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The problem of infinite plate loaded with normal force following a complex trajectory[*]

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Задача о бесконечной пластине, нагруженной нормальной силой, движущейся по сложной траектории[***]

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Introduction. A method for solving the problem of an infinite plate on an elastic foundation is proposed. The plate is affected by a periodic load in the form of a force following an arbitrary closed path. The work objective is to develop a numerical method for solving problems of the elasticity theory for bodies under a moving load.
Materials and Methods. Given the periodicity of the load under consideration, it is decomposed in a Fourier series in a time interval whose length is equal to the load period. The solution to the original problem is constructed by superposition of the solutions to the problems corresponding to the load specified by the terms of the Fourier series described above. The final solution to the problem is presented as a segment of a series. In this case, each term corresponds to the solution of the problem of the impact on an infinite plate of a load distributed along a closed curve (the trajectory of the force motion). To find these solutions, the fundamental solution to the equation of vibration of an infinite plate lying on an elastic base is used.
Research Results. A new method is proposed for solving problems on the elasticity theory for bodies with a load following a closed path of arbitrary shape. The problem of an infinite plane along which a concentrated force moves at a constant speed is solved. It is determined that the trajectory of motion is a smooth closed curve consisting of circular arcs. The behavior of displacements and stresses near a moving force is considered. The energy propagation of the elastic waves is studied. For this purpose, the coordinates of the Umov – Poynting vector are calculated. The effect of the force motion speed on the length of the Umov – Poynting vector is investigated.
Discussion and Conclusions. The method is applicable when considering more complex objects (plates of complex shape, layered plates, viscoelastic plates). Its advantage is profitability since the known problem solutions are used to build the solution.

Введение. Предлагается метод решения задачи о бесконечной пластине, лежащей на упругом основании. На пластину действует периодическая нагрузка в виде силы, перемещающейся по произвольной замкнутой траектории. Цель исследования — разработка численного метода решения задач теории упругости для тел, находящихся под действием подвижной нагрузки.
Материалы и методы. Учитывая периодичность рассматриваемой нагрузки, она раскладывается в ряд Фурье на временном отрезке, длина которого равна периоду нагрузки. Решение исходной задачи строится посредством суперпозиции решений задач, соответствующих нагрузке, задаваемой слагаемыми описанного выше ряда Фурье. Окончательное решение задачи представляется в виде отрезка ряда. Каждое слагаемое при этом соответствует решению задачи о воздействии на бесконечную пластину нагрузки, распределенной по замкнутой кривой (траектории движения силы). Для нахождения этих решений используется фундаментальное решение уравнения колебания бесконечной пластины, лежащей на упругом основании.
Результаты исследования. Предложен новый метод решения задач теории упругости для тел с нагрузкой, движущейся по замкнутой траектории произвольной формы. Решена задача о бесконечной плоскости, по которой с постоянной скоростью движется сосредоточенная сила. Определено, что траектория движения представляет собой гладкую замкнутую кривую, состоящую из дуг окружностей. Рассмотрен характер изменения перемещений и напряжений вблизи движущейся силы. Изучено распространение энергии упругих волн. С этой целью выполнено вычисление координат вектора Умова — Пойтинга. Исследовано влияние скорости движения силы на длину вектора Умова — Пойтинга.
Обсуждение и заключения. Метод применим и при рассмотрении более сложных объектов (плиты сложной формы, слоистые плиты, вязкоупругие плиты). Его преимущество — экономичность, так как для построения решения используются уже известные решения задач. Окончательное решение выра-

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The final decision is expressed in a convenient form – as the sum of curvilinear integrals. The results obtained can be used in the road design process. Studying the energy propagation of elastic waves from moving vehicles will enable to evaluate the impact of these waves on buildings near the road. The wear of the pavement is estimated considering data on the behavior of displacements and stresses.

жается в удобном виде — как сумма криволинейных интегралов. Полученные результаты могут быть использованы в процессе проектирования дорог. Изучение распространения энергии упругих волн от движущихся транспортных средств позволит оценить воздействие указанных волн на строения, расположенные вблизи дороги. С учетом данных о характере изменения перемещений и напряжений оценивается износ дорожного покрытия.

Keywords: infinite plate, moving force, arbitrary closed path, energy of elastic waves

Ключевые слова: бесконечная пластина, движущаяся нагрузка, произвольная замкнутая траектория, энергия упругих волн.

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Introduction. The study on dynamic phenomena caused by the action of a moving load is an urgent task that has application significance (for example, when solving transport development issues). In the papers devoted to this problem, various tasks with a moving load were considered. In particular, it is shown how a load moving in an infinite straight line at a constant speed acts on a half-plane or half-space (elastic isotropic, transversal isotropic, viscoelastic). In this case, when solving the problem, a moving coordinate system associated with a moving force is introduced; this enables to exclude time from the number of independent variables [1–5]. Some papers consider the action on an infinite plate or strip (elastic or viscoelastic) moving uniformly along a rectilinear load path. In this case, the same method of eliminating a temporary variable is used, or a quasistatic formulation of the problem is considered [6–12]. The tasks in which the length of the load path is finite and the path itself is a curved line are of main interest. In this case, finite element modeling of a moving load is frequently used [11–13]. In a number of papers, when solving such problems, variational methods are used (in particular, the Rayleigh – Ritz method) [14–16] or a variation of the Galerkin method, which makes it possible to reduce the problem to ordinary differential equations. In this case, various objects of the application of a moving load are considered (plates, layered plates, viscoelastic plates, half-spaces – both isotropic and anisotropic) [17–19]. This paper presents a method that develops the ideas described in [20–23].

Materials and Methods. Consider an infinite plate lying on an elastic Winkler base, which is under the action of a normally applied force moving along a closed path.

The problem is reduced to the integration of the equation of motion of a plate lying on an elastic Winkler base [14]:

$$\Delta^2 W + c^{-2}\partial_t^2 W + kW = \frac{P}{D}, \tag{1}$$

where W is the plate deflection; $D = \dfrac{Eh^3}{12(1-\nu^2)}$; E is Young's modulus; ν is Poisson's ratio; h is the plate thickness; $c^{-2} = \dfrac{\rho h}{D}$; ρ is the density of the material; $k = \dfrac{K}{D}$; K is the compliance coefficient of the elastic base; P is the concentrated force moving along a closed curve γ with the constant speed a.

Let us introduce the coordinate s counted from some fixed point of the curve γ. Then the force P moving along the curve γ with the velocity a will be described by the relation $P = P(s-at)$. The function $P(s-at)$ is periodic in t, with period $T = \dfrac{L}{a}$, where L is the length of the curveγ.

Solution. Consider the steady state. We expand the function $P(s-at)$ in the Fourier series in the variable t on the segment $\left[-\dfrac{L}{2a};\dfrac{L}{2a}\right]$. In this case, the expansion coefficients appear as:

$$c_k = \int_{-L/2a}^{L/2a} P(s-at)e^{2iak\pi t/L}dt.$$

After changing the integration variable in the integral $s-at=z$, we obtain:

$$c_k = \int\limits_{-L/2}^{L/2} P(z) e^{-2ik\pi z/L} dz \frac{e^{2ik\pi s/L}}{a} = d_{-k} \frac{e^{2ik\pi s/L}}{a},$$

where d_k are Fourier series expansion coefficients on a function segment $\left[-\frac{L}{2};\frac{L}{2}\right]$ of the function $P(z)$.

Then the moving load can be presented as a Fourier series:

$$P(s-at) = \frac{1}{a}\sum_{k=-\infty}^{\infty} d_{-k} e^{2ik\pi(s-at)/L}.$$

Given the linearity of the problem, its solution can be presented as:

$$W = \sum_{k=-\infty}^{\infty} d_{-k}\hat{W}_k, \quad (2)$$

where $\hat{W}_k$ are plate deflections caused by the action of a vertical load whose distribution along the curve γ is described by the function $e^{2ik\pi s/L}$ that varies in time according to the law $e^{-2ika\pi t/L}$.

To determine $\hat{W}_k$, we use the fundamental solution to the equation (1), which corresponds to $P = \delta(x-x_0)\delta(y-y_0)e^{-i\omega_k t}$, где $\omega_k = \frac{2ika\pi}{L}$.

Using the limiting absorption principle and traditional methods for constructing solutions to differential equations, we can obtain a fundamental solution to the equation (1), which at $k > \frac{\omega_\kappa^2}{c^2}$, has the form:

$$w_k(x, y, x_0, y_0) = \frac{i}{4\pi\varepsilon^2 D}\left[K_0(\alpha_1 R) - K_0(\alpha_2 R)\right],$$

where $R = \left[(x-x_0)^2 + (y-y_0)^2\right]^{1/2}$, $\varepsilon = \sqrt[4]{k - \omega_k^2/c^2}$, $\alpha_1 = \varepsilon e^{i\pi/4}$, $\alpha_1 = \varepsilon e^{-i\pi/4}$, $K_0(z)$ is the Macdonald function.

At $k < \frac{\omega_k^2}{c^2}$, the solution is given by:

$$w_k(x, y, x_0, y_0) = \frac{i}{4\pi\gamma^2 D}\left[\frac{\pi i}{2} H_0^{(1)}(\gamma R) - K_0(\gamma R)\right],$$

where $\gamma = \sqrt[4]{\frac{\omega_k^2}{c^2} - k}$, $H_0^{(1)}(\gamma R)$ is the Hankel function.

Then $\hat{W}_k = \oint\limits_\gamma w_k(x, y, x_0(s), y_0(s)) e^{2ik\pi(s-at)/L} ds$. Using the well-known formulas of the thin plate theory and the formula obtained from the above relations determining the deflection W (2), we can calculate the displacements u_x, u_y and the stresses σ_x, σ_y and σ_{xy} at any point on the plate.

For large k, it is necessary to calculate the integral of fast oscillating functions. For this, a quadrature formula based on replacing the weakly oscillating part of the integrand by a cubic spline was used, and the highly oscillating factor $e^{2ik\pi s/L}$ was considered as a weight function [15].

Research Results. So, an infinite plate lies on an elastic Winkler base. A normal force acts on it moving along the trajectory shown in Fig. 1.

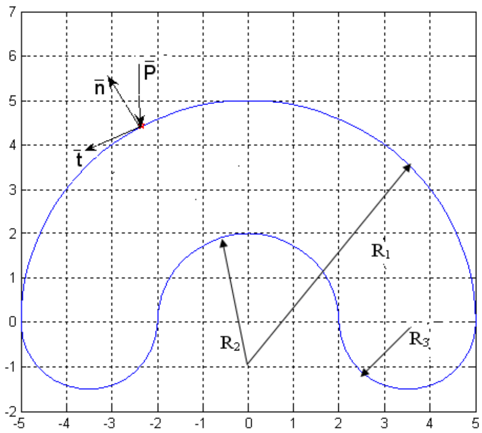

Fig. 1. Load motion trajectory

The computation was performed at the following initial data: h = 0.25 m; s = 221 m/s; E = 232469·10 N/m2; ν = 0.36. Fig. 2 shows the calculation results corresponding to K = 1.864 m^{-4} and a = 125 m/s.

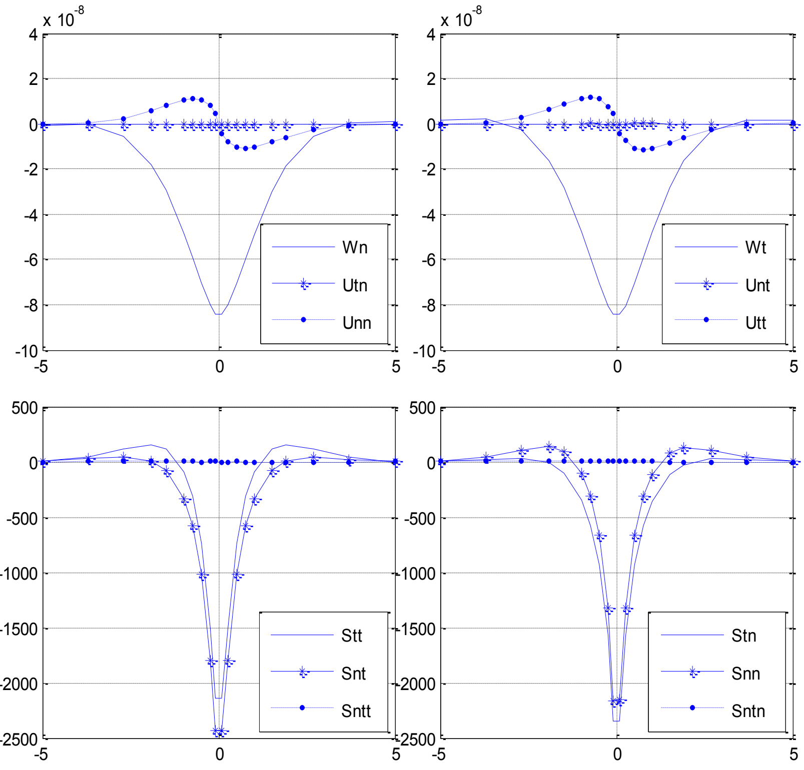

Fig. 2. Variation of displacements and stresses

These graphs describe changes in displacements and stresses in the coordinate system associated with the point of application of the moving force P. The t axis is directed tangentially to the motion path $\bar{t}$, and the n axis is directed along the external normal to the region bounded by the path $\bar{n}$ (see Fig. 1) at $z = h/2$.

In this case, the displacement vector and stress tensor, respectively, were represented as:

$$\bar{U} = Ut \cdot \bar{t} + Un \cdot \bar{n} + W \cdot \bar{k}\ ,\ \bar{\bar{S}} = St \cdot \overline{tt} + Sn \cdot \overline{nn} + Stn \cdot \left(\overline{tn} + \overline{nt} \right),$$

where $\bar{k}$ is normal to the plate.

Fig. 2 shows the change along the t axis of the displacement vector components Wt, Utt, Unt , the stress tensor $Stt, Snt, Stnt$, and the variation of these values along the n axis— Wn, Utn, Unn and $Stn, Snn, Stnn$.

According to the calculations, the behavior of these values at all points of the trajectory remains unchanged. The computation also showed that with a change in the velocity of the load a in the range from 0 to 125 m/s, the components of the displacement vector and the stress tensor increased moderately (by 3-4%).

To study the elastic energy propagation, the components of the Umov-Poynting vector $e_i = -S_{ij} \cdot \partial_t u_i$ (S_{ij} are the components of the stress tensor, u_i are the coordinates of the displacement vector) whose direction indicates the direction of energy propagation, and the length describes the amount of energy transferred through a surface unit perpendicular to this vector line per unit time.

Fig. 3 shows the elastic energy propagation near a moving concentrated force (the position of the force is marked with a red asterisk.

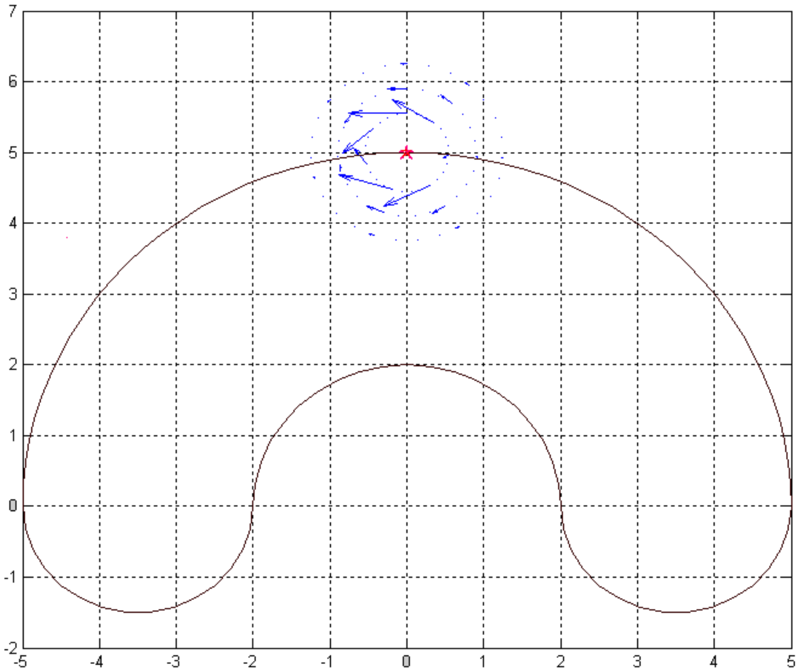

Fig. 3 Elastic energy propagation

Discussion and Conclusions. The analysis of the results obtained shows the following: at the speed variation limits indicated above, the length of the Umov-Poynting vector is almost proportional to the load speed and a sound energy pattern near the force changes slightly during the movement. The behavior of the displacements and stresses calculated above at all points of the trajectory remains unchanged, and their values weakly depend on the load speed a when this velocity varies from 0 to 125 m/s.

Application of the proposed method to an infinite plate lying on an elastic base does not exhaust its possibilities. It can be used when considering more complex objects (plates of complex shape, layered plates, viscoelastic plates). The considered method differs from the mentioned above in greater efficiency since it uses the known problem solving to construct the solution. The final decision is expressed in a convenient form – as a sum of curvilinear integrals.

The results can be used in the road design process. Studying the elastic energy propagation from moving vehicles will provide evaluating the impact of these waves on buildings near the road. The wear of the pavement is estimated considering data on the behavior of displacements and stresses

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