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Formation of similarity criteria for physical objects and processes based on NonDimCritFormer 1.0 computer program *

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Формирование критериев подобия физических объектов и процессов на основе компьютерной программы NonDimCritFormer 1.0***

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Introduction. The problem of adequate modeling of physical processes and objects is an urgent task of modern science. Automation of work in this area increases the accuracy of modeling and saves money on full-scale modeling of objects under study. The research objective was to create a computer program that would automatically form dimensionless similarity criteria based on the initial set of parameters.
Materials and Methods. The calculation algorithm is based on π-theorem of similarity theory and the basic provisions of the dimensional theory.
Research Results. The computer program is developed. It enables to get all the theoretically possible similarity criteria for the simulation of the physical process or object of interest.
Discussion and Conclusions. The results obtained can be used in the industrial and scientific modeling of physical objects of research, calculation of new similarity criteria, solving problems of describing complex processes, etc.

Keywords: similarity criterion, theory of similarity, dimensional theory, π-theorem, dimensionless complex, Kirpichev-Gukhman theorem.

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Введение. Современная наука признает актуальной проблему адекватного моделирования физических процессов и объектов. Автоматизация работ в этой сфере повышает точность моделирования и экономит средства на натурном моделировании объектов изучения. Целью данного исследования было создание компьютерной программы, которая по исходному набору параметров автоматически формирует безразмерные критерии подобия.
Материалы и методы. В основе расчетного алгоритма лежит π-теорема теории подобия и основные положения теории размерностей.
Результаты исследования. Создана компьютерная программа, которая позволяет получить все теоретически возможные критерии подобия для моделирования изучаемого физического процесса или объекта.
Обсуждение и заключение. Полученные результаты могут быть использованы в промышленном и научном моделировании физических объектов исследования, расчете новых критериев подобия, решении задач описания сложных процессов и т. д.

Ключевые слова: критерий подобия, теория подобия, теория размерностей, π-теорема, безразмерный комплекс, теорема Кирпичева — Гухмана.

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Introduction. Similarity criteria are dependencies between dimensionless combinations of parameters describing an object or physical process. On the basis of similarity criteria, it is possible to create mathematical models of the objects of interest [1-4].

Similarity criteria enable to establish the correspondence of the model and the object under study. They become irreplaceable when the mathematical description of processes according to experiments or observations is not

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formed. In fact, similarity criteria are dimensionless power complexes of sets of physical parameters describing objects and processes. They are identical in form and numerically equal for similar objects or processes.

The algorithm is based on the second theorem of similarity theory, the so-called π-theorem. In the source [5], it is formulated as follows*: any complete equation of the physical process, written in a certain system of units, can be represented by a functional dependence between the similarity criteria obtained from the parameters involved in the process.* From this, it follows that analyzing and combining with each other the physical parameters describing the object of study, it is possible to make the similarity criteria of the object or physical process [5, 6].

When modeling objects of research on the basis of similarity theory, the analysis of the dimensions of the physical parameters describing the object at its primary points [6, 7] is most frequently used.

Researchers select the determining physical parameters that describe the object of interest, in particular, pressure, viscosity, density, speed, temperature, resistance, current, and, working with their dimensions, obtain a theoretically possible number of similarity criteria [8]. These criteria can then be used as the basis for mathematical or full-scale modeling.

In the case when the initial object under study and the model have similarity criteria that are numerically equal in the critically important zones, then, according to the third similarity theorem (Kirpichev -Gukhman theorem) [9], it implies their physical similarity [6]. Therefore, to create, for example, a model of flow along the contour of pipes, it is necessary to find the appearance and numerical value of the similarity criteria describing the real object, and then to build a model or laboratory installation so that the values of the similarity criteria in it are numerically equal to the values of the same criteria on the real object.

The NonDimCritFormer 1.0 program presented in this paper enables a person who studies a physical object or process to select significant physical parameters (see Fig. 1), which determine its basic parameters and calculate automatically all theoretically possible similarity criteria and dimensionless complexes (see Fig. 2) formed on their basis. The basis of this algorithm is the methods of analysis of dimensions of the selected physical parameters.

2. Description of the program algorithm

The basis of the algorithm implemented in the program of formation of similarity criteria based on the dimensions of the selected physical parameters is a technique that includes five steps.

Step 1. Determination of physical parameters that affect significantly the physical properties of the object under study. The accuracy of the results depends on the correct choice of parameters, neglecting nonessential ones. A large number of selected parameters causes an increase in the number of obtained dimensionless complexes, but does not increase the accuracy of the model formed on their basis.

Step 2. Compiling a matrix of dimensions for the selected physical parameters. In it, the dimensions of the selected parameters in the corresponding degrees are arranged in rows. The dimension of the matrix corresponds to the number of parameters and their units of measurement, that is, in the matrix there are as many rows as the selected parameters, and there are as many columns as the units of measurement. For example, a string corresponding to the physical parameter “electrical resistance, R”, which has a dimension of four units $\frac{m^2 kg}{s^3 A^2}$, в матрице размерностей будет иметь вид (2 1 -3 -2).

Step 3. Determination of the number of independent parameters. The number of independent parameters *k* corresponds to the rank of the dimension matrix, i.e. the largest order of the non-zero determinant which is based on the rows of the matrix under study.

Step 4. Formation of groups of independent parameters. At this step, combinations of *k* pieces of *m* parameters that make up a dimensionless complex are formed. That is, the units that are included in the dimension of the selected parameters are mutually reduced, which leads to the fact that this combination of the selected physical parameters has no dimension. Generally speaking, the complex consists of such parameters, the partial matrix of dimensions of which has at least one determinant of *k-th* order unequal to zero.

According to the combinatorial placement formula, the theoretically possible number of such complexes can be [1]

$$C_m^k = \frac{m!}{k! \cdot (m-k)!},$$

where *m* is the total number of parameters, *k* is the rank of the complete matrix of dimensions.

Not all obtained complexes of the selected parameters make up a complex with a matrix of *k-th* order, therefore, very often the number of groups after the test is less than C_m^k.

Step 5. Determination of expressions for similarity criteria based on calculated groups of independent parameters. Expressions are formed by the following algorithm.

1. Take another group of parameters. For example, set #1- $P_1 \ldots P_k$.

2. We begin the calculation of similarity criteria with dividing the parameters of the next group by the parameters of the current group. In particular, the following criteria can be drawn up for the set #1

$$\pi_1 = \frac{P_{k+1}}{P_1^{x_{1.1}} \cdot \ldots \cdot P_k^{x_{1.k}}}, \quad \ldots \quad \pi_{m-k} = \frac{P_m}{P_1^{x_{m-k.1}} \cdot \ldots \cdot P_k^{x_{m-k.k}}}.$$

Ordinary division of a parameter by a set does not produce a dimensionless expression. To do this, you need to calculate the values of powers $x_{i.1} \ldots x_{i.k}$ for each of the criterion expression.

3. Considering that the dimensions of the parameters in the numerators and denominators of the criterion complexes should be equal, we form and solve a system of linear equations. Obtaining such a system is due to the equalization of degrees at the same units of measurement in the numerators and denominators of the criteria, which are the products of combinations of dimensions selected at the beginning of the calculation of physical parameters. As a result, we obtain a system of q equations with $k = q$ unknowns.

We obtain sets of m-k criteria for each group of parameters. The maximum possible number of dimensionless complexes, which can be calculated on the basis of m parameters, is

$$\pi_s = C_m^k \cdot (m-k).$$

This value can be quite large. For example, for $m = 7$ and $k = 3$, the total number is $\pi_s = 140$. But, due to the fact that the list of calculated criteria has a lot of the similar and reciprocal ones, the total number of selected criteria is less than π_s. The proportion of criteria which failed verification depends on the initial set of physical parameters selected for study.

3. Program interface and functionality

The program NonDimCritFormer 1.0 is written in the language VisualBasic.Net. It has an intuitive Windows interface and is easy to operate.

The user chooses physical parameters, which are essential, in his opinion, for the simulated object or process (see Fig. 1), then, by clicking the mouse on the button “Generate all criteria” and “Remove repetitions of criteria”, all similarity criteria (dimensionless complexes), which can be made on the basis of the selected physical parameters (see Fig. 2) are displayed.

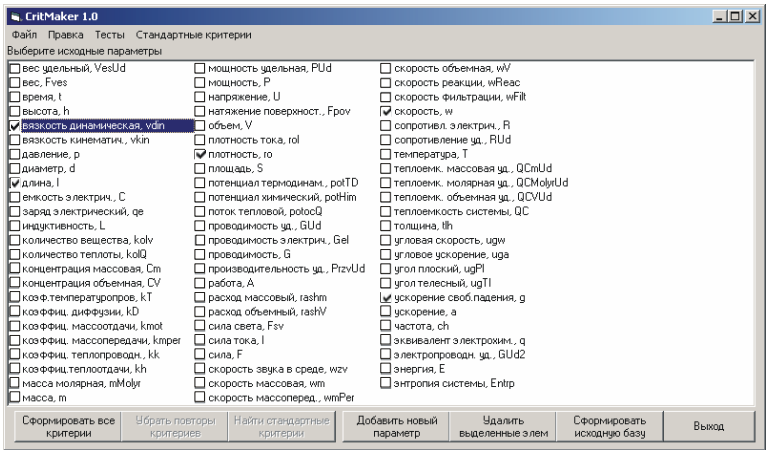

Fig. 1. Selection of initial physical parameters

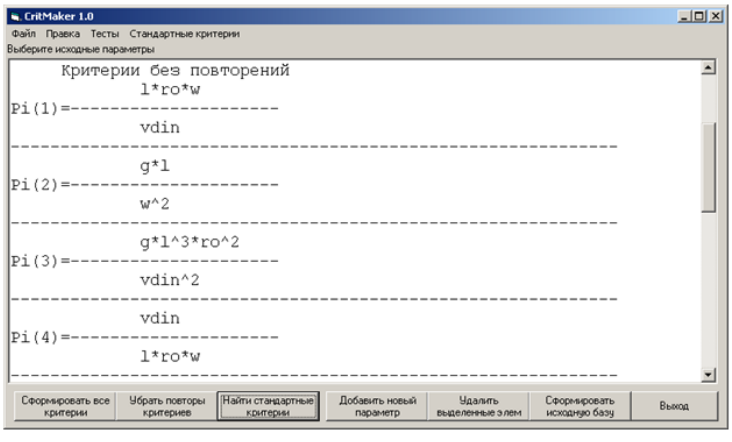

Fig. 2. Calculated similarity criteria

Using the "Find standard criteria" button, you can detect the existing (named) similarity criteria in the list. NonDimCritFormer 1.0 also enables to print the entire list of criteria or part of it, to save the list as a separate file, to edit the base of standard criteria.

4. Examples of using the program

Calculation example 1. Under constant voltage conditions, we find similarity criteria describing the current flow in a resistor, choke, and capacitor connected in series [8].

Step 1. Select the physical parameters essential for the simulated object. They include: current, I, A ; *inductance, L,* $\frac{\text{kg}\cdot m^2}{s^2\cdot A^2}$; capacitance, C, $\frac{\text{s}^4\cdot A^2}{kg\cdot m^2}$; resistance, R,; $\frac{\text{kg}\cdot m^2}{s^3\cdot A^2}$; voltage U, $\frac{\text{kg}\cdot m^2}{s^3\cdot A}$ and time t, s. Total parameters are $m = 6$ and base unit are $q = 4$.

Step 2. The matrix of dimensions of the specified parameters has the form:

$$\begin{array}{c|cccc} & m & kg & s & A \\ t & 0 & 0 & 1 & 0 \\ C & -2 & -1 & 4 & 2 \\ L & 2 & 1 & -2 & -2 \\ U & 2 & 1 & -3 & -1 \\ I & 0 & 0 & 0 & 1 \\ R & 2 & 1 & -3 & -2 \end{array}$$

Step 3. The rank of this matrix is $k = 3$. It follows that the number of independent parameters in the group will be three, and the number of criteria for each independent group will be $m - k = 3$.

Step 4. The total number of groups of independent parameters can be: $C_m^k = C_6^3 = 20$. Combinations of parameters are made automatically with the help of the described program. The algorithm checks the rank of the partial matrix for each composed combination.

For example, for an ILU combination, a partial matrix would look like this:

$$ILU = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 2 & 1 & -2 & -2 \\ 2 & 1 & -3 & -1 \end{vmatrix}$$

The rank of this matrix is 3, therefore, this combination forms a group of independent parameters. Not all of the possible 20 combinations are dimensionless similarity criteria. For example, *tCL* combination is not a criterion because the rank of the matrix is not equal to k. In the context of the initial set of physical parameters under consideration, only 15 can be considered as similarity criteria.

Step 5. Drawing up similarity criteria passes through the solution of systems of linear equations, which are formed on the basis of criterion complexes. For example, for a combination of *ILU*:

$$\pi_1 = \frac{t^{x_{1.1}}}{I^{x_{1.2}}\cdot L^{x_{1.3}}\cdot U^{x_{1.4}}},\quad \pi_2 = \frac{C^{x_{2.1}}}{I^{x_{2.2}}\cdot L^{x_{2.3}}\cdot U^{x_{2.4}}},\quad \pi_3 = \frac{R^{x_{3.1}}}{I^{x_{3.2}}\cdot L^{x_{3.3}}\cdot U^{x_{3.4}}},$$

after solving the system, which will result in finding the values of powers of dimensions x, the following criteria will be obtained:

$$\pi_1 = \frac{t}{I\cdot L\cdot U^{-1}},\quad \pi_2 = \frac{C}{I^2\cdot L\cdot U^{-2}},\quad \pi_3 = \frac{R}{I^{-1}\cdot L^0\cdot U}.$$

For each of 15 possible combinations of initial physical parameters, 3 criteria will be obtained. Among these 45 criteria, there are often repetitive and reciprocal ones. For this example, NonDimCritFormer 1.0 gives 11 unique criteria:

$$\pi_1 = \frac{t}{C^{0,5}\cdot L^{0,5}};\quad \pi_2 = \frac{I\cdot L^{0,5}}{C^{0,5}\cdot U};\quad \pi_3 = \frac{C^{0,5}\cdot R}{L^{0,5}};$$

$$\pi_4 = \frac{C\cdot L}{t^2};\quad \pi_5 = \frac{I\cdot t}{C\cdot U};\quad \pi_6 = \frac{C\cdot R}{t};$$

$$\pi_7 = \frac{C \cdot U^2}{I^2 \cdot L}; \quad \pi_8 = \frac{R \cdot I}{U}; \quad \pi_9 = \frac{L}{C \cdot R^2};$$

$$\pi_{10} = \frac{U \cdot t}{I \cdot L}; \quad \pi_{11} = \frac{R \cdot t}{L}.$$

Calculation example 2. Consider the process of fluid motion in a horizontal pipe. To do this, we form similarity criteria that describe the pressure drop in this process [10]. The most important parameters that describe the pressure drop in the pipe during the passage of the liquid medium, the following values can be called:

$$\Delta p = f(\omega, \rho, \mu, L),$$

where Δp is the pressure, $\frac{\text{kg}}{\text{m} \cdot \text{s}^2}$; ω is the flow rate of the fluid, $\frac{\text{m}}{\text{s}}$; ρ is fluid density, $\frac{kg}{m^3}$; μ is the coefficient of dynamic viscosity, $\frac{kg}{m \cdot s}$.

Total parameters are $m = 5$, and the base units are $q = 3$. The complete matrix of dimensions of the selected parameters has the form:

	m	kg	s
μ	-1	1	-1
Δp	–1	1	–2
L	1	0	0
ρ	–3	1	0
ω	1	0	–1

The rank of the reduced matrix is $k = 3$. The number of independent parameters in group is three and the number of criteria for each independent group is $m - k = 2$. Total groups can be $C_m^k = C_5^2 = 10$.

According to the calculations of the program, in this example, out of ten possible combinations, only nine are independent groups. It follows that a total of 18 dimensionless complexes can be obtained. After removing the repetitive and reciprocal ones, the program gives 6 unique combinations.

$$\pi_1 = \frac{L \cdot \Delta p}{\mu \cdot \omega}; \quad \pi_2 = \frac{\Delta p}{\rho \cdot \omega^2}; \quad \pi_3 = \frac{L \cdot \Delta p^{0,5} \cdot \rho^{0,5}}{\mu};$$

$$\pi_4 = \frac{\rho^{0,5} \cdot \omega}{\Delta p^{0,5}}; \quad \pi_5 = \frac{L \cdot \rho \cdot \omega}{\mu}; \quad \pi_6 = \frac{L^2 \cdot \Delta p \cdot \rho}{\mu^2}.$$

It is important to note that the complexes π_2 and π_5 are expressions of the Euler and Reynolds criteria, respectively [3].

4. Conclusion

The computer program NonDimCritFormer 1.0 is proposed. With its help, on the basis of selected physical parameters that determine the characteristics of the object under study or the physical process in its characteristic spatial and temporal points, it is possible to form dimensionless complexes – similarity criteria describing the object of study.

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