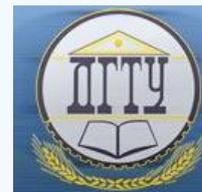


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### On joint efficiency of composite anisotropic plate rigidly fixed along outside edges\*

A. G. Akopyan<sup>1\*\*</sup>

<sup>1</sup>Moscow Automobile and Road Construction State Technical University (MADI), North Caucasus branch, Lermontov, Russian Federation

### О прочности соединения составной анизотропной пластины, жестко заземленной по внешним краям\*\*\*

A. G. Акопян<sup>1\*\*</sup>

<sup>1</sup>Московский автомобильно-дорожный государственный технический университет (МАДИ) Северо-Кавказский филиал, Лермонтов, Российская Федерация

*Introduction.* Modern processes of welding, surfacing, soldering and bonding provide producing structural elements of monolithic interconnected dissimilar anisotropic materials. The combination of different materials with qualities corresponding to certain operating conditions offer comprehensive facilities to improve the technical and economic characteristics of machines, equipment and structures. It can contribute to a significant increase in their reliability, durability, and to reduction of the production and operation costs.

*Materials and Methods.* The work objective is to study the boundary state of stress of anisotropic composite plates in the framework of the classical theory of plate bending. The outer edges of the plate are considered free. Using the classical theory of bending of an anisotropic plate in the space of physical and geometric parameters, hypersurface equations are obtained that define low-stressed zones for the contact surface edge of a cylindrical orthotropic composite plate.

*Research Results.* Finding the criteria for engineering structures to determine the limiting strength characteristics of structural elements is one of the urgent tasks of the deformable solid mechanics. Strength problems in structures are often reduced to elucidating the nature of the local stress state at the tops of the joints of the constituent parts. This paper is devoted to solving this problem for composite anisotropic plates in the area of their bending.

*Discussion and Conclusions.* The solution proposed in this paper may be useful for increasing the strength of composite products.

**Keywords:** low-stressed level, plate bending, anisotropic, composite, rigidly fixed, angle rib, classical theory of bending, linearly elastic.

*Введение.* Современные технологические процессы сварки, наплавки, пайки и склеивания позволяют изготавливать элементы конструкций из монолитно соединенных между собой разнородных анизотропных материалов. Комбинирование различных материалов, обладающих качествами, соответствующими тем или иным условиям эксплуатации, открывает большие возможности для повышения технических и экономических характеристик машин, оборудования и сооружений. Оно может способствовать значительному увеличению их надежности, долговечности, уменьшению расходов на изготовление и эксплуатацию.

*Материалы и методы.* Целью работы является изучение предельного напряженного состояния анизотропных составных пластин в рамках классической теории изгиба пластин. Внешние края пластины считаются свободными. Используя классическую теорию изгиба анизотропной пластины в пространстве физических и геометрических параметров, получены уравнения гиперповерхности, определяющие зоны малонапряженности для края контактной поверхности составной цилиндрически ортотропной пластины.

*Результаты исследования.* Нахождение критериев инженерных сооружений, позволяющих определить предельные прочностные характеристики элементов конструкций, является одной из актуальных задач механики деформируемого твердого тела. Проблемы прочности в конструкциях часто сводятся к выяснению характера местного напряженного состояния у вершин стыков составляющих частей. Данная статья посвящена решению этой проблемы для составных анизотропных пластин в области их изгиба.

*Обсуждение и заключения.* Решение, предлагаемое в данной работе, может быть полезным для повышения прочности композитных изделий.

**Ключевые слова:** малонапряженность, изгиб пластин, анизотропный, составной, жестко заземленный, угловое ребро, классическая теория изгиба, линейно упругий.

\* The research is done within the frame of the independent R&D.

\*\* E-mail: manakofoto@yandex.ru

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**Introduction.** A low-stressed state near the angle rib of the edge of the contact joint surface of two different, cylindrical orthotropic plates of the same thickness [1–5], rigidly fixed along the outside edges, in the framework of the classical bending theory of linear-elastic anisotropic plates [6, 7], rigidly fixed at the outside edges, is considered.

The behaviour of stresses at the angular vertex under bending a homogeneous isotropic plate that has an angle rib was studied in [8] using the classical theory of plate bending. The subsequent consideration of this problem through the refined Reissner theory has shown that the shearing forces in this edge are finite [9]. The existence and location of zones of low tension and stress concentration at the compound plate corners were experimentally shown in [10]. The case of bending of an inhomogeneous compound plate was considered in [11].

The surface connecting two plates is vertical to the median plane. Such a compound plate is subject to bending under total shear loading. The neighbourhood of the angle rib of the joint contact surface is free from external forces. We place the cylindrical coordinate system origin at the corner point of the median plane of the plate. Fig. 1 shows the plane  $z=0$ . Assume the main anisotropic axes coincide with the axes of this cylindrical coordinate system. The thickness of the plate is denoted by  $h$ , and the values in the vicinity of the point  $r=0$  referring to the regions  $0 \leq \theta \leq \alpha$ ,  $-h/2 \leq z \leq h/2$  and  $-\beta \leq \theta \leq 0$ ,  $-h/2 \leq z \leq h/2$ , note by the indices  $i=1, 2$ , respectively.

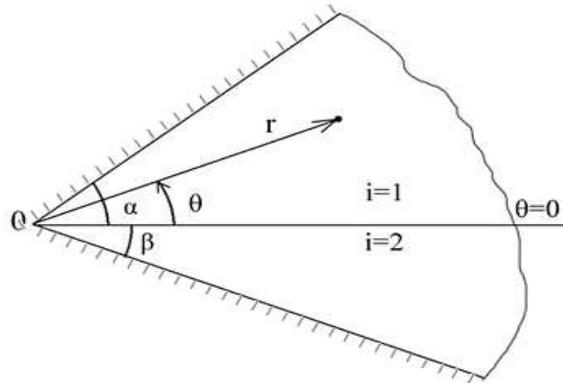


Fig. 1. Compound plate schematic

**Materials and Methods.** The deflection  $w_i$  of each region of the orthotropic plate about the point  $r=0$  is determined from the equation [3]:

$$D_{ri} \frac{\partial^4 w_i}{\partial r^4} + 2D_{r\theta i} \frac{1}{r^2} \frac{\partial^4 w_i}{\partial r^2 \partial \theta^2} + D_{\theta i} \frac{1}{r^4} \frac{\partial^4 w_i}{\partial \theta^4} + 2D_{ri} \frac{1}{r} \frac{\partial^3 w_i}{\partial r^3} - 2D_{r\theta i} \frac{1}{r^3} \frac{\partial^3 w_i}{\partial r \partial \theta^2} - D_{\theta i} \frac{1}{r^2} \frac{\partial^2 w_i}{\partial r^2} + 2(D_{\theta i} + D_{r\theta i}) \frac{1}{r^4} \frac{\partial^2 w_i}{\partial \theta^2} + D_{\theta i} \frac{1}{r^3} \frac{\partial w_i}{\partial r} = 0, \quad (1)$$

where  $D_{ri}, D_{\theta i}, D_{r\theta i}$  are stiffness of each region of the anisotropic plate:

$$D_{ri} = \frac{E_{ri}}{12(1-\nu_{ri}\nu_{\theta i})} h^3, \quad D_{\theta i} = \frac{E_{\theta i}}{12(1-\nu_{ri}\nu_{\theta i})} h^3, \quad D_{r\theta i} = D_{ri}\nu_{\theta i} + 2D_{ki}; \quad D_{ki} = \frac{G_i}{12} h^3$$

Here,  $E_{ri}, E_{\theta i}, \nu_{ri}, \nu_{\theta i}, G_i$  are anisotropic parameters of each region.

Presenting the plate deflection in the form

$$w_i(r, \theta) = r^{\lambda+1} f_i(\theta, \lambda), \quad (2)$$

where  $f_i$  and  $\lambda$  are the required functions and the constant,

$$f_i'''' + 2(k_{1i}\lambda^2 + 1) f_i'' + (\lambda^2 - 1)(k_{2i}\lambda^2 - 1) f_i = 0, \quad (3)$$

will follow from the equation (1), where  $k_{1i} = D_{r\theta i} / D_{\theta i}, k_{2i} = D_{ri} / D_{\theta i}$ .

The roots of the corresponding characteristic equation for (3) are determined from the following expression

$$r_{(1,2,3,4)i} = \pm \sqrt{-(k_{1i}\lambda^2 + 1) \pm \lambda \sqrt{(k_{1i}^2 - k_{2i})\lambda^2 + 2k_{1i} + k_{2i} + 1}} = \pm \sqrt{-a \pm b} \quad (4)$$

The following three cases are wanted to be considered:

1) All four roots (4) are imaginary ( $a \geq b, b$  is real value)

$$r_{(1,2,3,4)i} = \pm \omega_{ki} i,$$

Where the case  $k=1$  corresponds to the lower character under the radical (4), and  $k=2$  - to the upper one.

2) All roots (4) are complex ( $b$  is imaginary value).

$$r_{(1,2,3,4)i} = \pm(\xi_i \pm i\eta_i).$$

3) One pair of roots is real and the other is imaginary ( $a < b$ ,  $b$  are real).

$$r_{(1,2)i} = \pm\xi_i, r_{(3,4)i} = \pm\eta_i i.$$

For each of the cases, we write the general solution of the equation (3):

$$\begin{aligned} 1) f_i &= A_i \cos \omega_{1i} \theta + B_i \sin \omega_{1i} \theta + C_i \cos \omega_{2i} \theta + E_i \sin \omega_{2i} \theta \\ 2) f_i &= A_i \cosh \xi_i \theta \cos \eta_i \theta + B_i \sinh \xi_i \theta \cos \eta_i \theta + C_i \cosh \xi_i \theta \sin \eta_i \theta + E_i \sinh \xi_i \theta \sin \eta_i \theta \\ 3) f_i &= A_i \cosh \xi_i \theta + B_i \sinh \xi_i \theta + C_i \cos \eta_i \theta + E_i \sin \eta_i \theta, \end{aligned} \quad (5)$$

where  $A_i, B_i, C_i, E_i$  are arbitrary constants.

Then we have for the moments:

$$\begin{aligned} M_{ri} &= -D_{ri} r^{\lambda-1} [v_{\theta i} f_i'' + (\lambda + 1)(\lambda + v_{\theta i}) f_i] \\ M_{\theta i} &= -D_{\theta i} r^{\lambda-1} [f_i'' + (\lambda + 1)(v_{ri} \lambda + 1) f_i] \\ M_{r\theta i} &= -2D_{ki} r^{\lambda-1} f_i' \end{aligned} \quad (6)$$

The shearing forces will be calculated by the formulas:

$$\begin{aligned} Q_{ri} &= -r^{\lambda-2} [(D_{r\theta i} \lambda - D_{\theta i}) f_i'' + (\lambda + 1)(D_{ri} \lambda^2 - D_{\theta i}) f_i] \\ Q_{\theta i} &= -r^{\lambda-2} [D_{\theta i} f_i''' + (\lambda + 1)(D_{r\theta i} \lambda + D_{\theta i}) f_i'] \end{aligned} \quad (7)$$

For a generalizing cutting force, we will have

$$V_{\theta i} = Q_{\theta i} + \frac{\partial M_{r\theta i}}{\partial r} = -r^{\lambda-2} (D_{\theta i} f_i''' + g_i f_i'), \quad (8)$$

where

$$g_i = (\lambda + 1)D_{\theta i} + \lambda[(\lambda + 1)D_{r\theta i} + 2(\lambda - 1)D_{ki}].$$

On the contact surface ( $\theta = 0$ ), the conditions of deflection continuity, rotation angle, bending moment and generalized shear force should be observed.

$$\begin{aligned} f_1 &= f_2, f_1' = f_2', D_{\theta 1} f_1''' + g_1 f_1' = D_{\theta 2} f_2''' + g_2 f_2', \\ D_{\theta 1} [f_1'' + (\lambda + 1)(v_{r1} \lambda + 1) f_1] &= D_{\theta 2} [f_2'' + (\lambda + 1)(v_{r2} \lambda + 1) f_2]. \end{aligned} \quad (9)$$

Consider the boundary conditions at the outside edges ( $\theta = \alpha, \theta = -\beta$ ) of the plate. In case of rigid restraint

$$f_i' = f_i = 0, \quad (10)$$

Substituting the value  $f_i$  from (5) into the boundary conditions (9) and (10), we obtain three systems of eight linear equations relatively eight constants  $A_i, B_i, C_i, E_i$  for each of the three cases in (5)

For case 1:

$$\begin{aligned} A_1 + C_1 - A_2 - C_2 &= 0 \\ \omega_{11} B_1 + \omega_{21} E_1 - \omega_{12} B_2 - \omega_{22} E_2 &= 0 \\ D_{\theta 1} q_{11} A_1 + D_{\theta 1} q_{21} C_1 - D_{\theta 2} q_{12} A_2 - D_{\theta 2} q_{22} C_2 &= 0 \\ \omega_{11} p_{11} B_1 + \omega_{21} p_{21} E_1 - \omega_{12} p_{12} B_2 - \omega_{22} p_{22} E_2 &= 0 \\ A_1 \cos \omega_{11} \alpha + B_1 \sin \omega_{11} \alpha + C_1 \cos \omega_{21} \alpha + E_1 \sin \omega_{21} \alpha &= 0 \\ A_1 \omega_{11} \sin \omega_{11} \alpha - B_1 \omega_{11} \cos \omega_{11} \alpha + C_1 \omega_{21} \sin \omega_{21} \alpha - E_1 \omega_{21} \cos \omega_{21} \alpha &= 0 \\ A_2 \cos \omega_{12} \beta - B_2 \sin \omega_{12} \beta + C_2 \cos \omega_{22} \beta - E_2 \sin \omega_{22} \beta &= 0 \\ A_2 \omega_{12} \sin \omega_{12} \beta + B_2 \omega_{12} \cos \omega_{12} \beta + C_2 \omega_{22} \sin \omega_{22} \beta + E_2 \omega_{22} \cos \omega_{22} \beta &= 0 \end{aligned} \quad (11)$$

The following notation is used here:

$$\begin{aligned} p_{ji} &= (\lambda + 1 - \omega_{ji}^2) D_{\theta i} + \lambda[(\lambda + 1)D_{r\theta i} + 2(\lambda - 1)D_{ki}] \\ q_{ji} &= (\lambda + 1)(v_{ri} \lambda + 1) - \omega_{ji}^2, j=1, 2 \end{aligned}$$

For case 2:

$$\begin{aligned} A_1 - A_2 = 0, \xi_1 B_1 + \eta_1 C_1 - \xi_2 B_2 - \eta_2 C_2 &= 0 \\ \omega_1 D_{\theta 1} A_1 + 2D_{\theta 1} \xi_1 \eta_1 E_1 - \omega_2 D_{\theta 2} A_2 - 2D_{\theta 2} \xi_2 \eta_2 E_2 &= 0 \\ p_1 B_1 + q_1 C_1 - p_2 B_2 - q_2 C_2 &= 0 \\ A_1 \cosh \xi_1 \alpha \cos \eta_1 \alpha + B_1 \sinh \xi_1 \alpha \cos \eta_1 \alpha + C_1 \cosh \xi_1 \alpha \sin \eta_1 \alpha + E_1 \sinh \xi_1 \alpha \sin \eta_1 \alpha &= 0 \\ A_1 (\xi_1 \sinh \xi_1 \alpha \cos \eta_1 \alpha - \eta_1 \cosh \xi_1 \alpha \sin \eta_1 \alpha) &+ B_1 (\cosh \xi_1 \alpha \cos \eta_1 \alpha - \eta_1 \sinh \xi_1 \alpha \sin \eta_1 \alpha) \\ + C_1 (\xi_1 \sinh \xi_1 \alpha \sin \eta_1 \alpha + \eta_1 \cosh \xi_1 \alpha \cos \eta_1 \alpha) &+ E_1 (\xi_1 \cosh \xi_1 \alpha \sin \eta_1 \alpha + \eta_1 \sinh \xi_1 \alpha \cos \eta_1 \alpha) = 0 \\ A_2 \cosh \xi_2 \beta \cos \eta_2 \beta - B_2 \sinh \xi_2 \beta \cos \eta_2 \beta - C_2 \cosh \xi_2 \beta \sin \eta_2 \beta + E_2 \sinh \xi_2 \beta \sin \eta_2 \beta &= 0 \\ A_2 (\xi_2 \sinh \xi_2 \beta \cos \eta_2 \beta - \eta_2 \cosh \xi_2 \beta \sin \eta_2 \beta) &= 0 \end{aligned} \quad (12)$$

$$\begin{aligned}
 & -B_2(\xi_2 \cosh \xi_2 \beta \cos \eta_2 \beta - \eta_2 \sinh \xi_2 \beta \sin \eta_2 \beta) \\
 & - C_2 (\xi_2 \sinh \xi_2 \beta \sin \eta_2 \beta + \eta_2 \cosh \xi_2 \beta \cos \eta_2 \beta) + E_2 (\xi_2 \cosh \xi_2 \beta \sin \eta_2 \beta + \eta_2 \sinh \xi_2 \beta \cos \eta_2 \beta) \\
 & = 0,
 \end{aligned}$$

where the following is indicated:

$$\begin{aligned}
 \omega_i &= \xi_i^2 - \eta_i^2 + (\lambda + 1)(\nu_{ri} \lambda + 1) \\
 p_i &= \xi_i \{ (\xi_i^2 - 3\eta_i^2 + \lambda + 1) D_{\theta i} + \lambda [(\lambda + 1) D_{r\theta i} + 2(\lambda - 1) D_{ki}] \} \\
 q_i &= \eta_i \{ (3\xi_i^2 - \eta_i^2 + \lambda + 1) D_{\theta i} + \lambda [(\lambda + 1) D_{r\theta i} + 2(\lambda - 1) D_{ki}] \}
 \end{aligned}$$

For case 3:

$$\begin{aligned}
 A_1 + C_1 - A_2 - C_2 &= 0 & (13) \\
 \xi_1 B_1 + \eta_1 E_1 - \xi_2 B_2 - \eta_2 E_2 &= 0 \\
 a_1 D_{\theta 1} A_1 - b_1 D_{\theta 1} C_1 - a_2 D_{\theta 2} A_2 + b_2 D_{\theta 2} C_2 &= 0 \\
 \xi_1 p_1 B_1 - \eta_1 q_1 E_1 - \xi_2 p_2 B_2 + \eta_2 q_2 E_2 &= 0 \\
 A_1 \cosh \xi_1 \alpha + B_1 \sinh \xi_1 \alpha + C_1 \cos \eta_1 \alpha + E_1 \sin \eta_1 \alpha &= 0 \\
 A_1 \xi_1 \sinh \xi_1 \alpha + B_1 \xi_1 \cosh \xi_1 \alpha - C_1 \eta_1 \sin \eta_1 \alpha + E_1 \eta_1 \cos \eta_1 \alpha &= 0 \\
 A_2 \cosh \xi_2 \beta - B_2 \sinh \xi_2 \beta + C_2 \cos \eta_2 \beta - E_2 \sin \eta_2 \beta &= 0 \\
 A_2 \xi_2 \sinh \xi_2 \beta - B_2 \xi_2 \cosh \xi_2 \beta - C_2 \eta_2 \sin \eta_2 \beta - E_2 \eta_2 \cos \eta_2 \beta &= 0,
 \end{aligned}$$

where

$$\begin{aligned}
 a_i &= \xi_i^2 + (\lambda + 1)(\nu_{ri} + 1), \quad b_i = \eta_i^2 - (\lambda + 1)(\nu_{ri} + 1) \\
 p_i &= (\xi_i^2 + \lambda + 1) D_{\theta i} + \lambda [(\lambda + 1) D_{r\theta i} + 2(\lambda - 1) D_{ki}] \\
 q_i &= (\eta_i^2 - \lambda - 1) D_{\theta i} - \lambda [(\lambda + 1) D_{r\theta i} + 2(\lambda - 1) D_{ki}]
 \end{aligned}$$

For a nontrivial solution to the homogeneous systems (11), (12) and (13) of linear algebraic equations with respect to the coefficients  $A_i, B_i, C_i, E_i$ , it is necessary that the determinants of these systems be equal to zero

$$\Delta(\lambda, \alpha, \beta, \nu_{ri}, \nu_{\theta i}, E_{ri}, E_{\theta i}, G_i) = 0 \tag{14}$$

From (2) and (6), it follows that if  $0 < \text{Re} \lambda_1 < 1$ , then under approaching the edge of the joint surface ( $r \rightarrow 0$ ) the stresses (moments) increase unlimitedly, and the order of the singularity is  $|\text{Re} \lambda_1 - 1|$ . And if  $\text{Re} \lambda_1 > 1$ , the stresses decrease to zero when approaching the vertex of the angle.

**Research Results.** Thus, the study of the nature of the stressed state near the rib of the edge of the joint surface of a compound anisotropic plate under bending involves finding the root  $\lambda$  of transcendental equation (14) with the least positive part for constraint angles and mechanical characteristics of the materials being joined.

Setting the determinants of these new systems to zero, we obtain equations relatively  $\lambda$  for each of the three cases, respectively. A numerical solution of these equations is carried out for the following groups of parameters:

- 1)  $\gamma = 1, G_i = \mu_i$ ; 2)  $\gamma = 1, G_i = 4\mu_i$ ; 3)  $\gamma = 1, G_i = \mu_i/4$ ;
- 4)  $\gamma = 1/2, G_i = \mu_i$ ; 5)  $\gamma = 1/2, G_i = 4\mu_i$ ; 6)  $\gamma = 2, G_i = 4\mu_i$ .

In the numerical calculations, Voigt's remark [6] on equality  $E_{ri} = E_{\theta i}$  is accepted.

Some results of a numerical study of the root  $\lambda$ , depending on the angle  $\varphi = \alpha + \beta$ , are shown in the table where  $\alpha = 10^\circ$ .

Table 1

Parameter  $\lambda$  values depending on the angles  $\alpha$  and  $\beta$

$\varphi$	1	2	3	4	5	6
140	1.533	0.845	2.34	1.574	0.784	0.88
160	1.288	0.703	1.72	1.146	0.653	0.73
180	1.000	0.596	1.51	0.910	0.556	0.61
200	0.816	0.516	1.039	0.756	0.485	0.53
230	0.652	0.436	0.817	0.614	0.416	0.447
290	0.519	0.373	0.60	0.508	0.367	0.375
360	0.500	0.325	0.563	0.474	0.311	0.333

The table shows that for these angles, depending on the anisotropy parameters, there may or may not be stress concentration at the vertex.

We can solve the inverse problem [1–5] as well. We construct curves that, for fixed values of the mechanical characteristics of materials on the  $\alpha \beta$  plane, separate the regions of finite and infinite stresses (moments).

Assuming that the smallest root of the equation (11) is real near the boundary of high stress concentration region, we put  $\lambda=1$  in this equation (preliminarily clearing of the double root  $\lambda=1$ ) and find the smallest positive values of the angles  $\alpha$  and  $\beta$  depending on the anisotropy parameters. The geometrical loci in the  $\alpha\beta$  plane form those limiting curves that separate the concentration region (above the curves) from the low-stressed regions (below the curves). The numerical implementation of the obtained equation allows for the determination of a low-stressed region for the edge that provides the joint strength in the space of the parameters  $\alpha, \beta, \nu_{ri}, \nu_{\theta i}, E_{ri}, E_{\theta i}, G_i$ .

Fig. 2 shows these curves for various values of the anisotropy parameters. Lines 1–9 correspond to the following parameters: 1)  $\gamma = 1, G_i = \mu_i$ ; 2)  $\gamma = 1/2, G_i = \mu_i$ ; 3)  $\gamma = 2, G_i = \mu_i$ ; 4)  $\gamma = 1, G_i = 4\mu_i$ ; 5)  $\gamma = 1/2, G_i = 4\mu_i$ ; 6)  $\gamma = 2, G_i = \mu_i$ ; 7)  $\gamma = 1, G_i = \mu_i/4$ ; 8)  $\gamma = 1/2, G_i = \mu_i/4$ ; 9)  $\gamma = 2, G_i = \mu_i/4$ .

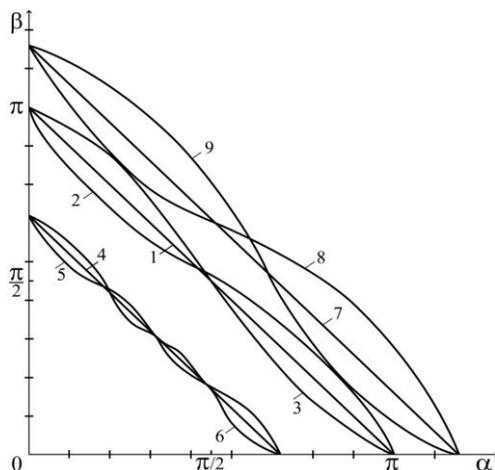


Fig. 2. Distribution of low-stressed zones

In the graphs, straight lines correspond to a homogeneous plate, and curves correspond to a compound plate.

**Discussion and Conclusions.** If for a homogeneous isotropic plate with a corner angle larger than  $\pi$  there is always a stress concentration at the apex, and with an angle smaller than  $\pi$  there is no stress concentration, then for a homogeneous anisotropic plate and compound isotropic and anisotropic plates, this pattern is violated as it is shown in the graphs (Fig. 2).

It can be seen that the degree of concentration of the shearing forces near the angular point is higher by 1 as compared to the moments, which is explained by the imperfection of the classical theory of plate bending.

In a similar way, we can consider the boundary conditions when the plate is freely supported along the outside edges, the outside edges are free, as well as the mixed boundary conditions.

The problem considered here can also be investigated using the refined theory of bending of anisotropic plates [12, 13], which enables to go beyond the restrictions imposed by the Kirchhoff approximation and to compare the results.

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**Author:**

**Akopyan, Ashot G.,**

associate professor of Moscow Automobile and Road Construction State Technical University (MADI), North Caucasus branch, (20, Promyshlennaya St., Lermontov, Stavropol region, 357340, RF), Cand.Sci. (Phys.-Math.), associate professor,

ORCID: <https://orcid.org/0000-0002-2921-5334>

[manakofoto@yandex.ru](mailto:manakofoto@yandex.ru)