# Information technology, computer science, and management

# INFORMATION TECHNOLOGY, COMPUTER SCIENCE, AND MANAGEMENT



UDC 519.2, 51-74

https://doi.org/10.23947/2687-1653-2020-20-4-405-413

# An approach to forecasting damage due to unfavorable circumstances associated with indistinguishability of source data

V. F. Zolotukhin<sup>1</sup>, A. V. Matershev<sup>2</sup>, L. A. Podkolzina<sup>3</sup>

1,2 SC "VNII 'Gradient'" (Rostov-on-Don, Russian Federation)



Introduction. When administering complex multi-parameter systems, management decisions are often made under uncertainty. There is an acute problem of reduction of the likelihood of unwanted events and mitigation of possible damage. The efficiency of predicting damage to complex systems depends directly on the quality of processing methods, systematization, and the amount of input data. It is required to improve methods for assessing and predicting damage and to develop new approaches and criteria for statistical forecasting of damage and evaluating the system reliability. The solution to such problems is complicated by a large number of indicators, data uncertainty, short series of observations, incomplete initial information, insufficiently developed scientific methodological apparatus. Existing methods for predicting damage in the systems of potentially dangerous objects do not take into account the causes of accidents that happened due to unfavorable circumstances. As a consequence, management decisions are made upon unreliable forecasting results. In this regard, an urgent scientific task is the development of methods and techniques for the formation of viable management decisions, free from this shortcoming. The major study objective is to consider a particular problem for predicting damage due to unfavorable circumstances associated with the indistinguishability of the initial data. The tasks are to consider this kind of uncertainty which includes indistinguishability of the true system condition and the real value of its quantitative characteristics; to formulate a combinatorial problem for the case when a rather dangerous composite feature is determined by the joint manifestation of two or more simple features.

Materials and Methods. Under the conditions of multiple indistinguishability, the following was used as the source data: a set of indistinguishable outcomes with reliable information on the event instance and the uncertainty of assigning the event to a certain type; a family of sets having the same number of elements. The Cartesian product of the families of the corresponding sets and the actual value of the group of a compound potentially dangerous factor with a compound rather dangerous feature are taken into account. The resulting mono-element fuzzy group is presented, which is also a possible event resulting from the intersection of two necessary events.

Results. It is established that the problem of predicting damage due to unfavorable circumstances corresponds to a combinatorial-type problem, which consists in enumerating all sets of arguments. The resulting range, which is an elemental group of indistinguishability, characterizes the smaller and larger possible values of the size of the group of a potentially dangerous factor with a composite rather dangerous feature. It is shown that the formulated combinatorial problems without significant changes are applicable to problems in a generalized form, when composite rather dangerous features are determined using not only the operation of intersection, but also uniting and difference; thereby, the initial groups are not necessarily the objects with simple features.

*Discussion and Conclusions*. The results obtained are focused on the construction of analytical algorithms for establishing indistinguishability under the monitoring, modeling, forecasting state-related processes and complex dynamic multiparameter objects.

Keywords: indistinguishability, probability, mathematical model, risk, random event, accident potential.

*For citation:* V. F. Zolotukhin, A. V. Matershev, L. A. Podkolzina. An approach to forecasting damage due to unfavorable circumstances associated with indistinguishability of source data. Advanced Engineering Research, 2020, vol. 20, no. 4, p. 405–413. <a href="https://doi.org/10.23947/2687-1653-2020-20-4-405-413">https://doi.org/10.23947/2687-1653-2020-20-4-405-413</a>

**Funding information:** the research is supported by RFFI grant no. 19-01-00357.

© Zolotukhin V. F., Matershev A. V., Podkolzina L. A., 2020



<sup>&</sup>lt;sup>3</sup> Don State Technical University (Rostov-on-Don, Russian Federation)

**Introduction.** Activities in the technogenic sphere involve risks and critical situations. This can be the destruction of systems (engineering), loss of control (military matters), bankruptcy (economy). The prediction problem is a challenge for many industries and is directly related to the need to improve, develop, and apply the mathematical apparatus of control tools for complex multiparametric systems.

Issues of simultaneous processing of dynamic arrays of varying degrees of granularity remain urgent. The prototypes of mathematical models containing such structures are also optimization problems of practical resource allocation under the conditions of possible hard-to-formalize impacts [1]. A high degree of process uncertainty reduces the feasibility of using resource-intensive distribution algorithms. At the same time, it is necessary to obtain many alternative solutions. This is particularly important in situations when information on existing threats is uncertain and inconsistent [2]. If an object or process changes unexpectedly, an adequate mathematical model may not be available at the time of decision-making. At the same time, the failure of complex production facilities often causes man-made emergencies with severe economic, environmental and social consequences, which makes it necessary to improve the mathematical foundations of risk analysis [3, 4].

Strategic decisions on managing complex multiparameter systems are made under conditions of uncertainty. The purpose of modern risk management is to avoid a critical situation. In the case of a negative scenario, it is required to minimize losses. To predict damage and make adequate decisions, constant monitoring is carried out, which identifies the major factors of the implementation of the critical situation [5].

It is necessary to rank and systematize risks according to the degree of impact on the protected object activity. Any system is characterized by many parameters, which are often random and changeable. The efficiency of predicting damage to complex systems depends directly on the quality of processing methods, systematization, and the amount of input data. Therefore, to estimate the time and condition of the occurrence of a critical situation, combined forecasting methods are used, which contain expert, analytical, and simulation parts and use the apparatus of probability theory [6, 7]. Critical values can be determined analytically based on the experimental data processing results. This is often performed under the conditions of lack of information, so errors are possible.

A quantitative characteristic of hazard (or security) of systems and situations is the risk. Hereinafter, the risk will be considered damage (consequences) from the implementation of a possible event in a complex system [8]. Due to the complexity and heterogeneity of the factors that affect the system and should be taken into account in the decision-making process, it is required to use a set of methods for analyzing and processing information to assess the damage caused by system failures.

Often, due to the complexity and high cost of monitoring, some of the parameters associated with the technical condition of the system (hereinafter — TCS) are indistinguishable. In general cases, indistinguishability is considered as the state uncertainty, which can manifest itself not only in the future, but also in the present (or near past) time [9–10]. Indistinguishability is described in terms of the theory of possibilities, in which methods for obtaining simple numerical characteristics — estimates of various TCS are developed. It also corresponds to the decision- making principle based on facts. According to this principle, the indistinguishability of the system states is caused by the uncertainty of knowledge on it and represents the concretization of the knowledge uncertainty on the operation, physical behavior. Uncertainties can be exogenous and endogenous, due to external and internal actions, respectively [11]. Each type of uncertainty can significantly impair the decision accuracy.

**Current state of the problem.** A significant number of papers are devoted to certain aspects of the problem of forecasting and risk assessment in case of data indistinguishability. However, these studies lack a general methodological basis. In the papers [12–27], particular solutions are proposed, but the general mathematical apparatus is not developed yet. Thus, in [14–15], it is assumed that the evaluation of each of the indistinguishable criteria is equal to the arithmetic mean of their numbers. The number of the entire group as a whole object in the ordering is taken for the rank of each of the indistinguishable criteria.

In the paper [16], it is offered to unify approaches to management of complex security of various systems; and also, the situation when the expert does not distinguish some criteria is shown.

In [17], the problem of constructing interval estimates for an unknown probability in the presence of multiple indistinguishable outcomes in the experimental results is considered. Two solution paths are proposed: to take into account all unobservable outcomes or to discard them. In both cases, roughening up the result, and errors are possible.

The paper [18] clarifies the uncertainty and indistinguishability that arise when diagnosing the state of power plants. Indistinguishability is understood as the uncertainty of the state of the managed object for the observer. At the same time, it is assumed that uncertainty decreases as we move down the hierarchical ranks of energy system management. In addition, it is proposed to introduce a certain threshold and consider the solutions indistinguishable if the square of the difference of the desired value does not exceed the set threshold.

The indistinguishability aspect is mentioned in light of the development of approximate set theory in [19]. The concepts of lower, upper approximation and border area are revealed. This provides creating decision rules "if ... then" and using facts only, without assumptions. Referring to [20], the author clarifies that one of the key decision-making methods in the field of multi-criterion optimization has been developed on the basis of the theory of approximate sets.

In the paper [21] devoted to the safe operation of lifting cranes, a method of expert assessment of the frequency of adverse events is proposed, which provides the development of recommendations for reducing the risk.

In [22], the problems on military operational research are considered in the following context: warships seek to contain and prevent sea robbery, and the model of pirate movement is based on forecasting the probability of piracy and on the Markov assumption. The author uses the solution to the flow problem with minimal costs. The number of search engines does not matter. It is assumed that they are identical and indistinguishable.

In the study [23], the T-indistinguishability operators stated geometrically are studied as a special case of generalized metric spaces for further application under researching fuzzy subgroups.

It is shown in [24] that the existing indistinguishability of data from a set of observations does not provide accurate estimates of the system state. Hence, an accurate prediction should be based on the probability density of indistinguishable states. This density can be calculated as follows: first, through calculating the maximum likelihood estimate of the state, and then – through an ensemble estimate of the density of states that are indistinguishable from the maximum likelihood state.

In [25], a characteristic of functions that provide combining partial T-indistinguishability (relations) operators into a new set, is presented. In [26], an aggregation of partial T-indistinguishability operators and partial pseudometric is considered. The aggregation of a set of partial T-distinguishability operators is analyzed, and the relationship between functions is shown. These relations:

- combine partial T-indistinguishability operators,
- preserve partial T-pseudometric under the aggregation process.

In [27], the connection of indistinguishability and fuzzy subsets is validated. It is proved that the basic relation for them is lattice isomorphism.

So, the known approaches to the problem of indistinguishability of influencing factors are functionally limited. They do not provide the required level of reliability when making management decisions. The weak points of their research and methodological procedure are obvious. This generally hinders the development of forecasting systems taking into account the factor of indistinguishability, and proves the demand for the development and improvement of the mathematical apparatus.

Thus, an important task of risk management is to predict the damage caused by adverse circumstances due to the indistinguishability of the initial data. Its solution is of considerable theoretical and practical interest for many complex parametric dynamical systems.

Existing methods of predicting damage in the systems of potentially hazardous objects do not take into account the causes of accidents that occurred due to adverse circumstances. As a result, management decisions are made on the basis of unreliable forecasting results. In this regard, it is an urgent scientific task to develop methods and techniques for the formulation of appropriate management decisions that are devoid of this disadvantage.

**Research Problem Statement.** Damage caused by the hazards and attacks is not always possible to predict in advance due to the difficulty of distinguishing between the usual states of multiparametric objects and systems whose

behavior is not determined. As a result, there is a need to study and evaluate two independent types of uncertainty: indistinguishability and nondeterminism. Consider a prediction model due to an unfavorable set of circumstances associated with the indistinguishability of the source data.

We show that in terms of set-theoretic modeling, a mathematical model of adverse circumstances represents a combinatorial problem with common source data. We formulate this problem for the case when a very dangerous compound feature is determined by the joint manifestation of two or more simple features.

## Materials and Methods. Source data:

- a set of indistinguishable outcomes U with the number of elements n containing reliable information on the fact of the event implementation and the uncertainty of assigning the event to a certain type;
  - family  $\overline{A}_i$  of sets of the type  $A_i$ , having the same number  $n_i$  of elements.

Under the conditions of multiple indistinguishability, we are forced to consider all sets of type  $A_i$ , for each i, because any of them can represent a group of objects that have acquired the i-th simple feature.

Consider the Cartesian product of families  $\overline{A}_1 \times \overline{A}_2 \times ... \times \overline{A}_k$ . Its elements are all element sets  $(A_1, A_2, ..., A_k)$  of sets of type  $A_1, A_2, ..., A_k$ .

Let us consider the case when the set  $U = \{1, 2, 3, 4\}$ , k = 2 of an element set of type  $A_1$  contains  $n_1 = 3$  elements, and of type  $A_2 - \pi o$   $n_2 = 2$  elements.

Assume, 
$$A_{1\phi} = \{1, 2, 3\}, A_{2\phi} = \{1, 4\}.$$
 Then

— the true value of the group of a compound potentially hazardous factor with a very dangerous compound feature. In this case, the families  $\overline{A}_1$  and  $\overline{A}_2$  have the form:

$$\overline{A}_{1} = \{\{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}\},$$
 (1)

$$\overline{A}_2 = \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}.$$
(2)

The composition of groups of a potentially hazardous factor with simple features has the form (3, 4), which represents mono multiple indistinguishability groups:

$$A_{1,0}:\{1,2,\underline{3},\underline{4}\},$$
 (3)

$$A_{2\phi}: \{1,2,3,4\}.$$
 (4)

As can be seen, the obtained families (1) and (2) contain  $C_4^3 = 4$ ,  $C_4^3 = 6$  elements, represent multielement groups of indistinguishability, and are *necessary events*.

The Cartesian product of these families will represent a collection containing the following  $n_{1,2} = C_4^3 \cdot C_4^3 = 24$  distinct pairs of elements  $\overline{A}_1$  in  $\overline{A}_2$ :

$$(\{1,2,3\}\{1,2\}), (\{1,2,3\}\{1,3\}), ..., (\{1,2,3\}\{3,4\}), (\{1,2,4\}\{1,2\}), ..., (\{1,2,4\}\{1,4\}), ..., (\{1,3,4\}\{3,4\}), ..., (\{2,3,4\}\{1,2\}), ..., (\{2,3,4\}\{3,4\}).$$

$$(\{2,3,4\}\{1,2\}), ..., (\{2,3,4\}\{3,4\}).$$

You can see that  $\overline{A}_1 \times \overline{A}_2$  is a family of distinct pairs of sets of  $A_1$ ,  $A_2$ , type that have the same number of elements — 3 and 2, respectively. At k > 2 the Cartesian product  $\overline{A}_1 \times \overline{A}_2 \times ... \times \overline{A}_k$  is a family of sets, each of which contains one set of type  $A_1, A_2, ..., A_k$ .

We will call the mentioned sets — elements  $\overline{A}_1 \times \overline{A}_2 \times ... \times \overline{A}_k$  — k-sets. Each k-set will correspond to the intersection of the sets included in it, that is, we select common elements in such sets. For example, the intersections for the

sixth and ninth pairs in (5), which have the form  $(\{1,2,3\}\{3,4\})$  and  $(\{1,2,4\}\{1,4\})$ , will be the sets  $\{1,2,3\} \cap \{3,4\} = \{3\}$  and  $\{1,2,4\} \cap \{1,4\} = \{1,4\}$ , respectively.

The general result (5) will be:

$$\begin{cases}
\{1,2\} & \{1,3\} & \{1\} & \{2,3\} & \{2\} & \{3\} \\
\{1,2\} & \{1\} & \{1,4\} & \{2\} & \{2,4\} & \{4\} \\
\{1\} & \{1,3\} & \{4\} & \{3\} & \{1,4\} & \{3,4\} \\
\{2\} & \{3\} & \{4\} & \{2,3\} & \{2,4\} & \{3,4\}
\end{cases}
\end{cases}$$
(6)

The expression (6) is the resulting mono-element group of indistinguishability, as well as a *possible event* obtained as a result of the intersection of two necessary events (1) and (2).

Now consider function f with arguments that are the mentioned intersections of sets of type  $A_1 \cap A_2 \cap ... \cap A_k$ , and with values equal to the number of elements of these intersections:

$$f(A_1 \cap A_2 \cap \dots \cap A_k) = |A_1 \cap A_2 \cap \dots \cap A_k| = r, \tag{7}$$

where k — the number of elements of the corresponding set.

For the sixth and ninth pairs discussed above:

$$f(\{1,2,3\} \cap \{3,4\}) = f(\{3\}) = 1,$$
  
 $f(\{1,2,4\} \cap \{1,4\}) = f(\{1,4\}) = 2.$ 

Now you can see that we are talking about a combinatorial type problem, which consists in enumerating all sets-arguments to obtain different values from them. Such an enumeration can be found after determining the smaller and larger values f, that coincide with the smaller and larger possible values of the population of a group of objects with a compound potentially dangerous factor.

**Research Results.** Consider the following definition: C — a possible event under the implementation of E event, if  $E \cap C \neq \emptyset$ .

We will proceed from the fact that the concept of the possibility of an event is usually associated with assumptions under uncertain circumstances. Therefore, the application of this concept is inappropriate if E, C are known, and the realized outcome  $x \in E$ . With the appearance of E, it is clear that C either takes place, if  $x \in C$ , or it does not — otherwise. At the same time, there is no uncertainty.

The most common, productive, and sufficient concept for our purposes is the concept of a possible event, due to the uncertainty in the form of indistinguishability of the outcome that has appeared among other outcomes E. In this regard, C is referred to as a possible event under the additional conditions listed below.

Condition 1. The events E and C are known, and, therefore, their intersection  $E \cap C \neq \emptyset$  (by definition, this is not an empty set).

Condition 2. The outcome  $x'_i$  is not set, because of which E occurs if more than one element belongs to  $E = \{x_1, x_2, ..., x_n\}$ . It is only known that  $x'_i$  — one of the elements of E, but it is not established which one exactly, i.e.,  $\exists! \ x'_i \in E$ .

With account for conditions 1 and 2, the generally accepted meaning of the statement that C is an important event is specified. So, C either was realized, if indistinguishable actual outcome  $x_i' \in E \cap C$  (and thus,  $x_i' \in C$ ), or it was not realized — otherwise (if  $x_i' \notin E \cap C$ ). Consider an example that illustrates a possible event of this kind.

Let E and C consist, respectively, in the appearance of an even number and more than three points when throwing a dice, that is  $E = \{2, 4, 6\}$ ,  $C = \{4, 5, 6\}$ . Here,  $E \cap C = \{4, 6\} \neq \emptyset$ . It is known that E occurred, but it was not established which of its outcomes took place. As a result, the above conditions are met, according to which C is a possible event: it was realized if either the outcome  $x_2' = 4$ , or the outcome  $x_3' = 6$ , and it was not realized if the outcome  $x_1' = 2$  was realized.

An important special case of the possible event is necessary event C. It takes place if  $E \cap C \neq \emptyset$  (condition 1), condition 2 is met, and  $E \subset C$ , that is, in the case when all the outcomes of E belong to C. Whichever of them appears, it causes the implementation of C.

In this example, from n = 4,  $n_1 = 3$ ,  $n_2 = 2$ , we find that f takes two values: 1 and 2. This indicates that there are only one- and two-element intersections  $A_1 \cap A_2$  of sets of the type  $A_1, A_2 \subset U, U = \{1, 2, 3, 4\}$ , i.e., in a group of potentially dangerous objects with a compound very dangerous feature, there can be either 1 or 2 objects, and the indistinguishability group (8) is a numerical mono-element indistinguishability group:

$$\{1,\underline{2}\}. \tag{8}$$

The next specific task that follows from the considered model of adverse circumstances, consists in the fact that each obtained above value f should be divided by n, i.e., finding its normalized values — a set of alternative frequencies for an arbitrary object to acquire a compound very dangerous feature under the conditions of multiple indistinguishability and nondeterminism. The latter consists in the fact that quotients  $n_i \mid n$  represent the frequencies of occurrence on an arbitrary object of the i-th simple feature for all i = 1, ..., k.

We formulate the purpose of solving the following combinatorial problem corresponding to the frequency problem: to determine number s of elements  $\overline{A}_1 \times \overline{A}_2 \times ... \times \overline{A}_k$ , to find s(r)/s for all r.

Quotients s(r)/s for all r represent the probabilities that the number of objects with a compound very dangerous feature will be equal to a certain value r. To calculate such probabilities in this regard, we use the dependence:

$$s = C_n^{n_1} \cdot C_n^{n_2} \cdot \dots \cdot C_n^{n_k} . \tag{9}$$

Similarly, by expanding the scope of a specific problem, it is possible to determine a set of alternative frequencies of particular interest (absence of a compound very dangerous feature on an arbitrary object). It can be found through replacing sets of type  $A_1$ ,  $A_2$  with their complements of type  $UA_1$ ,  $UA_2$  with the number of elements  $n - n_1 = 1$  and  $n - n_2 = 2$ , respectively, in the source data. In this case, at the preliminary stage, families, the Cartesian product of families whose elements are assigned intersections of sets of the type  $(UA_1 \cap UA_2)$ , and the function whose values are the numbers of elements of such intersections, are defined in the same way. Having determined these numbers, we solve a combinatorial problem corresponding to one specific problem, and after normalization — to another one. In terms of modeling the first problem in relation to the new source data, the second one will represent:

- set U with the number of elements n,
- various sets  $U \setminus A_1$  with the same number of elements  $n n_1$ ,
- various sets  $U \setminus A_2$  with the number of elements  $n n_2$  each.

The combinatorial problem corresponding to the problem with new source data is of great importance for practice. To solve it, it is advisable to use the result of the following arguments. Let along with a set of ascending-sorted values f, we obtain a set of similarly ordered values — the same as f of function g, whose arguments correspond to the new data. It can be shown that the first and second sets have the same number of elements. Let  $f_h$  and  $g_h$  be elements of these sets with number h. We can make sure that element  $g_h$  of the second set corresponds to the same number of arguments of function g, as element  $f_h$  of the first set, and the number of all arguments (variants) f and g are the same, that is, equal to g.

Therefore, the quotients of s(r)/s type, where  $r = f_h$  correspond to arbitrary value  $g_h$  of function g in terms of the model. These quotients represent the corresponding probabilities of the presence of groups of objects without a compound very dangerous feature of different numbers, if the probabilities of the appearance of different groups of objects of the same number with a simple feature i for all i = 1...k are the same.

It can be shown that the formulated combinatorial problems are applicable without significant changes to problems in a generalized form, when compound very dangerous features are determined using not only the intersection operation, but also the population  $A_1 \cup A_2 \cup ... \cup A_k$ , differences  $A_1 \setminus A_2$ ,  $A_2 \setminus A_1$ , etc., and the source groups are not necessarily only those of objects with simple features.

**Discussion and Conclusions.** Thus, it is established that the problem of predicting damage due to unfavorable circumstances corresponds to a combinatorial type problem. It consists in:

- enumeration of all sets-arguments,
- derivation of different values f,
- determination of smaller and larger values f,
- obtaining the range of possible values of function f, in which its true value is found.

This range is an element group of indistinguishability, it characterizes the smaller and larger possible value of the group size of a potentially hazardous factor with a compound very dangerous feature, and includes several operations.

The results obtained are focused on the construction of analytical algorithms for establishing indistinguishability under the monitoring, modeling and forecasting processes related to the state, and complex dynamic multiparameter objects.

### References

- 1. Ventsov NN, Chernyshev YuO. Formirovanie startovoi populyatsii v usloviyakh neopredelennostei [Formation of the starting population under conditions of uncertainty]. In: Proc. Int. Sci.-Tech. Conf. on Intelligent Systems and Information Technologies 2019. Rostov-on-Don: DSTU Publ. Centre; 2019. P. 25–30. (In Russ.)
- 2. Podinovsky VV. Potentsial'naya nedominiruemost' v zadachakh vybora neskol'kikh luchshikh variantov [Potential nondomination in problems of choice of several best options]. Moscow Witte University Bulletin. Educational Resources and Technologies. 2013;2:57–63. (In Russ.)
- 3. Chernyshev YuO, Ventsov NN, Pshenichnii IS. Ehvolyutsionnyi algoritm poiska mnozhestva al'ternativnykh marshrutov v usloviyakh vozmozhnykh vozdeistvii [Evolutionary algorithm of search of a set of alternative routes in the conditions of possible influences] Engineering Journal of Don. 2018;4(51):42–56. (In Russ.)
- 4. Kravtsova MV. Otsenka tekhnogennogo riska tekhnicheski slozhnykh proizvodstvennykh ob"ektov mashinostroeniya [Risk assessment of technogenic technically complex production facilities engineering]. Proceedings of the Samara Scientific Center of RAS. 2012;14(1–3):877–884. (In Russ.)
- 5. Bunkovsky DV. Instrumenty upravleniya predprinimatel'skimi riskami [Entrepreneurial risks management tools]. Management Issues. 2019;1(37):65–76. DOI: 10.22394/2304-3369-2019-1-65-76 (In Russ.)
- 6. Belykh AA. Osnovy metodologii prognozirovaniya i otsenki ehffektivnosti informatsionnykh sistem [The foundations for prognoses methodology and for information system efficiency estimation]. Scientific Journal of KubSAU. 2011;71:11–133. (In Russ.)
  - 7. Orlov AI. Menedzhment [Management]. Moscow: Izumrud; 2003. 298 p. (In Russ.)
- 8. Ostreikovsky VA, et al. Matematicheskoe modelirovanie tekhnogennogo riska [Mathematical modeling of technogenic risk]. Surgut: SurSU Publ. Center; 2010. 96 p. (In Russ.)
- 9. Germeier YuB. Vvedenie v teoriyu issledovaniya operatsii [Introduction to the theory of operations research]. Moscow: Nauka; 1971. 384 p. (In Russ.)
- 10. Zolotukhin VF, Zakharov AA, Reva VYu. Kharakteristiki tekhnogennoi bezopasnosti v usloviyakh nerazlichimosti [Characteristics of the technogenic safety under the conditions of indistinguishability]. Izvestiya SFedU. Engineering Sciences. 2009;91(2):54–58. (In Russ.)
- 11. Tseligorov NA, Tseligorova EN, Mafura GM. Matematicheskie modeli neopredelennostei sistem upravleniya i metody, ispol'zuemye dlya ikh issledovaniya [Mathematical model uncertainties management systems and methods used for their research]. Engineering Journal of Don. 2012;23(4–2):48. (In Russ.)
- 12. Zolotukhin VF. Fundamental'nye chislovye kharakteristiki, vozmozhnosti, vozmozhnye raspredeleniya i mery [Fundamental numerical characteristics of contingency, contingent distributions and measures]. Automation and Remote Control. 2003;12:152–159. (In Russ.)
- 13. Gusev LA, Khutorskaya OYe. Ob odnoi otsenke ehffektivnosti mashinnoi diagnostiki dvigatel'nykh narushenii [On an estimation of the effectiveness of computer diagnosis of movement disorders]. Automation and Remote Control 2003;12:112–121. (In Russ.)

- 14. Litvak BG. Ehkspertnaya informatsiya: metody polucheniya i analiza [Expert information: methods of obtaining and analysis]. Moscow: Radio i svyaz'; 1982. 184 p. (In Russ.)
- 15. Azhmukhamedov IM. Modelirovanie na osnove ehkspertnykh suzhdenii protsessa otsenki informatsionnoi bezopasnosti [Modeling based on the expert judgments in the process of informational safety evaluation]. Vestnik of Astrakhan State Technical University. Series: Management, Computer Sciences and Informatics. 2009;2:101–109. (In Russ.)
- 16. Azhmukhamedov IM. Analiz i upravlenie kompleksnoi bezopasnost'yu na osnove kognitivnogo modelirovaniya [Cognitive-modeling-based integrated security analysis and management]. Large-scale Systems Control. 2010;29:5–15. (In Russ.)
- 17. Gusev LA. Interval'nye otsenki veroyatnosti pri nalichii nerazlichimosti [Interval estimates of probability in the presence of indistinguishability]. Control Sciences. 2013;4:16–22. (In Russ.)
- 18. Krokhin GD. Istochniki informatsii i prichiny ee neopredelennosti, vyyavlennye pri diagnostike sostoyaniya ehnergoustanovok [Information sources and causes of its uncertainty revealed under diagnosis of powerplant status]. Vestnik NSUEM. 2014;1:292–311. (In Russ.)
- 19. Fatuev VA, Safronova MA. Upravlenie dinamicheskimi sistemami s ispol'zovaniem situatsionnykh i regressionnykh modelei [Management of dynamic systems using situational and regression models]. Izvestiya Tula State University. 2012;2:118–127. (In Russ.)
- 20. Greco S, Matarazzo B, Slowinski R. Rough sets theory for multicriteria decision analysis. European Journal of Operational Research. 2001;129(1):1–47.
- 21. Korotkiy AA, Yegelskaya EV, Sherstyuk AP. Obosnovaniya bezopasnosti gruzopod"emnykh kranov [Safety case: cargo cranes]. Vestnik of DSTU. 2017;91(4):136–143. (In Russ.)
- 22. Bourque F-A. Solving the moving target search problem using indistinguishable searchers. European Journal of Operational Research. 2019;275(1):45–52. DOI:10.1016/j.ejor.2018.11.006
- 23. Jacas J, Recasens J. The group of isometries of an indistinguishability operator. Fuzzy Sets and Systems. 2004;146(1):27–41. DOI:10.1016/j.fss.2003.11.004
- 24. Judd K, Smith L. Indistinguishable states: I. Perfect model scenario. Physica D: Nonlinear Phenomena. 2001;151(2-4):125-141. DOI:10.1016/s0167-2789(01)00225-1
- 25. Calvo T, Fuster P, Valero O. On the problem of aggregation of partial T-indistinguishability operators. Atlantis Studies in Uncertainty Modelling. 2019;1:52–59. DOI: 10.2991/eusflat-19.2019.8
- 26. Sánchez TC, Fuster-Parra P. Aggregation of partial *T*-indistinguishability operators and partial pseudometrics. Fuzzy Sets and Systems. 2021;403:119–138. DOI: 10.1016/j.fss.2019.10.009
- 27. Mattioli G, Recasens J. Structural analysis of indistinguishability operators and related concepts. Information Sciences. 2013;241:85–100.

Submitted 29.09.2020

Scheduled in the issue 02.11.2020

About the Authors:

**Zolotukhin, Vladimir Ph.,** employee, SC "VNII "Gradient" (96, Sokolov Pr., Rostov-on-Don, 344010, RF), Dr.Sci. (Tech.), professor, ORCID: <a href="http://orcid.org/0000-0002-8449-5826">http://orcid.org/0000-0002-8449-5826</a>, ScopusID: <a href="mailto:7006603308">7006603308</a>, <a href="mailto:chita1983@rambler.ru">chita1983@rambler.ru</a>

**Matershev, Alexander V.,** postgraduate student of the Radiophysics Department, Southern Federal University (105/42, Bolshaya Sadovaya St., Rostov-on-Don, 344006, RF), Cand.Sci. (Eng.) applicant, ORCID: <a href="http://orcid.org/0000-0002-9533-3785">http://orcid.org/0000-0002-9533-3785</a>, <a href="mailto:matershev.aleksandr@mail.ru">mail.ru</a>

**Podkolzina, Lubov A.,** postgraduate student of the Information Technologies Department, Don State Technical University (1, Gagarin sq., Rostov-on-Don, 344003, RF), ORCID: <a href="http://orcid.org/0000-0001-9476-5802">http://orcid.org/0000-0001-9476-5802</a>, ScopusID: 57200151503, Researcher ID: M-5035-2019, podkolzinalu@gmail.com

Claimed contributorship

V. F. Zolotukhin: basic concept formulation; academic advising; analysis of the research results; the text revision; correction of the conclusions. A. V. Matershev: research objectives and tasks setting; computational analysis; formulation of conclusions. L. A. Podkolzina: literature review; analysis of calculations; text preparation; formulation of conclusions.

All authors have read and approved the final manuscript.