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Transverse vibrations of a circular bimorph with piezoelectric and piezomagnetic layers

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Introduction. Transverse axisymmetric oscillations of a bimorph with two piezo-active layers, piezoelectric and piezomagnetic, are studied. This element can be applied in an energy storage device which is in an alternating magnetic field. The work objective is to study the dependence of resonance and antiresonance frequencies, and electromechanical coupling factor, on the geometric parameters of the element.
Materials and Methods. A mathematical model of the piezoelement action is a boundary value problem of linear magneto-electro-elasticity. The element consists of three layers: two piezo-active layers (PZT-4 and $CoFe_2O_4$) and a centre dead layer made of steel. The finite element method implemented in the ANSYS package is used as a method for solving a boundary value problem.
Results. A finite element model of a piezoelement in the ANSYS package is developed. Problems of determining the natural frequencies of resonance and antiresonance are solved. Graphic dependences of these frequencies and the electromechanical coupling factor on the device geometrics, the thickness and radius of the piezo-active layers, are constructed.
Discussion and Conclusions. The results obtained can be used under designing the working element of the energy storage device due to the action of an alternating magnetic field. The constructed dependences of the eigenfrequencies of the resonance and antiresonance on the geometric parameters of the piezoelement provide selecting the sizes of the piezo-active layers for a given working frequency with the highest electromechanical coupling factor.

Keywords: energy storage device, piezoelectrics, piezomagnetics, finite element method, natural oscillation frequencies.

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Introduction. In the development of sensor and measuring systems, modern small-sized household appliances, cellphones and wireless sensor systems, powerful sources of energy are not required for monitoring and diagnosing the technical condition of various objects, but mobility and nonvolatility of the above devices are mandatory. Energy storage devices with piezo-active elements that directly convert the energy of mechanical vibrations into electrical energy are widely used to power this kind of apparatus. In [1-3], energy storage devices using piezoelectric generators under the action of mechanical loads are studied.

If the system is in an alternating magnetic field created by permanent magnets mounted on rotating parts of the machine, then the piezomagnetic layer is deformed along with the piezoelectric element. Due to this, an electric current is generated. In [7,8], Guo-Liang Yu and others discussed theoretical models of multilayer magnetoelectric composites for the magnetoelectric response at resonant frequencies corresponding to vibrations of the bending and extensional modes. A theoretical model of magnetic energy collectors using a functionally gradient composite cantilever was analyzed to improve the collecting ability and adjust the resonance frequency in [9]. In [10], Y.F. Zhanga and others studied bifurcations, periodic and chaotic dynamics of a tetrahedral composite multilayer piezoelectric rectangular plate with simple supports. In [11], bending and free vibrations of a magnetoelectroelastic plate with surface effects were studied.

In this paper, axisymmetric vibrations of the device are considered. The effect of the radius and thickness of the piezoelectric and piezomagnetic plates on the frequency characteristics of the device (eigenfrequencies of resonance and antiresonance) and on the efficiency of the conversion of vibratory energy into electrical energy, which is characterized by an electromechanical coupling factor, is studied. As a method for solving the problem, the finite element method implemented in the ANSYS package is selected.

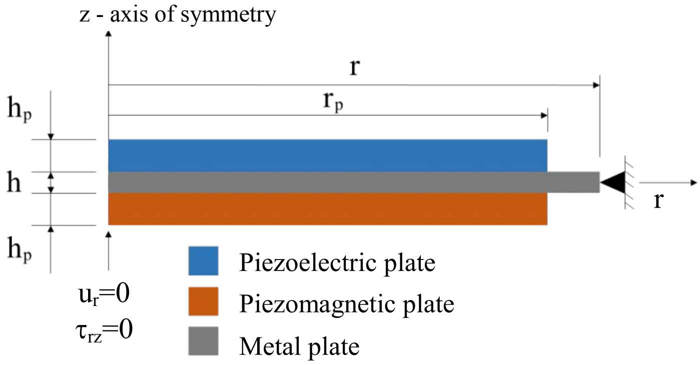


Fig. 1. Scheme of half axial section of piezoelectric generator

The energy storage device under consideration is an axisymmetric design, which consists of a metal disk (substrate) with two piezoelectric layers glued on it (Fig. 1). The top layer is piezoelectric; the bottom one is piezomagnetic. The flat surfaces of the piezoelectric layer are covered with electrodes that are connected to an external electrical circuit, or one of the electrodes is free while the other is set to zero electric potential. The piezomagnetic layer is affected by an alternating magnetic field according to the harmonic law; the outer radius of the substrate is pivotally fixed. The mathematical model of transverse steady-state vibrations of the described construction is the boundary-value problem of the linear theory of piezo-magnetoelectric elasticity [4]. The ANSYS package, which implements a piezoelectric model, is selected as a solution tool.

The boundary value problem for a piezo-magnetoelectric body consists of a system of equations and boundary conditions [4]:

$$\nabla\cdot\sigma+\rho f=\rho\ddot{u},\quad \nabla\cdot D=\sigma_\Omega,\quad \nabla\cdot B=0 \tag{1}$$

$$\begin{gathered}\sigma=c:\varepsilon-e^T\cdot E-h^T\cdot H\\ D=e:\varepsilon+\kappa\cdot E+\alpha\cdot H\\ B=h:\varepsilon+\alpha^T\cdot E+\mu\cdot H\end{gathered} \tag{2}$$

$$\varepsilon=\frac{1}{2}\left(\nabla u+(\nabla u)^T\right),\quad E=-\nabla\varphi,\quad B=-\nabla\phi, \tag{3}$$

where σ and ε are tensors of mechanical stress and strain; D and E are vectors of electric induction and electric field strength; B and H are vectors of magnetic induction and magnetic field strength; ρ is density of the material; c is tensor of elastic modules; e is tensor of piezoelectric modules; h is tensor of piezomagnetic modules; κ is tensor of dielectric permeabilities; α is tensor of magnetoelectric modules; μ is permeability tensor; f is vector of the density of mass forces; σ_Ω is volume density of electric charges; u is displacement vector; φ and ϕ are electric and magnetic potentials.

Boundary conditions are specified for the mechanical, electrical, and magnetic components of the fields.

Mechanical boundary conditions. Let the surface S consist of two parts Γ_1 and Γ_2, so that $S=\Gamma_1\cup\Gamma_2$, moreover $\Gamma_1\cap\Gamma_2=\varnothing$.

$$u=U \text{ on } \Gamma_1,\quad n\cdot\sigma=p \text{ on } \Gamma_2. \tag{4}$$

Electrical boundary conditions. Let the surface S consist of two parts Γ_3 and Γ_4, so that $S=\Gamma_3\cup\Gamma_4$, moreover $\Gamma_3\cap\Gamma_4=\varnothing$.

$$\varphi=\varphi(x,t) \text{ on } \Gamma_3,\quad n\cdot D=-\sigma_0 \text{ on } \Gamma_4, \tag{5}$$

where σ_0 is surface-charge density. In addition, if the electrodes are connected to an external circuit, two conditions should be added:

$$\varphi\big|_{S_E} = v, \quad \iint\limits_{S_E} \mathrm{n} \cdot \mathrm{D} dS = I , \tag{6}$$

where s_E is electrode area; v is unknown potential which is found from the second condition; I is current.

Magnetic boundary conditions. Let the surface S consist of two parts Γ_5 and Γ_6, so that $S = \Gamma_5 \cup \Gamma_6$, moreover $\Gamma_5 \cap \Gamma_6 = \varnothing$.

$$\phi = \phi(\mathrm{x}, t) \text{ on } \Gamma_5, \ \mathrm{n} \cdot \mathrm{B} = \sigma_1 \text{ on } \Gamma_6 \tag{7}$$

where σ_1 is density of free surface currents along the boundary.

For the elastic layer, the first equations in the system (1)–(3) are used; the components of the displacement vector u are unknown. For the piezoelectric layer, the second equations are added to them; u and the electric potential φ are unknown. For the piezomagnetic layer, the third equations are added to the first equations, and the magnetic potential ϕ is added to the unknown displacements. In this case, the relations (2) are transformed through zeroing the corresponding constants.

Finite element modeling. The computer model of the device is built in the ANSYS finite element package. The metal substrate (steel) has thickness *h* and radius *r*. The piezo-active layers consist of one piezoceramic and one piezomagnetic plates which are polarized in thickness, have thickness h_p and radius r_p (Fig. 1).

In the finite element model, devices for the metal and piezoelectric layers are used as finite elements PLANE42 and PLANE13, respectively. In this work, the piezomagnetic layer is modeled by the finite element PLANE13, in which the piezoelectric properties of the material are replaced by piezomagnetic ones. This can be done for two reasons: the piezoelectric layers do not contact each other; qualitatively, the equations for the electric and magnetic potentials coincide.

The properties of the piezoelectric layers used in the calculations are presented in Tables 1–2: piezoceramics — PZT-4, piezomagnetic material — CoFe2O4 [5, 6], adhesive layers are not taken into account.

Table 1

Material properties of PZT-4 piezoceramics

C_{11}^E, Hpa	C_{12}^E, Hpa	C_{13}^E, Hpa	C_{33}^E, Hpa	C_{44}^E, Hpa	e_{31}, CL/m^2	e_{33}, CL/m^2	e_{15}, CL/m^2	k_{11} / ε_0	k_{33} / ε_0
139	77.8	74.3	115	25.6	–5.2	15.1	12.7	730	635

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, PZT-4 piezoceramic density $\rho = 7500$ kg/m^3

Table 2

Material properties of $CoFe_2O_4$ piezomagnetic element

C_{11}^M, Hpa	C_{12}^M, Hpa	C_{13}^M, Hpa	C_{33}^M, Hpa	C_{44}^M, Hpa	Q_{31}, N/Am	Q_{33}, N/Am	Q_{15}, N/Am	λ_{11}, N s^2/Kl^2	λ_{33}, N s^2/Kl^2
286	173	170	269.5	45.3	580.3	699.7	550	5.9x10^{-4}	1.57x10^{-4}

$CoFe_2O_4$ piezomagnetic element density is $\rho = 5290$ kg/m^3

The elastic properties of the isotropic substrate material are characterized by the Young's modulus *E* and the Poisson's ratio ν, $E = 200$ Hpa, $\nu = 0.29$, density $\rho = 7860$ kg/m^3 (steel) were used in the calculations.

To achieve high accuracy in the calculations, the size of the final element of the metal layer is set to the value not higher than $1/5$ its thickness; the size of the final element of the piezoelectric layers is automatically set. The finite element grid of the energy storage device is shown in Fig. 2.

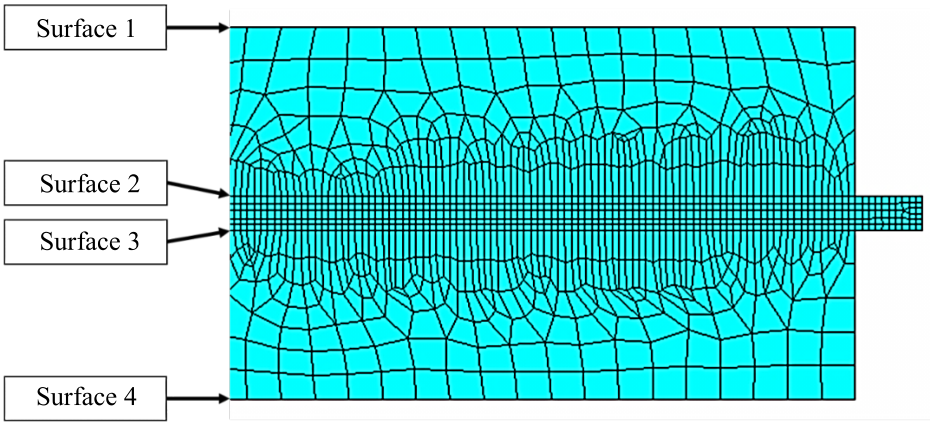


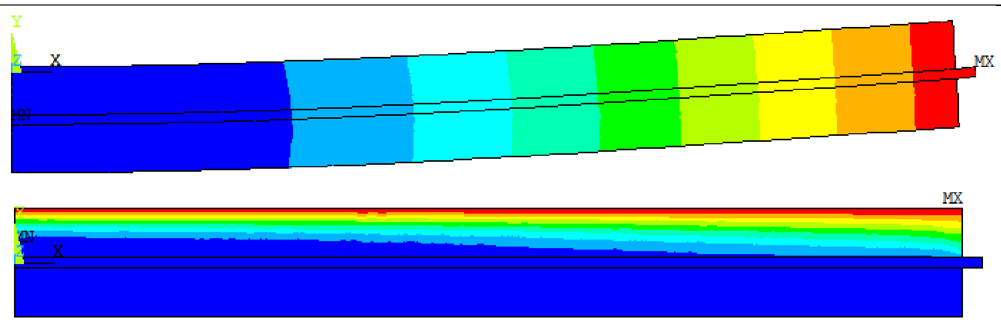
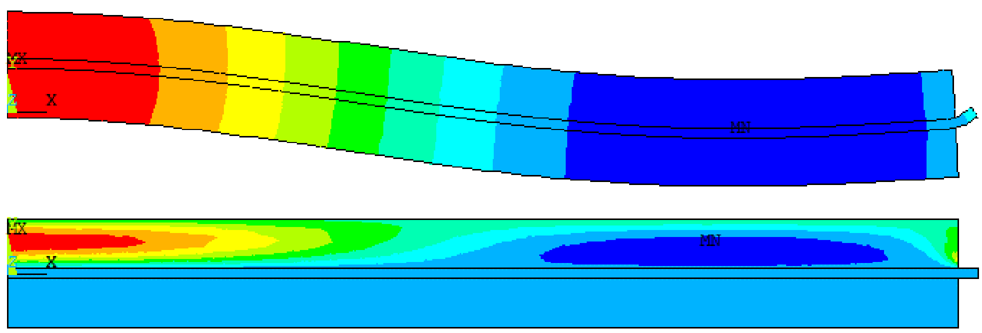
Fig. 2. The finite element grid of the energy storage device.
Surfaces 1, 4 are outer surfaces of piezoelectric layers;
surfaces 2, 3 are contact surfaces between piezoelectric layers and metal layer.

When analyzing the natural vibrations of a piezo-electromagnetic energy storage device, it is assumed that the following mechanical and electromagnetic boundary conditions are met. At the left end, symmetry conditions are specified, the right end is pivotally fixed (Fig. 1). To calculate the resonance frequency of the device, electric potentials are set on surfaces 1, 2; on surface 3, the magnetic potential is set; the magnetic-flux density is set on surface 4. In the case of calculating the antiresonance frequency of the device, the same boundary conditions as for calculating the resonance frequency are set on surfaces 1–4. However, the electric potential on surface 1 is unknown and is found from the condition (6).

Numerical results. The natural vibrations of a piezoelectric element are considered, the radius of which is r_p=9.8 mm, the thickness of the piezoelectric layers is h_p=0.5 mm, the radius of the substrate is r=10 mm, and the thickness of the substrate is h=0.1 mm. Table 3 presents the first three natural frequencies of antiresonance.

Table 3

Antiresonance natural frequencies

No.	Antiresonance frequency kHz	Distribution of vertical displacement on deformed state and distribution of electric and magnetic potential on undeformed sample
1	11.675	
2	56.748	

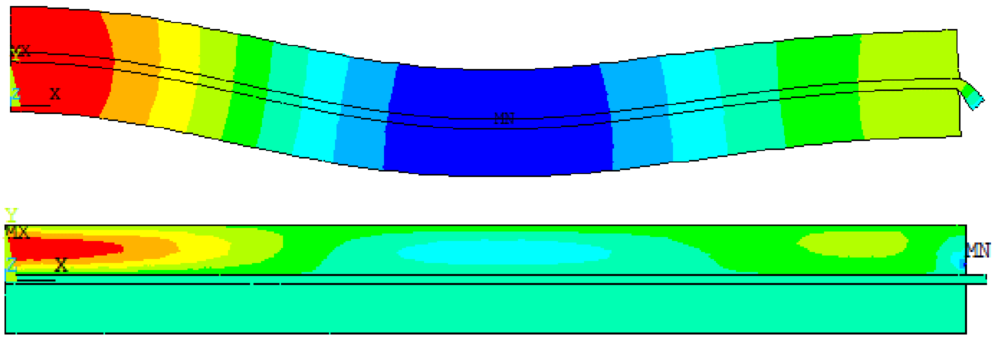
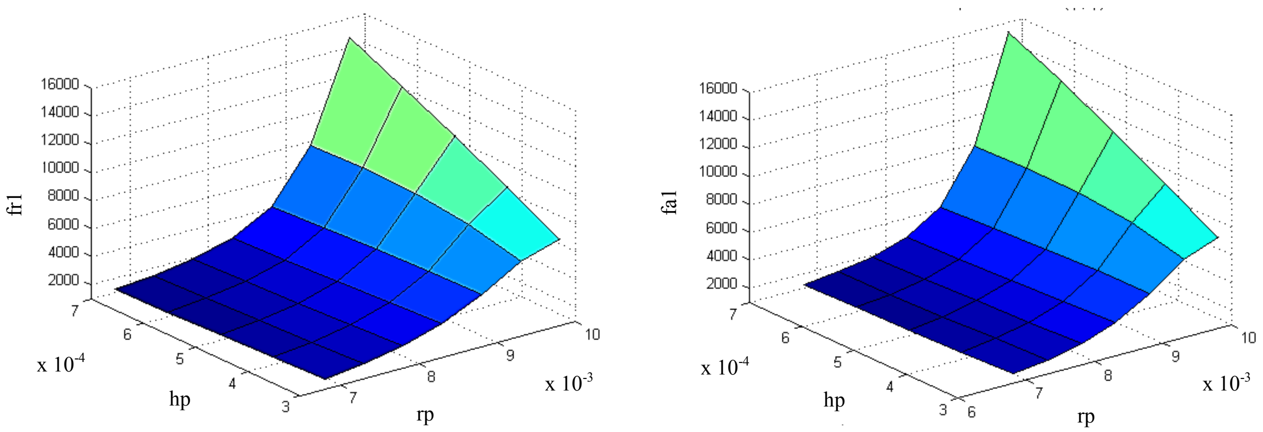
No.	Antiresonance frequency kHz	Distribution of vertical displacement on deformed state and distribution of electric and magnetic potential on undeformed sample
3	115.572	

Table 3 presents the distribution of vertical displacement, electric and magnetic potential while the latter are presented in a single scale, here, the display of the magnetic potential is not visual. To build an approximate theory of calculating the vibrations of the considered bimorph, an analysis of the stress-strain state, electric and magnetic fields, which shows that for lower bending modes, there can be accepted hypotheses about their distribution corresponding to the bending of the plates.

In the results of numerical calculations presented below, the dependence of the eigenfrequencies of resonance and antiresonance, and the electromechanical coupling factor on geometric parameters is investigated.

When the number of alternating piezoelectric and piezomagnetic layers becomes sufficiently large, it is possible to use an approach based on the effective properties of the piezo-magnetoelectric composite [5, 6]. In this case, all equations of the problem (1) - (3) are used.

The thickness value of the piezoelectric layers h_p varies in the range of 0.3 ÷ 0.7 mm, and the radius value r_p – in the range of 6.8 ÷ 9.8 mm.



a) *b)*

Fig. 3. Natural frequencies: a) resonance frequency,
b) antiresonance frequency

Fig. 3 shows the natural frequency dependences on the thickness h_p and the radius r_p of the piezoelectric layers. Fig. 3 shows that the values of the natural frequencies increase with increasing the radius.

Fig. 4 shows the dependence of the electromechanical coupling factor on the thickness h_p and the radius r_p of the piezoelectric layers. Fig. 4 shows that the value of the natural frequency increases with increasing the radius of the piezoelectric layers r_p, but decreases with increasing thickness h_p of the piezoelectric layers.

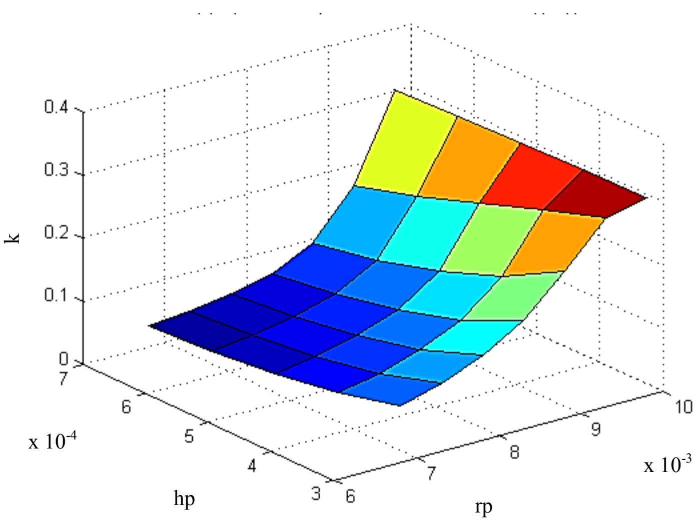


Fig. 4. Electromechanical coupling factor

Conclusion. An axisymmetric finite element model of an energy storage device based on round plates in the ANSYS package is considered. The active elements of the energy storage device are the piezoelectric and piezomagnetic plates fixed on a metal plate. The effect of the geometric characteristics of the piezoelectric layers under certain restrictions on the device dimensions, on the electromechanical coupling factor, which shows the efficiency of the energy storage device, is studied. The calculation results presented in the paper allow selecting rational sizes of piezoelectric elements operating at a certain frequency and having the greatest efficiency. This analysis shows that the maximum value of the electromechanical coupling factor is achieved when the thickness and radius of the piezoelectric layers take the largest and smallest values, respectively, within the considered limits.

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Claimed contributorship

A.N. Solov'ev: task formulation; discussion of the results. Do Thanh Binh: survey conducting; selection of a solution method for constructing a mathematical and computer model; computational analysis; discussion of the results. O.N. Lesnyak: discussion of the results.

All authors have read and approved the final manuscript.