi_2}{dt} = -\Psi_1 + 4{,}6\Psi_3 \\ \dfrac{d\Psi_3}{dt} = -\Psi_2 + 2\Psi_3 \end{cases} \qquad (9)$$

Eigenvalue matrix for (9) case:

$$A^{\Psi-D} = \begin{pmatrix} 1.2666 & 0 & 0 \\ 0 & 0.3667+1.8806i & 0 \\ 0 & 0 & 0.3667+1.8806i \end{pmatrix}$$

The solution to the system (9) for the diagonalized case of $A^{\psi-D}$ matrix takes the form:

$$\begin{cases} \Psi_1 = C_1 e^{1.2666t} \\ \Psi_2 = C_2 e^{(0.3667+1.8806i)t} \\ \Psi_3 = C_3 e^{(0.3667-1.8806i)t} \end{cases}. \qquad (10)$$

Considering V^{ψ} matrix, the solution in the initial basis will be:

$$\begin{cases} \Psi_1 = -0.9474C_1 e^{1,.666t} + (0.5812-0.3347i)C_2 e^{(0.3667+1.8806i)t} + (0.5812+0.3347i)C_3 e^{(0.3667-1.8806i)t} \\ \Psi_2 = -1.1893C_1 e^{1.2666t} + 0.6883C_2 e^{(0.3667+1.8806i)t} - 0.6883C_3 e^{(0.3667-1.8806i)t} \\ \Psi_3 = -0.2581C_1 e^{1.2666t} + (0.1812+0.2086i)C_2 e^{(0.3667+1.8806i)t} + (0.1812-0.2086i)C_3 e^{(0.3667-1.8806i)t} \end{cases},$$

where C_1, C_2, C_3 are integration constants.

The general expression describing the Hamilton function:

$$H = \Psi_1 f_1 + \Psi_2 f_2 + \Psi_3 f_3 \qquad (11)$$

In the expression (11), an important – from the point of view of the method of synthesis – role is played by the member, which includes the control:

$$H^* = \Psi_3 U = (-C_1 0.2581 e^{1.2666t} + C_2(0.1812+0.2086i) + C_3(0.1812-0.2086i)e^{(0.3667-1.8806i)t})bU.$$

In order for H Hamiltonian calculated by the formula (11) to take the maximum positive value, H^* term must be always positive and the greatest. For this, the optimal control algorithm should be $u(t)=\sigma U_{max}$, where:

$$\sigma = sign(-0.2581C_1 e^{1.2666t} + (0.1812+0.2086i)e^{(0.3667+1.8806i)t} + (0.1812-0.2086i)e^{(0.3667-1.8806i)t}).$$

$u(t)$ optimal control is a piecewise constant function, taking $\pm U_{max}$ values, and it has no more than two intervals of constancy, since the nonlinear function

$$-0.2581C_1 e^{1.2666t} + (0.1812+0.2086i)e^{(0.3667+1.8806i)t} + (0.1812-0.2086i)e^{(0.3667-1.8806i)t}$$

changes the sign no more than once. In this case, the possible sign change occurs from plus to minus, that is, to fulfill the maximum principle, it is required to first apply $U= +U_{max}$, to the engine, and then $U= -U_{max}$. Let us verify these arguments through constructing Ψ_3 obtained functional dependence in the *Matlab* package (Fig. 2).

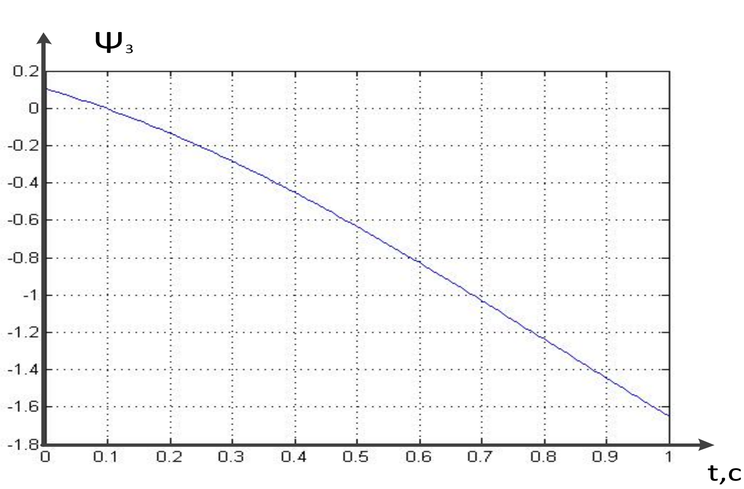

Fig. 2. Graph of Ψ_3 variance

As can be seen from Fig. 2, Ψ_3 does change the sign from (+) to (–) only once. With this in mind, we define an optimal equation algorithm that ensures the transfer of an object from $x_1(0)=0$, $x_2(0)=0$, $x_3(0)=0$ initial state to $x_1(T)=50$, $x_2(T)=0$, $x_3(T)=0$ final state for *T* minimum time. Here we note that under modern conditions, there is no need to obtain an analytical solution to the original system of equations. Using the available capabilities of modern software packages like *Matlab*, we straightforward and clearly can obtain a numerical solution to the case under consideration.

The numerical simulation results for the calculated time optimal control algorithm are presented in Fig. 3. The model parameters are selected in such a way that it fulfills the boundary condition required by the right-hand border.

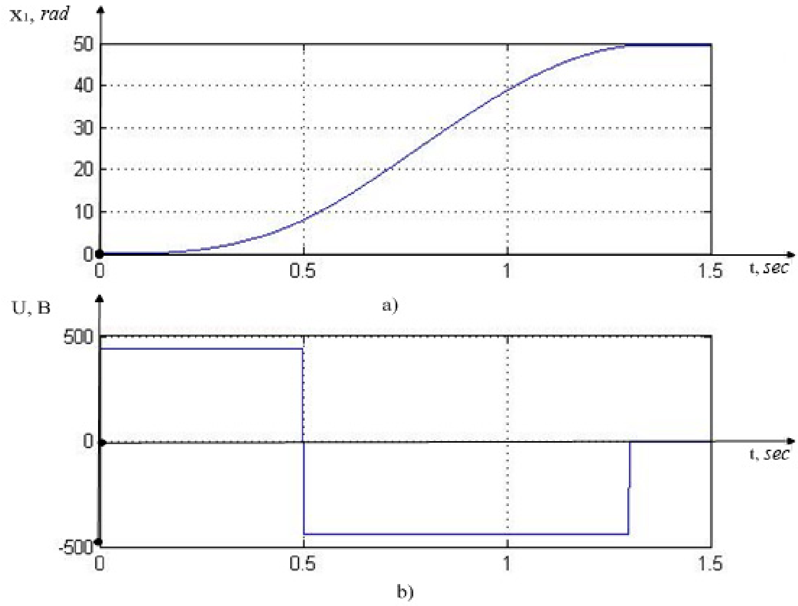

Fig. 3. Graphs of output coordinate (a) and control switching at $t_1 = 0.5$ (b)

Fig. 3 shows that the control switching time from ($+U_{max}$) to ($-U_{max}$) is selected as $t_1 = 0,5$ seconds, and the total control time is $T = 1.3$ seconds. In this case, the control system comes to the required output level, that is, the right-hand boundary of the boundary conditions is reached.

3. Synthesis of control system through ADAR method

To synthesize a control system using the ADAR method, it is possible not to make a transition to the abstract case of the state space (see (5)), but it is easy to use the original system (2). However, it needs to be translated to the following form:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= -a_{21}x_1 + a_{23}x_3 \\ \frac{dx_3}{dt} &= -a_{32}x_2 - a_{33}x_3 + bU \end{aligned} , \quad (12)$$

where $\alpha = x_1, \omega = x_2, i = x_3$ are variables; $\frac{c_m}{J} = a_{23}, \frac{C}{J} = a_{21}, \frac{c_e}{L} = a_{32,} \frac{R}{L} = a_{33}, \frac{1}{L} = b$ are introduced as constants. Considering the previously defined values, we receive: $a_{23} = 1.515, a_{21} = 2.325, a_{32,} = 1.5, a_{33} = 2, b = 1$ and the system (12) takes the form:

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= -2.325x_1 + 1.515x_3 , \\ \frac{dx_3}{dt} &= -1.5x_2 - 2x_3 + U \end{aligned} \quad (13)$$

With this in mind, *A* matrix takes the form:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ -2.325 & 0 & 1.515 \\ 0 & -1.5 & -2 \end{pmatrix}.$$

From knowing A matrix, we find λ eigenvalues of A matrix: λ1 = –1.2674 + 0.0000i; λ2 = –0.3663 + 1.8801i; λ3 = –0.3663 - 1.8801i. As is clear from these results, it is referred to the control system case considered in the synthesis through the maximum principle.

The coordinate characterizing the angle of rotor rotation (x_1) is the output coordinate of the system. Therefore, in order to form requirements for the desired system behavior in the state space, we introduce a macrovariable of $\Psi_1 = x_1 - x_{01} \Rightarrow 0$ order, where x_{01} is the specified target value of x_1 coordinate. The velocity value of the angle change must ensure that the angle of rotor rotation tends to the value specified above. To this end, we introduce another macrovariable – $\Psi_2 = x_2 - \phi_2(x_1) \Rightarrow 0$, where $\phi_2(x_1)$ is a certain function describing $x_2 \Rightarrow \phi_2(x_1)$ tendency in the stationary state. Further, we introduce a macrovariable of order in $\Psi_3 = x_3 - \phi_3(x_1, x_2) \Rightarrow 0$ coordinate, where $\phi_3(x_1, x_2)$ is a certain function that describes the relationship between the coordinates in the stationary state of the system and, therefore $x_3 \Rightarrow \phi_3(x_1, x_2)$.

For newly introduced Ψ_1, Ψ_2, Ψ_3 macrovariables, we require the asymptotically stable law of change, that is, execution of the following system of differential equations:

$$\begin{cases} T_1 \dfrac{d\Psi_1}{dt} + \Psi_1 = 0 \\ T_2 \dfrac{d\Psi_2}{dt} + \Psi_2 = 0 \\ T_3 \dfrac{d\Psi_3}{dt} + \Psi_3 = 0 \end{cases}, \qquad (14)$$

where T_1, T_2, T_3 are time constants that determine the vanishing rate of the introduced macrovariables, or, in other words, the shrinking rate of an arbitrary path of the system in the state space to the desired attractor.

The system of equations introduced in the expression (14) has expanded the system state space from the 3rd to the 6th, which is one of the main provisions of the ADAR method. Further synthesis of the control system is reduced to a stage-by-stage process of decomposition (compression) of this space to the initial level [1–2]. We start the decomposition process with the substitution of the system (14) to the first equation instead of Ψ_1 macrovariable of its value in the coordinates of the controlled process. As a result, we obtain:

$$T_1 \frac{dx_1}{dt} + (x_1 - x_{01}) = 0, \qquad (15)$$

Considering (13), the equation (15) is written as:

$$T_1 x_2 + (x_1 - x_{01}) = 0. \qquad (16)$$

Considering the next step of the synthesis algorithm, because of which it was assumed that $x \Rightarrow \phi_2(x_1)$, we find $\phi_2(x_1)$ value as:

$$\phi_2(x_1) = -\frac{(x_1 - x_{01})}{T_1} \qquad (17)$$

With this context, Ψ_2 macrovariable is as follows:

$$\Psi_2 = x_2 - \phi_2(x_1) = x_2 + \frac{(x_1 - x_{01})}{T_1} \qquad (18)$$

Validity of the expression (17), in terms of the control objectives, is confirmed by the fact that the value in the steady state is as follows: $x_2 \Rightarrow \phi_2(x_1) \Rightarrow 0$. Then, considering (18), the second equation of the system (14) will take the following form:

$$T_2 \frac{dx_2}{dt} - T_2 \frac{d\phi_2(x_1)}{dt} + x_2 - \phi_2(x_1) = 0. \qquad (19)$$

Or, considering $\phi_2(x_1)$:

$$T_2 \frac{dx_2}{dt} + \frac{T_2}{T_1}\frac{dx_1}{dt} + x_2 + \frac{(x_1 - x_{01})}{T_1} = 0. \qquad (20)$$

At the next step of the synthesis algorithm, using the same reasoning as before, and the fact that we determine $x_3 \Rightarrow \phi_3(x_1, x_2)$ from (20), considering (13), $\phi_3(x_1, x_2)$ value in coordinates of the controlled process:

$$\phi_3(x_1, x_2) = 1,535x_1 - 0,66\frac{(T_2 + T_1)}{T_1}x_2 - 0,66\frac{(x_1 - x_{01})}{T_2T_1}0 . \quad (21)$$

Having obtained $\phi_3(x_1, x_2)$ value in the coordinates of the controlled process, we can determine the value of Ψ_3 macrovariable in the coordinates of the system state and solve the third equation of the system (14).

$$T_3[\frac{dx_3}{dt} - \frac{d\phi_3(x_1, x_2)}{dt}] + x_3 - \phi_3(x_1, x_2) = 0 . \quad (22)$$

After substituting the previously obtained $\phi_3(x_1, x_2)$ values from (21) into (22) and using the system (13) at this step of decomposing the state space of the control system, we define the control value in the coordinates of the controlled process:

$$U = 1,5x_2 + 2x_3 + 1,535x_2 + 1,5x_1\frac{(T_1T_3 + T_2T_3 + T_1T_2)}{T_1T_2T_3} - x_3\frac{(T_1T_3 + T_2T_3 + T_1T_2)}{T_1T_2T_3} - \Leftrightarrow$$
$$\Leftrightarrow 0,66x_2\frac{(T_1 + T_2 + T_3)}{T_1T_2T_3} - 0,66\frac{(x_1 - x_{01})}{T_1T_2T_3} \quad (23)$$

The expression (23) determines the asymptotically stable control action on the control system described by the equations (13).

The simulation results of the system of differential equations (13) with the required value of the engine rotor angle: x_{10}=50 rad, and the values of $T_1 = 0.1$, $T_2 = 0.2$, $T_3 = 0.3$ time constants entered are shown in Fig. 4.

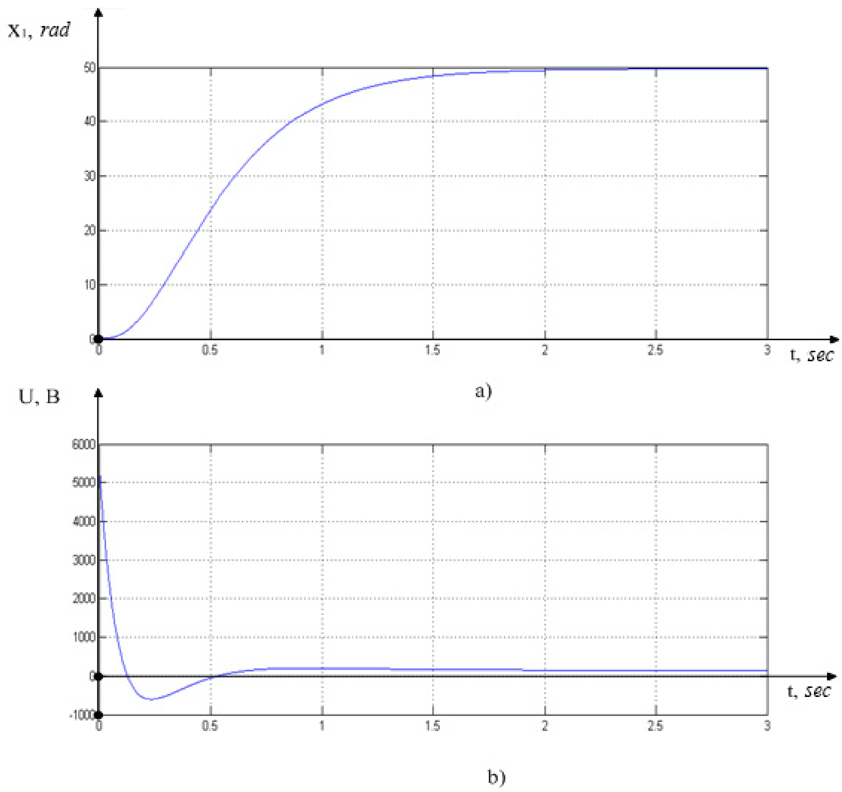

Fig. 4. Simulation results with control at $T_1 = 0.1$, $T_2 = 0.2$, $T_3 = 0.3$: transition process on output coordinate (a); control change schedule (b)

Fig. 4 shows that, in comparison with Fig. 3, the control structure is the same in both cases, that is, the point is that at the beginning, a positive control is applied, and then its sign changes to the opposite one. The settling time increases dramatically to the value close to 2.5 seconds, but with this, the maximum control reaches values greater than 5,000 V. This is unacceptable according to the maximum method. To limit the maximum control value, we introduce a nonlinear link of the "saturation" type with the same threshold as in the case of the maximum into the control loop. The results of the system simulation, with this threshold and with selected values of T_1, T_2, T_3 constants, are presented in Fig. 5.

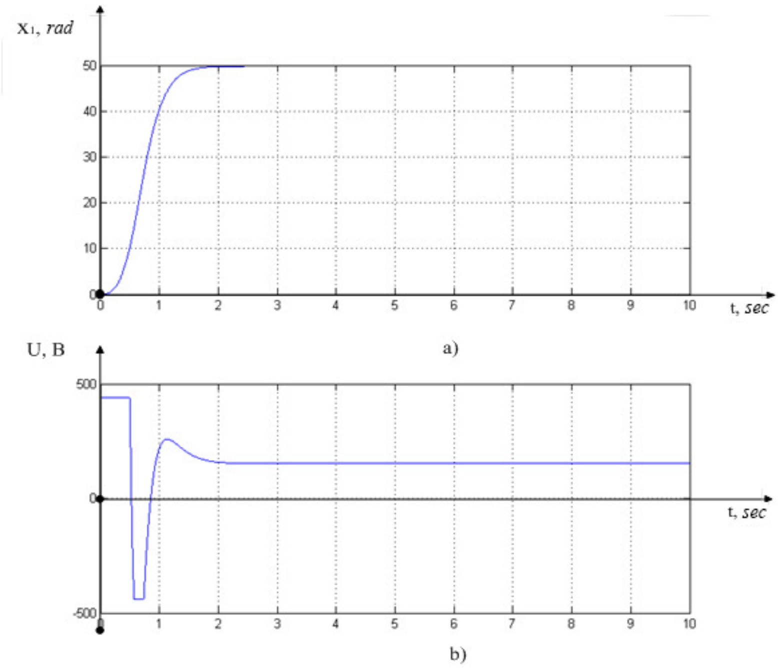

Fig. 5. Simulation results with control at $T_1 = 0.15$, $T_2 = 0.16$, $T_3 = 0.17$:
output coordinate transition (a); control change schedule (b)

Fig. 5 shows that the introduction of a nonlinear constraint on the control of the “saturation” type enables to obtain the desired result from the point of view of the control admissibility. In this case, the control structure determined by the maximum method is really observed, but the control is not time optimal. It should be noted here that such a modification of the ADAR method could also lead to the loss in system stability (Fig. 6).

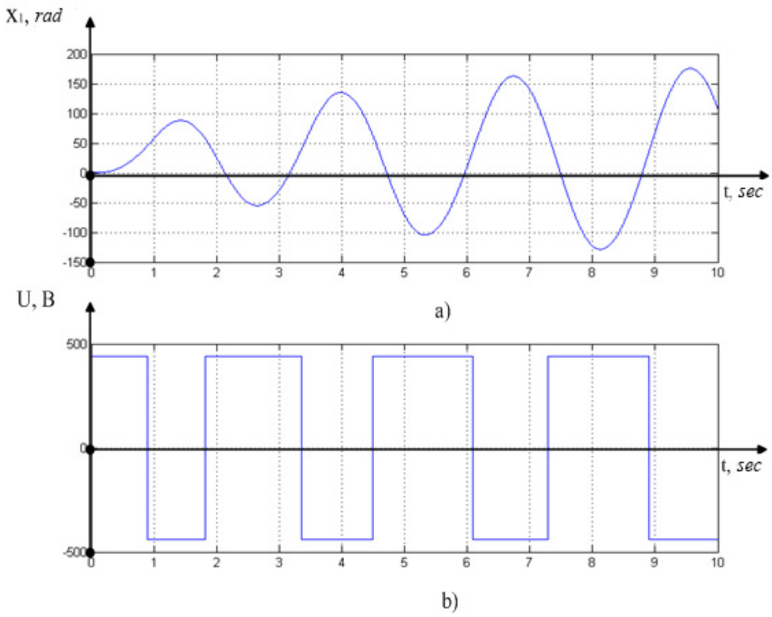

Fig. 6. Simulation results with control at $T_1 = 0.015$, $T_2 = 0.016$, $T_3 = 0.017$:
output coordinate transition (a); control change schedule (b)

However, despite the result presented in Fig. 6, the addition of amplitude constraint of the control signals to the control system obtained by the ADAR method essentially brings it closer to the solutions obtained through implementing the synthesis procedure by the maximum method.

Discussion and Conclusions. Thus, for the basic model of the control system (2), the time optimal control was synthesized by the maximum method. For the same case, in the third part of the paper, with the same system parameters and boundary conditions, the control of the ADAR method was synthesized. Both of these controls look different.

In case of the maximum method, this is a software piecewise-constant control where the optimal time instant of the sign change of the control signal is important. In ADAR case, it is a continuous smooth functional control dependence on the coordinates of the system state. However, their implementation shows the structural similarity expressed in changing the control sign. After the ADAR method modification, the structural similarity has become even more visible. Note that the parametric similarity of the implemented controls can be achieved through selecting the time constants introduced by the ADAR method during the synthesis. This allows the authors to formulate the following hypothesis.

- Selection of the values of T_1, T_2, T_3 time constants introduced by the ADAR method in the synthesis procedure, in case of modification of the obtained control by the method proposed in the article, makes it possible to get time optimality of the synergetic control. The authors failed to obtain a general proof from this intuitive hypothesis. However, this task was not set within the framework of this paper. The numerical experiments with T_1, T_2, T_3 variation helped us to achieve quasioptimality of the control obtained by the ADAR method.

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