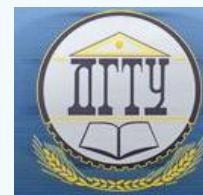


# МАШИНОСТРОЕНИЕ И МАШИНОВЕДЕНИЕ

## MACHINE BUILDING AND MACHINE SCIENCE



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### Development of a power model for large wave gear toothing \*

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### Разработка силовой модели зубчатого зацепления крупной волновой передачи \*\*\*

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*Introduction.* The development of computational-and-experimental methods for evaluating the force distribution pattern across the width of the toothed rim and in the circumferential direction of the toothing of a large wave gear is considered. The study is based on the results of the test tooth tensometry using scale modeling of prototype units. The work objective is to create a reliable experimental-theoretical model of the teeth force interaction in a large wave gearing. Such a solution involves the transformation of model sample deformations into a distributed load between teeth which will eliminate the basic uncontrollable nonlinear errors and improve the accuracy of estimation of force factors in the toothing area.

*Materials and Methods.* An improved power analysis procedure of a large wave gearing, optimized by accuracy criteria, is developed. The accuracy of the research results is enhanced through improving physical and computational models. This approach enables to obtain reasonable dependences of the power factors distribution in a large wave gearing.

*Research Results.* The design shape of the control tooth is simplified; an invariant profile is introduced over the full width of the ring gear. Thus, non-linear distortions of the experimental results introduced by a variable tooth shape across the width of the ring gear are excluded. In this case, the installation of tensoresistors across the full width of the test tooth is possible. In addition, the proposed solution can establish the dependence of the teeth deformation across the full width of the ring gear, and not only in the extreme areas as suggested by the well-known techniques. The development of perfect physical and mathematical models enables to increase accuracy of the results of theoretical and experimental studies on power processes in the large wave gearing. The scientific-based two-parameter dependences of the force distribution in gearing are obtained.

*Введение.* Статья посвящена разработке расчетно-экспериментальных методов оценки характера распределения сил по ширине зубчатого венца и в окружном направлении зубчатого зацепления крупной волновой передачи. Исследование основано на результатах тензометрии контрольного зуба с использованием масштабного моделирования опытных образцов. Цель работы — создание объективной экспериментально-теоретической модели силового взаимодействия зубьев в зацеплении крупной волновой передачи. Такое решение предполагает преобразование деформаций модельных образцов в распределенную нагрузку между зубьями, что позволит исключить основные неконтролируемые погрешности нелинейного характера, повысить точность оценки силовых факторов в поле зацепления.

*Материалы и методы.* Разработана уточненная методика силового анализа зубчатого зацепления крупной волновой передачи, оптимизированная по критериям точности. Точность результатов исследования повышена за счет совершенствования физических и расчетных моделей. Такой подход позволил получить обоснованные зависимости распределения силовых факторов в зубчатом зацеплении крупной волновой передачи.

*Результаты исследования.* Упрощена конструктивная форма контрольного зуба: по всей ширине зубчатого венца введен неизменный профиль. Таким образом исключены нелинейные искажения результатов экспериментов, вносимые переменной формы зуба по ширине зубчатого венца. В таком случае возможен монтаж тензорезисторов по всей ширине контрольного зуба. Кроме того, предлагаемое решение позволяет установить зависимость деформации зубьев по всей ширине зубчатого венца, а не только на крайних участках, как предлагают известные методики. Разработка совершенных физических и математических моделей позволила повысить точность результатов теоретических и экспериментальных исследований силовых процессов в зубчатом зацеплении крупной волновой передачи. Получены научно обоснованные двухпараметрические зависимости рас-

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**Discussion and Conclusions.** Approximation of the involute-tooth profile in the trapezoidal profile has simplified evidence of identity of the elasticity equations and the boundary conditions of mathematical models. The results obtained are applicable in the mathematical simulation of the planar stress state of teeth with nonlinear profiles. Comparative evaluation of errors introduced by deviations of geometry and dimensions of physical models and mathematical analogues supports the experiment correctness and the validity of the quantitative data obtained. The research results can be used in the improved calculation of the design parameters of the gear components in the engineering process of large heavily loaded wave reducers.

**Keywords:** wave gear, gearwheel, power analysis, load distribution

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пределения сил в зубчатом зацеплении.

**Обсуждение и заключения.** Аппроксимация эвольвентного профиля зуба в трапецидальный профиль упростила доказательство тождественности уравнений упругости и граничных условий математических моделей. Полученные результаты применимы при математическом моделировании плосконапряженного состояния зубьев с нелинейными профилями. Сравнительная оценка погрешностей, вносимых отклонениями геометрических форм и размеров физических моделей и математических аналогов, подтверждает корректность постановки эксперимента и обоснованность полученных количественных данных. Результаты работы могут быть использованы при уточненном расчете конструктивных параметров элементов зубчатого зацепления в процессе проектирования крупных тяжело нагруженных волновых редукторов.

**Ключевые слова:** волновая передача, зубчатое зацепление, силовой анализ, распределение нагрузки.

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**Introduction.** The intermediate transformation of the rotational motion into a continuous wave deformation of the flexible wheel has changed the established principles and forms of conjugation of the engagement elements of higher kinematic pairs. At this, a small tooth difference in the internal gearing of wave transmission (two teeth) differs from the traditional ideas on the conditions of geometric synthesis and functioning of a gear pair. Therefore, from the viewpoint of interference of the teeth, the engagement of the wave transmission is beyond the permissible limits of the existence of an involute internal gear with rigid tooth-wheels. The interference of second-type teeth arising in clamping is enhanced by the deformation of the flexible wheel from the transmitted load, which limits the carrying capacity of wave gear. The negative impact of the scale factor contributes to the occurrence of tooth interference in large wave transmissions, which causes jamming and slipping of teeth in the gearing.

The basic concept of wave gear (strain wave gearing, SWG) was patented by K. W. Musser in 1957. The solution remains unchanged for the stock-produced designs of limited capacity despite many design improvements. This is confirmed by wave gears manufactured by the leading companies in the USA (Harmonic Drive Technologies Inc., United Shoe Machinery Corp.), Japan (Harmonic Drive Systems Inc.), Germany (Harmonic Drive AG), and others including licensed products of Harmonic Drive Systems Inc., China [1, 2]. In large wave gearheads, high capacities are realized, and torques exceed  $(0.3 \dots 1.5) \times 10^6$  Nm. Mining, metallurgical equipment and large machines produced by Novokramatorsk Machine Building Plant (NKMZ) are completed with such parts. The standard design of a cam wave generator with a flexible ball bearing is inoperative and inapplicable in such products of heavy engineering [3]. Large wave gears use a disk wave generator. In contrast to the cam generator which simultaneously contacts along the entire perimeter of the flexible wheel, the disk wave generator interacts with the flexible wheel in diametrically opposite areas. This creates a higher level of freedom of deformation of the flexible wheel and increases possible deviations from the specified position.

Under the action of the wave generator discs and torque, the flexible wheel becomes curved in the form of a slightly twisted taper. This causes tooth misalignment over the width of the ring gear relative to the rigid wheel. The uniformity of the gap setting in the gearing across the width of the gear rims is disturbed.

Under heavy-duty service, the deformations of the flexible wheel exceed the gaps in the gearing and create interference conditions for the teeth of the second kind. With wide toothed crowns, a small module of the teeth under heavy loads, the gaps in the local zones of the deformed gearing take on negative values, activate the interference, cause jamming and breakthrough. Constructive features of large wave gears are mainly due to the scale. This factor exacerbates the negative processes observed in the gearing of higher kinematic pairs, and reduces the specifications and performance of heavily loaded wave gearboxes.

The work objective is to increase the load capacity, the specifications and performance of large wave gear reducers that meet the technical requirements of heavy engineering.

Theoretical and practical developments in the study of large wave transmissions are limited, the results obtained are contradictory, and they cannot always be repeated. Besides, we note significant construction features, lack of geometric similarity in the size series of kinematic and large wave transmissions among others. All this does not allow using well-known computational methods in the design of wave transmissions for the purposes of heavy engineering [4–6]. Synthesis of gearing is subject to the condition of constancy of a given gear ratio, governed by the Euler – Savary formula. Permanent deformation of the flexible wheel does not agree with the fundamental gearing theorem, which contributed to the development of conflicting methods for the synthesis of wave gearing [7–11]. In some papers, a general case of gearing is taken as the basis [12], while others consider a wedge mechanism with a complex relative movement of links [13–15]. In some techniques, the geometry and kinematics of wave gearing are simplified [16, 17]. The combined approach is synthesis of the general theory of gearing with elements of the wedge mechanism [1].

To satisfy the conditions of the fundamental theorem of gearing in relation to the synthesis of wave transmission, E.G. Ginzburg used the kinematic approach [18]. The concept of “angular velocity of a point” introduced by him is wrong, the arguments based on it are incorrect.

In the monograph by N.A. Kovalev [19], complexity of the force and kinematic processes occurring in wave transmissions, absence of reliable dependencies that can adequately reflect the impact of external factors on internal processes in kinematic pairs are specified. It prevents from developing an effective method for synthesis of wave gear that optimizes the basic parameters of the bearing links. According to N.A. Kovalev, with an inextensible middle surface, only the point of intersection of the tooth axis and the middle surface moves uniformly, which contradicts the conclusions of E.G. Ginzburg [18].

Maximum radial elastic displacement ( $w_o$ ) on the middle surface of the flexible wheel is set by the wave generator in the absence of torque. The  $w_o$  offset must exceed half the working height of the tooth. The tooth dimensions should be adjusted by  $w_o$  value, not vice versa [19].

There is no consensus on the value of  $w_o$  parameter. Some authors associate maximum radial deformation ( $w_{max}$ ) with the transmission ratio of wave gear [20] and a disk wave generator. To increase efficiency, it is recommended to assume  $w_{max} = m$ . E.G. Ginzburg recommends to determine  $w_{max}$  through  $m_y$  conditional module, different from  $m$  module [18].

**Materials and Methods.** To prevent the interference of teeth, the following well-known techniques are used in the stock-produced wave gearboxes:

- tooth correction,
- widening of the tooth space,
- angle increase in the original tooth contour.

This, along with a narrow width of the gear rims and relatively small transmitted torques, in many ways settles practical issues of the approximate gearing synthesis in terms of eliminating the tooth interference for the stock-produced low-loaded wave gears. However, theoretical studies on these issues are far from complete. So far, there is no satisfactory solution regarding the engagement of higher kinematic pairs of large wave gears with the gear rim width of 100–200 mm, a tooth module of 1.5–3 mm and a disk wave generator. Large wave gearboxes, designed by well-known methods and manufactured at NKMZ, turned out to be inoperable due to the tooth interference. Solving such problems requires new technical solutions based on reliable results of the theoretical and experimental research.

A scientific research has been carried out to assess the boundary conditions for the interference phenomena and tooth breakthrough in the gearing, as well as to prevent these negative developments. Their results enable to create a generalized model of the tooth interaction in the engagement of a large wave transmission. A refined power analysis procedure of such toothing is developed. The research results accuracy is enhanced through improving the physical and computational models. This allows obtaining reasonable dependences of the distribution of power factors in the gearing of a large wave transmission.

Deformation of stressed structural elements, including teeth, is usually measured using strain gauge methods. However, it is impossible to fix strain gages on the wave transmission teeth with a small module. According to the known methods [20, 21], two parallel slots are made in the rim of the hard wheel removing two adjacent teeth (Fig. 1).

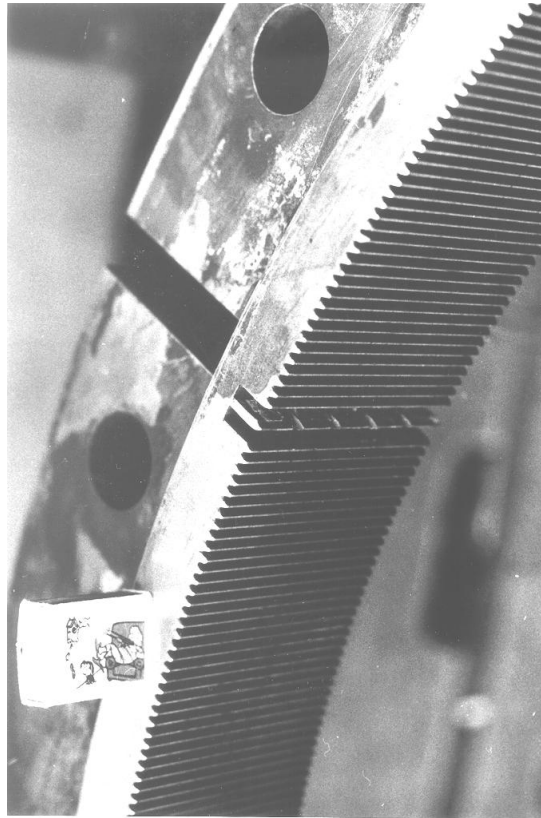


Fig. 1. Localized tooth elements of rigid wheel tilt gear of MP-600AC mobile mixer

Across the wheel width, the slots can be end-to-end or blind, made at both ends of the rim so that the middle part remains intact. According to the authors of common methods, through deep slots can reduce significantly the wheel stiffness, which is unacceptable. In the known papers, an experimental model is used, where the middle part of the teeth is not removed and it works alongside with all others.

The ideas of an unacceptable reduction in wheel stiffness through end-to-end slots [22] are inconclusive, since the thickness of the rim of a rigid wheel is not regulated; and it can be taken sufficient under the experimental conditions that the slots have no real impact on rigidity. In addition, the hard wheel is pressed into the hull design and, when evaluating stiffness, is considered together with the gearbox housing. .

According to the methods [20, 21], the slots of the rigid wheel increase the deformation of the control tooth relative to its middle part. The change in the rigidity of the control tooth along the length increases the deformation of the selected elements at the ends, distorts the deformed state of the control tooth and does prevents with sufficient accuracy the estimation of the force characteristics of the gearing along the crown width and in the circumferential direction. A multiple decrease in the rigidity of the selected tooth elements is not considered by the well-known methods [22], which violates the objectivity of the experimental results and complicates their analysis.

Standard techniques for measuring the tooth deformation of the wave transmission [8] differ little from the method of conventional gears [23]. In [22], it is shown that the difference in rigidity of the model, due to blind slots on the hard wheel, distorts the deformation of the control tooth under load and is considered when conducting the experiment and processing of the data obtained. This reduces the accuracy and reliability of the results.

The strain gauge is not mounted in the middle of the control tooth, so it is impossible to get a full picture of its deformation, qualitative and quantitative characteristics of the force distribution in the toothing. Moreover, when considering the stepwise change in the rigidity of a physical model by the known methods, the boundary conditions are not taken into account, the impact of which is enhanced by the scale factor [18].

Consider VZ-1120 wave reducer of the tilt drive of MP-600AC mobile mixer with the capacity of 600 tons of molten metal. Assume that load in the gearing of the wave transmission is applied in the middle of the tooth height, because there is  $\sim 20\div 25\%$  of teeth in the internal gearing of one wave.

Let us determine a possible error of allowance in the experiment. Consider a rigid annulus internal gear:

- modulus of teeth  $m = 1.5$  mm;
- number of teeth of the rigid wheel  $Z_2 = 762$ ;
- number of cutter teeth  $Z_4 = 68$ ;



- wheel pitch circle diameter  $d = 1143$  mm;
- wheel tooth depth  $H = 2.985$  mm;
- wheel outside diameter  $D = 1155.12$  mm;
- root diameter of wheel teeth  $D_b = 1161.09$  mm;
- pressure angle at the tooth point on the pitch circle  $d$ ,  $\alpha_0 = 20^\circ$ ;
- tool addendum modification coefficient  $x = +4.953$  mm;
- face width  $b = 100$  mm;
- normal rack tooth profile as per GOST 13755-81 standard;
- accuracy degree 7D (GOST 1643-81).

The relative error of  $R$  assumption concerning the conditional application of normal force in the middle of the tooth depth:

$$R = \frac{2H}{D + D_b} \cdot 100\% = \frac{2 \cdot 2.985}{1155.12 + 1161.09} \cdot 100\% = 0.3\%.$$

The accepted assumption has no real impact on the transmission of forces in gearing, since it changes slightly the diameter of the loading appliance. For approximation of the involute tooth profile in a straight line shape, we determine the thickness of the tooth ( $S_x$ ) of the rigid wheel along an arbitrary circle of radius ( $r_x$ ):

$$S_x = m \frac{\cos \alpha_0 \pi}{\cos^2 \alpha_x} + \Delta_2 + Z_2 (\text{inv} \alpha_x - \text{inv} \alpha_0), \quad \cos \alpha_x = \frac{r}{r_x} \cos \alpha_0,$$

where  $\alpha_x$  is pressure angle at the tooth point, where radius is  $r_x$  vector;  $\Delta_2$  is ratio of the tooth thickness on any wheel circumference in the normal section under toothing by the cutter of medium wear:

$$\Delta_2 = (Z_2 - Z_4) (\text{inv} \alpha_0 - \text{inv} \alpha_{C2}). \quad (1)$$

The relative error is defined as the modulus of ratio of the absolute error to the approximation value. In this case, maximum possible absolute error is equal to  $H/2$ . The approximate value is the radius of the middle tooth circle of the rigid wheel:  $(D + D_b)/4$ . The angle cosine of the machine tool  $\alpha_{C2}$  in the expression (1) when cutting a rigid wheel by the cutter of medium wear is determined considering the addendum modification coefficient of the cutter:

$$\cos \alpha_{C2} = \cos \alpha_0 \frac{z_2 - z_4}{z_2 - z_4 + 2\varepsilon_2}.$$

The tooth thickness values ( $S_x$ ) along the arc of an arbitrary radius ( $r_x$ ) of the rigid wheel are given in Table 1.

Table 1

Arc tooth thickness  $S_x$  of rigid wheel in different sections with radius  $r_x$ , mm

$r_x$	579.239	579.426	579.612	579.799	579.985	580.172	580.358	580.545
$S_x$	2.613	2.765	2.918	3.071	3.223	3.377	3.530	3.685

Arc tooth thickness differs from the chordal tooth thickness at the given parameters of a rigid wheel by  $5 \cdot 10^{-5}$  mm, so with sufficient accuracy, we can equate the arc tooth thickness with  $r_x$  radius and the corresponding chordal tooth thickness.

We determine the tooth thickness  $S_{ax}$  approximated in a straight profile at the radius of  $r_x = 579.799$  mm. It is equal to the half-sum of the extreme tooth thickness values along the radii  $r_x = 579.053$  mm and  $r_x = 580.545$  mm:  $S_{ax} = 3.074$  mm.

The tooth thickness of the involute profile of the rigid wheel on the circle of radius  $r_x = 579.799$  mm is equal to  $S_x = 3.071$  mm. Absolute error of the approximation of the tooth profile of a rigid wheel in a straight-line profile:

$$A_x = S_x - S_{ax} = -0.003 \text{ mm}.$$

Relative error of the straight-line approximation of the involute tooth profile of a rigid wheel:

$$R = \left| \frac{S_x - S_{ax}}{S_{ax}} \right| \cdot 100 = 0.1\%.$$

The approximation of the involute tooth profile of a rigid wheel with a straight-line profile does not introduce a detectable error in the design scheme. The equation of the straight-line profile of the tooth model in the  $XOY$  coordinates is shown in Fig. 2.

$$\frac{Y - Y_2}{X - X_2} = \frac{Y_1 - Y_2}{X_1 - X_2}. \quad (2)$$

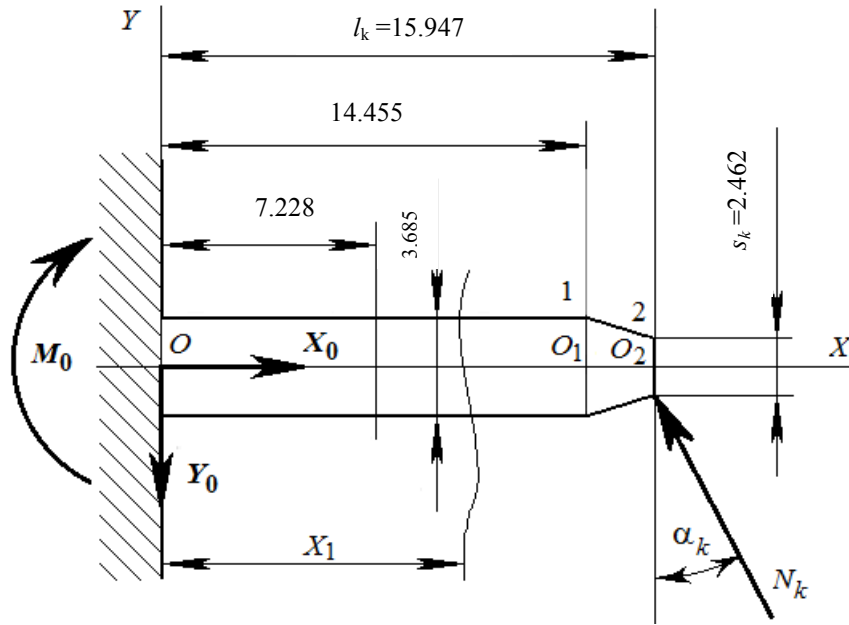


Fig. 2. Design model of control tooth of rigid wheels approximated by straight-line profile

In Fig. 2, 1 and 2 indices indicate membership of the coordinate functions  $X$  and  $Y$  to the extreme 1 and 2 points of the straight-line tooth profile.  $X$  and  $Y$  values are inserted in the equation (2) at 1 and 2 points as shown in Fig. 2, and the equation of the straight tooth profile with linear dimensions in millimeters is written:

$$Y = 7.767 - 0.410 X.$$

Thickness and rigidity of the approximated straight tooth of a rigid wheel in an arbitrary section, mm:

$$S_{ax} = 2Y = 15.534 - 0.820 X. \quad (3)$$

$$EI_z = 2.1 \cdot 10^5 \frac{I S_x^3}{12}. \quad (4)$$

Specific rigidity of the approximated straight tooth of a rigid wheel in an arbitrary cross section is determined by substituting the values (3) at  $l = 17.5$  mm into the expression (4):

$$EI_{z_0} = 0.169 \cdot 10^6 \cdot (18.943 - X)^3.$$

Deflection of the elastic line of the control tooth model under  $N_k$  force is determined at  $X = l_k = 15.947$  mm:

$$Y_2 = 0.819 \cdot 10^{-4} N_k \text{ [mm]}. \quad (5)$$

Tension stresses  $\sigma^+$  and compression  $\sigma^-$  of the control tooth model in the area of installation of strain gauges at the distance of  $a_r = 7.2275$  mm from the cantilever fastening (pinching) of the model:

$$\sigma^+ = \frac{M_H}{W_z} - \frac{X_0}{F} = 0.1867 N_k, \frac{H}{\text{mm}^2}, \quad (6)$$

$$\sigma^- = \frac{M_H}{W_z} + \frac{X_0}{F} = 0.1983 N_k, \frac{H}{\text{mm}^2}, \quad (7)$$

Here,  $M_H$  is bending moment in the area of installation of strain gauges;  $M_H = M_0 - Y_0 \cdot a_r$ ;  $W_z$  is section modulus of the linear tooth model:

$$W_z = \frac{bh^2}{6} = 39.61 \text{ mm}^3.$$

Using the ratio

$$\sigma = E\varepsilon,$$

where  $\varepsilon$  is the relative deformation of the strain gauge, as well as the formulas (5–7), a relationship is established between the relative deformation of the strain gauges on the stretched and compressed surfaces of the control tooth model and the deflection of the elastic line under the force:

$$Y_2 = 5013.33 \cdot \epsilon^+, Y_2 = 472.07 \cdot \epsilon^- . \quad (8)$$

**Research Results.** Theoretical and experimental studies, practical developments are mainly associated with the development, production and operation of relatively small wave gearboxes [25–32]. The accumulated experience cannot be used without considering the scale factor for large wave gears with torques of  $5 \cdot 10^5$  Nm and more.

The flexible link enhances the negative effect of the scale factor through disturbing the regular operation of the gearing of higher kinematic pairs. Under heavy-duty service, interference of the second kind can cause jamming and breakthrough of the teeth. Interference also activates force processes in the wave generator area increasing energy losses

The fragment of the oscillograph pattern corresponds to the turnover of  $2\pi$  wave generator (Fig. 3).

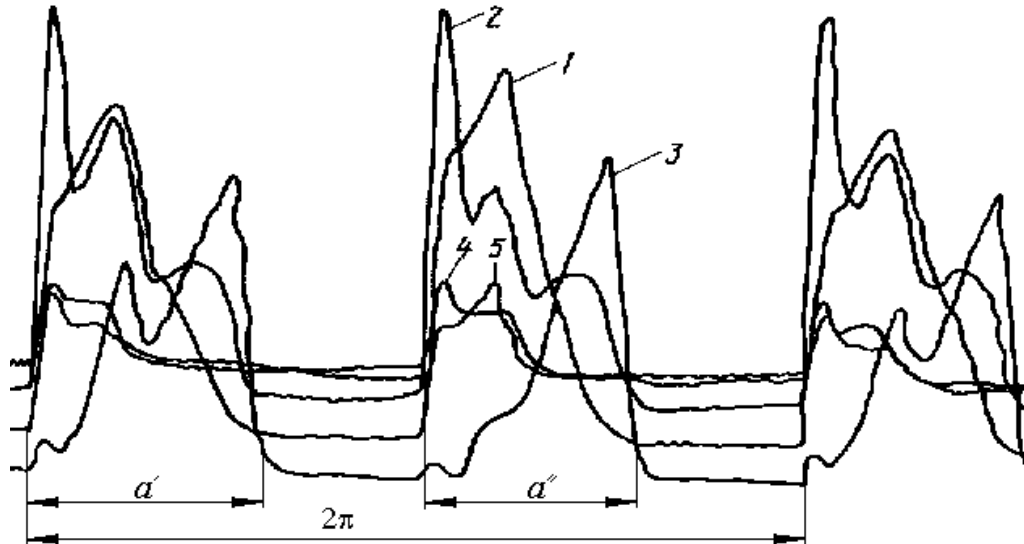


Fig. 3. Oscillogram of rigid wheel tooth deformation under load  $M_2 = 3 \cdot 10^5$  Nm and rotation frequency  $\omega_1 = 1.667 \text{ s}^{-1}$ ; 1, 2, 3, 4, 5 are deformation oscillograms of localized control tooth elements

The number of tooth pairs in  $Z_2$  gearing is determined by the duration of the engagement of the control tooth under a complete rotation of the wave generator ( $\alpha' + \alpha''$ ):

$$Z_{\Sigma} = \frac{\alpha' + \alpha''}{2\pi} Z_B \cdot 100 \% . \quad (9)$$

The number of simultaneously engaging tooth pairs and the load shape across the width of the ring gear depend on the design parameters of the wave transmission, which determine the conditions of toothing, and represent some torque functions.

The technique of power analysis of gearing for large wave transmission is developed. Two advantages differ it from the known solutions.

1) It enables to eliminate the distortion of the deformed state of the control tooth under load due to the stiffness constancy lengthwise. For this purpose, through parallel slots are made on a rigid wheel symmetrically to the axis of the control tooth.

2) It becomes possible to significantly improve the accuracy of the evaluation of gearing power characteristics across the width of the crown and in the circumferential direction due to the rigidity reduction of the control tooth. For this purpose, physical models of a control tooth and a scaled tooth of a rigid wheel are developed.

**Discussion and Conclusions.** The control tooth shape is taken invariant across the full width of the ring gear (this differs the presented approach from the techniques [20–22]). Hence, the distortion of the experimental results introduced by the variable width of the tooth shape is eliminated.

The solution provides the installation of strain gauges across the full width of the control tooth, which establishes a complete dependence of the tooth deformation across the full width of the ring gear, and not just in the outermost sections, as the well-known technique suggests.

The updating of physical and mathematical models made it possible to improve the accuracy of the theoretical and experimental results of force processes in the gearing of a large wave transmission [8–10]. The scientific-based two-parameter dependences of the force distribution in gearing are obtained.

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