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Thick-walled spherical shell problem

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Introduction. Cylindrical and spherical shells are extensively used in engineering. They face internal and/or external pressure and heat. Stresses and strains distribution in elastoplastic shells has been studied by many scientists. Numerous works involve the use of the von Mises yield conditions, maximum shear stress, maximum reduced stress. These conditions do not include the dependence on the first invariant of the stress tensor and the sign of the third invariant of the stress deviator. In some cases, it is possible to obtain numerical-analytical solutions for stresses, displacements and deformations for bodies with spherical and cylindrical symmetry under axisymmetric thermal and force action.
Materials and Methods. The problem on the state of a thick-walled elastoplastic shell is solved within the framework of the theory of small deformations. A plasticity condition is proposed, which takes into account the dependence of the stress tensor on three independent invariants, and also considers the sign of the third invariant of the stress deviator and translational hardening of the material. A disconnected thermoelastoplastic problem is being solved. To estimate the stresses in the region of the elastic state of a spherical shell, an equivalent stress is introduced, which is similar to the selected plasticity function. The construction of the stress vector hodograph is used as a method for verification of the stress state.
Results. The problem has an analytical solution for linear plasticity functions. A solution is obtained when the strengthening of the material is taken into account. Analytical and graphical relationships between the parameters of external action for the elastic or elastoplastic states of the sphere are determined. For a combined load, variants are possible when the plastic region is generated at the inner and outer boundaries of the sphere or between these boundaries.
Discussion and Conclusions. The calculation results have shown that taking into account the plastic compressibility and the dependence of the plastic limit on temperature can have a significant impact on the stress and strain state of a hollow sphere. In this case, taking into account the first invariant of the stress tensor under the plasticity condition leads to the fact that not only the pressure drop between the outer and inner boundaries of the spherical shell, but the pressure values at these boundaries, can vary within a limited range. In this formulation of the problem, when there is only thermal action, the hollow sphere does not completely pass into the plastic state. The research results provide predicting the behavior of an object (a hollow sphere) that experiences centrally symmetric distributed power and thermal external influences.

Keywords: hollow sphere, thick-walled spherical shell, thermoelastoplastic state, equivalent stress, associated plastic deformation law, stress hodograph, model behavior control parameters.

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Introduction. The solution to the problem of a thick-walled spherical shell experiencing different external influences is given in the monographs [1, 2] and a number of scientific papers on the theory of elasticity, plasticity, and thermoelastic plasticity [3-9]. Usually, the case is considered when the loading process is simple.

The problem of a thick-walled spherical shell is one of the simplest elastoplastic problems when the fields of external actions and internal parameters have central symmetry. Due to the central symmetry in the plastic region, the regime of complete plasticity is performed. For an ideal plastic body, the problem is statically definable, which allows it to be solved under any plasticity conditions. In the monograph [1], the most complete solution to the spherical shell problem is given, when the dependence of the plastic limit on temperature is not taken into account, and the plasticity condition does not depend on the first invariant of the stress tensor and the sign of the third invariant of the stress deviator. The cases of only thermal and combined loading are considered, when the temperature on the walls of the sphere is set, the pressure on the inner walls is set, and there is no pressure on the outer wall. In [10-13], thermoelastoplastic state of various objects was studied, and in [3-9], the process of thermal loading and unloading of a sphere free from external forces and a hollow sphere for the Tresca condition was considered with account for the dependence of the plastic limit on temperature. The solution to this and similar problems is of interest since it is possible to obtain an analytical or partially analytical solution for various mathematical models. An analytical solution can be obtained through selecting piecewise linear plasticity functions [11, 13]. Mathematical modeling of objects enables to predict their state and behavior depending on the values of the initial parameters [14, 15].

Materials and Methods. Problem Statement. We consider the problem of a thick-walled spherical shell (a hollow sphere) experiencing centrally symmetric external influences: pressure p_b on the outer wall at $\rho = b$ and pressure p_a on the inner wall at $\rho = a$. The thermal effect on the sphere is also considered: temperature T_a is maintained at the boundary $\rho = a$, temperature T_b is maintained at the boundary $\rho = b$. It is assumed that the sphere exhibits elastic and plastic properties. The desired state parameters at each point of the sphere are the components of the stress tensor, the components of the strain tensors, and the displacement vectors. In the elastic state region, the elastic deformations are complete (there are no residual deformations).

Basic Ratios. All relations are reduced to a dimensionless form. The outer radius of the sphere b is selected as a length scale. All values having the stress dimension are assigned to the plastic limit under uniaxial tension k. The scale unit for temperature is 1 C.

Due to the specified symmetry of external actions, in the spherical coordinate system ρ, θ, ϕ of the matrix, the components of the stress and strain tensor will have the form:

$$(\sigma) = \begin{pmatrix} \sigma_\rho & 0 & 0 \\ 0 & \sigma_\theta & 0 \\ 0 & 0 & \sigma_\phi \end{pmatrix}, \quad (\varepsilon) = \begin{pmatrix} \varepsilon_\rho & 0 & 0 \\ 0 & \varepsilon_\theta & 0 \\ 0 & 0 & \varepsilon_\phi \end{pmatrix}.$$

In this case, the equalities $\sigma_\theta = \sigma_\phi$, $\varepsilon_\theta = \varepsilon_\phi$ are fulfilled.

If the plasticity functions do not depend on the first invariant of the stress tensor and the sign of the third invariant of the stress deviator, then, when solving the sphere problem, the plasticity functions will be reduced to the form:

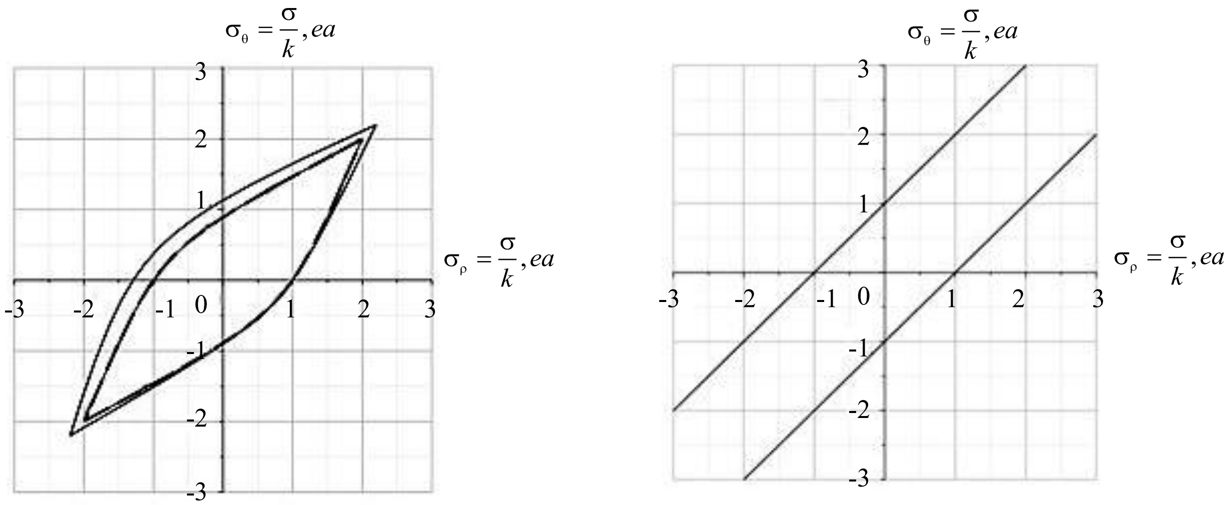
$$f = |\sigma_\theta - \sigma_\rho| = k. \quad (1)$$

Consider the plasticity condition:

$$f(\sigma_\rho, \sigma_\theta, \varepsilon_\rho^p, \varepsilon_\theta^p) = \frac{\varsigma((\sigma_\rho - \delta\varepsilon_\rho^p)^w + 2(\sigma_\theta - \delta\varepsilon_\theta^p)^w)^{\frac{1}{w}}}{\varsigma + \eta(1+\alpha)^{1/m}} + \\ + \frac{\eta((|\sigma_\theta - \sigma_\rho - \delta(\varepsilon_\theta^p - \varepsilon_\rho^p)|^m + \alpha(\sigma_\rho - \sigma_\theta - \delta(\varepsilon_\rho^p - \varepsilon_\theta^p)^m)^{\frac{1}{m}}}{\varsigma + \eta(1+\alpha)^{1/m}} = k(T), \quad (2)$$

where ε_ρ^p, ε_θ^p — components of the plastic strain tensor; T — temperature.

When the parameters have the values: $\varsigma = 0, \eta = 1, \delta = 0, \alpha = 0, m = 1, \mu = 0, k = k_0$, the condition (2) implies the condition (1). In Fig. 1, the plane $\sigma_\rho, \sigma_\theta$ shows the plasticity curves determined from the formula (2) for different values of the numerical coefficients in the plasticity function.



a) *b*)

Fig. 1. Plasticity curves: *a*) for parameters: $\varsigma = 0,2; w = 2; \eta = 0,5; \delta = 0; m = 3; k = 1$; solid line $\alpha = 0,5;$ dotted line $\alpha = 0$; *b*) for parameters: $\alpha = 0; \varsigma = 0; \delta = 0; k = 1$

The results presented in Fig. 1 show that when the first invariant of the stress tensor is taken into account, the radial and circumferential stresses can vary in a limited range when the point of the sphere is in an elastic state. Accordingly, the pressure on the boundaries of the sphere should also be limited. When the first invariant in the plasticity condition is not taken into account, the elastic state is possible for any pressure value at the boundaries of the sphere, but the pressure drop is limited $\Delta p = p_a - p_b$. Taking into account the sign of the third invariant of the stress deviator, as noted above, affects the values of the plasticity limits.

If the values of the state parameters $\sigma_\rho, \sigma_\theta$ determine the point of the region bounded by the plasticity curve, it is assumed that the defining equations connecting stresses and deformations are the relations of the Duhamel-Neumann law [1, 2]:

$$E\varepsilon_\theta = (1-\nu)\sigma_\theta - \nu\sigma_\rho + E\alpha T, \quad E\varepsilon_\rho = \sigma_\rho - 2\nu\sigma_\theta + E\alpha T, \tag{3}$$

where the Young's modulus E and the Poisson's ratio ν are constants.

If the state parameters $\sigma_\rho, \sigma_\theta$ determine the points on the plasticity curve, then an additive representation of the total deformations in terms of reversible and irreversible deformations is assumed:

$$\varepsilon_\theta = \varepsilon_\theta^e + \varepsilon_\theta^p, \quad \varepsilon_\rho = \varepsilon_\rho^e + \varepsilon_\rho^p. \tag{4}$$

Complete deformations are determined through displacements from the formulas:

$$\varepsilon_\theta = \frac{u}{\rho}, \quad \varepsilon_\rho = \frac{du}{d\rho}. \tag{5}$$

Complete deformations are bound by the condition of compatibility of deformations:

$$r\frac{d\varepsilon_\theta}{dr} + \varepsilon_\theta - \varepsilon_\rho = 0. \tag{6}$$

Increments of irreversible deformations are related to stresses by the normal law:

$$\frac{d\varepsilon_\theta^p}{\partial f / \partial\sigma_\theta} = \frac{d\varepsilon_\rho^p}{\partial f / \partial\sigma_\rho}. \tag{7}$$

The relation (7) is generally non-integrable when choosing nonlinear plasticity functions [16]. In the quasi-static approximation, the stresses must satisfy the equilibrium equation:

$$\rho\frac{d\sigma_\rho}{d\rho} + 2(\sigma_\rho - \sigma_\theta) = 0. \tag{8}$$

Equivalent Stress. The equivalent stress is the convex isotropic scalar functions of the stress tensor. In special cases, the term “equivalent stress” is synonymous with other terms, for example, “stress intensity” [17]. In this paper, the equivalent stress coincides with the plasticity function. In this case, the equivalent stress will not have a discontinuity at the elastic-plastic boundary.

Temperature Field. The temperature field in the sphere is found from the solution to the boundary value problem [1]:

$$\begin{cases} \rho \dfrac{d^2T}{d\rho^2} + 2\dfrac{dT}{d\rho} = 0, \\ T\,|_{\rho=a} = T_a,\ T\,|_{\rho=b} = T_b. \end{cases} \tag{9}$$

The solution to the problem (9) is presented in the form:

$$T = T_b + \frac{a\Delta T}{(b-a)}\left(\frac{b}{\rho} - 1\right), \Delta T = T_a - T_b. \tag{10}$$

Elastic Area. In the region of the elastic state of a hollow sphere, the formulas for stresses have the form:

$$\sigma_\rho = A + \frac{B}{\rho^3} - \frac{\lambda}{\rho}, \quad \sigma_\theta = A - \frac{B}{2\rho^3} - \frac{\lambda}{2\rho}, \quad \lambda = \frac{abE\alpha\Delta T}{(1-\nu)(b-a)}.$$

Plastic Area. We select the conditions (1). Let us consider the case of only the thermal effect (10). Then, the plastic region will originate at the inner boundary of the shell under the condition [1]:

$$\beta = \frac{E\alpha}{(1-\nu)}\,|\,\Delta T\,| = \beta_1 = \frac{2(a^2+ab+b^2)k}{b(a+2b)}.$$

Denote by c_1 — the radius of the elastic-plastic boundary $\rho = c_1$. During the loading process, when $\beta > \beta_1$, the plastic area $a \le \rho \le c_1$ increases. When the condition (1) is selected, the stresses in the region $a \le \rho \le c_1$ are calculated from the formulas:

$$\sigma_\rho^{(1)} = 2\kappa_1 k \ln(\rho / a), \quad \sigma_\theta^{(1)} = \sigma_\rho^{(1)} + \kappa_1 k, \quad \kappa_1 = sign(\sigma_\theta - \sigma_\rho),$$

where $\kappa_1 = sign(\sigma_\theta - \sigma_\rho)\,|_{\rho=c_1}$. If $\Delta T > 0$, then $\kappa_1 = -1$, if $\Delta T < 0$, then $\kappa_1 = +1$.

If the region $c_1 \le \rho \le b$ remains elastic, then the values A, B and radius of the elastic-plastic boundary c_1 are determined from the conditions of continuity of stresses at the elastoplastic boundary, and the boundary condition $\sigma\,|_{\rho=b} = 0$. So, if A and B are determined only from the conditions of continuity of stresses at the elastoplastic boundary, then, the following expressions take place:

$$A = 2\kappa_1 k\left(\ln\left(\frac{c_1}{a}\right) + \frac{1}{3}\right) + \frac{\lambda}{3c_1}, \quad B = \frac{\lambda c_1^2}{3} - \frac{2}{3}\kappa_1 k c_1^3. \tag{11}$$

The equation to calculate c_1 will have the form:

$$2\kappa_1 k\left(\ln\left(\frac{c_1}{a}\right) + \frac{1}{3} - \frac{2c_1^3}{3b^3}\right) + \left(\frac{2}{3c_1} + \frac{c_1^2}{3b^3} - \frac{1}{b}\right)\lambda = 0. \tag{12}$$

If A and B are determined from the conditions of continuity of stresses at the elastoplastic boundary, and the conditions $\sigma\,|_{\rho=b} = 0$, then the following expressions take place:

$$A = \left(1 - \frac{c_1^2}{3b^2}\right)\frac{\lambda}{b} + \frac{2\kappa_1 k c_1^3}{3b^3}, \quad B = \frac{\lambda c_1^2}{3} - \frac{2}{3}\kappa_1 k c_1^3. \tag{13}$$

The choice of formulas (11) or (13) affects the steps of the algorithm for solving the problem, but does not affect the final results.

A second plastic region will be generated at the boundary $\rho = b$ if the following condition is met:

$$(\sigma_\theta - \sigma_\rho)\,|_{\rho=b} = \kappa_2 k, \ \kappa_2 = -\kappa_1. \tag{14}$$

To determine the value $\Delta T = \Delta T_1$, when the condition (14) is satisfied, it is required to combine the system of equations (12), (14). Since the parameter β enters the equations (13) and (1) linearly, it is possible to obtain a separate equation for determining the radius of the elastoplastic boundary:

$$2\kappa_1 k \ln\left(\frac{c_1}{a}\right) + \frac{4\kappa_1 k_0 (b-c_1)(c_1^2-b^2)}{3(b+c_1)bc_1} = 0, \tag{15}$$

as well as the formula for calculating the parameter β:

$$\beta = \beta_2 = \frac{2k_0(\kappa_2 b^3 - \kappa_1 c_1^3)(b-a)}{(b^2-c_1^2)ab}. \tag{16}$$

Under further loading, when the inequality $\beta > \beta_2$ is satisfied, the sphere region $c_2 \le \rho \le b$ goes into a plastic state at the boundary $\rho = b$.

A hollow sphere when exposed to heat. Consideration of material hardening. Consider the case when the plasticity condition has the form:

$$f(\sigma_\rho, \sigma_\theta, \varepsilon_\rho^p, \varepsilon_\theta^p) = |\sigma_\theta - \sigma_\rho - \delta(\varepsilon_\theta^p - \varepsilon_\rho^p)| = k. \qquad (17)$$

If there are no residual deformations in the sphere before loading, then as a result of thermal heating, the plastic zone will be generated at the inner boundary $\rho = a$, when the conditions (15), (16) are met. With further loading, plastic region $a \le \rho \le c_1$ is formed. To find the stresses in this region, it is required to get the corresponding equations from the system of equations (3-6), (8), (17):

$$\begin{cases} \rho^2 \dfrac{d^2\sigma_\rho}{d\rho^2} + 4\rho \dfrac{d\sigma_\rho}{d\rho} - \dfrac{6}{1+3\delta(1-\nu)}\left(k + \dfrac{\delta abE\alpha\Delta T}{\rho(b-a)}\right) = 0, \\ \sigma_\theta = \dfrac{\rho}{2}\dfrac{d\sigma_\rho}{d\rho} + \sigma_\rho. \end{cases} \qquad (18)$$

The solution to the system (18) is written as:

$$\begin{aligned} \sigma_\rho &= \frac{1}{1+3\delta(1-\nu)}\left(2k\ln\rho - \frac{3\delta abE\alpha\Delta T}{\rho(b-a)}\right) - \frac{C_1}{\rho^3} + C_2, \\ \sigma_\theta &= \frac{1}{1+3\delta(1-\nu)}\left(k + 2k\ln\rho - \frac{3\delta abE\alpha\Delta T}{2\rho(b-a)}\right) + \frac{C_1}{2\rho^3} + C_2. \end{aligned} \qquad (19)$$

The values C_1, C_2, included in the formulas (19), are determined from the boundary condition $\sigma_\rho|_{\rho=a} = 0$ and the condition for the absence of plastic deformations at the elastic-plastic boundary $\rho = c_1$:

$$\begin{aligned} C_1 &= 2\kappa_1(k - k_\delta)c_1^3 - 3E\alpha N_\delta c_1^2, \\ C_2 &= 2\kappa_1 k_\delta \ln a + \frac{2\kappa_1(k-k_\delta)c_1^3}{3a^3} + \frac{E\alpha N_\delta}{a}\left(3 - \frac{c_1^2}{a^2}\right), \\ N_\delta &= \frac{\delta ab\Delta T}{(1+3\delta(1-\nu))(b-a)}. \end{aligned} \qquad (20)$$

As a result of substituting (20) in (19), we get:

$$\begin{aligned} \sigma_\rho &= \frac{1}{1+3\delta(1-\nu)}\left(2\kappa_1 k\ln\frac{\rho}{a} + \frac{3\delta abE\alpha\Delta T}{b-a}\left(\frac{1}{a} - \frac{1}{\rho}\right) - \right. \\ &\left. -\left(\frac{2\kappa_1 kc_1^3}{3} + \frac{\delta abc_1^2E\alpha\Delta T}{b-a}\right)\left(\frac{1}{a^3} - \frac{1}{\rho^3}\right)\right) + \frac{2\kappa_1 kc_1^3}{3}\left(\frac{1}{a^3} - \frac{1}{\rho^3}\right), \\ \sigma_\theta &= \frac{1}{1+3\delta(1-\nu)}\left(\kappa_1 k + 2\kappa_1 k\ln\frac{\rho}{a} + \frac{3\delta abE\alpha\Delta T}{b-a}\left(\frac{1}{a} - \frac{1}{2\rho}\right) - \right. \\ &\left. -\left(\frac{\kappa_1 kc_1^3}{3} + \frac{\delta abc_1^2E\alpha\Delta T}{2(b-a)}\right)\left(\frac{2}{a^3} + \frac{1}{\rho^3}\right)\right) + \frac{\kappa_1 kc_1^3}{3}\left(\frac{2}{a^3} + \frac{1}{\rho^3}\right). \end{aligned}$$

From the solution to the elastic problem, it follows: if $\Delta T > 0$, then $\kappa_1 = -1$; if $\Delta T < 0$, then $\kappa_1 = +1$.

Accounting for plastic compressibility. Consider the case when the plasticity function is linear with respect to the components of the stress tensor:

$$\begin{aligned} &\varsigma(2\sigma_\theta + \sigma_\rho) + \kappa(\sigma_\theta - \sigma_\rho) + \eta(\sigma_\theta - \sigma_\rho) = k_0(1 - \chi T), \\ &\kappa = sign(\sigma_\theta - \sigma_\rho). \end{aligned} \qquad (21)$$

The plasticity condition (21) can be represented as:

$$\begin{aligned} &\alpha\sigma_\theta + \beta\sigma_\rho = k(1-\chi T), \\ &\alpha = 2\varsigma + \kappa - \eta, \\ &\beta = \varsigma - \kappa + \eta. \end{aligned}$$

Taking into account the introduced notation, to determine the stresses in the plastic region, we obtain the problem:

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$$\begin{cases} \alpha\sigma_\theta + \beta\sigma_\rho = k(1-\chi T), \\ \rho \dfrac{d\sigma_\rho}{d\rho} + 2(\sigma_\rho - \sigma_\theta) = 0, \\ \sigma_\rho \,|_{\rho=a} = -p_a. \end{cases} \tag{22}$$

The solution to the problem (22) has the form:

$$\sigma_\rho = \left(-p_a + \frac{(M_\chi - 1)k_0}{\alpha+\beta} + \frac{2N_\chi k_0}{a(\alpha+2\beta)}\right)\left(\frac{a}{\rho}\right)^{2+2\beta/\alpha} - \left(\frac{M_\chi - 1}{\alpha+\beta} + \frac{2N_\chi}{\rho(\alpha+2\beta)}\right)k_0,$$
$$\sigma_\theta = -\frac{\beta}{\alpha}\left(-p_a + \frac{(M_\chi - 1)k_0}{\alpha+\beta} + \frac{2N_\chi k_0}{a(\alpha+2\beta)}\right)\left(\frac{a}{\rho}\right)^{2+2\beta/\alpha} - \left(\frac{M_\chi - 1}{\alpha+\beta} + \frac{N_\chi}{\rho(\alpha+2\beta)}\right)k_0, \tag{23}$$

where the notation is introduced: $M = T_b - \dfrac{a\Delta T}{b-a}$, $N = \dfrac{ab\Delta T}{b-a}$, $M_\chi = \chi M$, $N_\chi = \chi N$.

To get the correct result from (20), when, for example, $\alpha+\beta=0$, it is required to perform a limit transition when solving (22). It is easier to obtain the correct result directly in (23), while taking into account that the condition $\alpha+\beta=0$ is met. In this case $\varsigma = 0$, so we get:

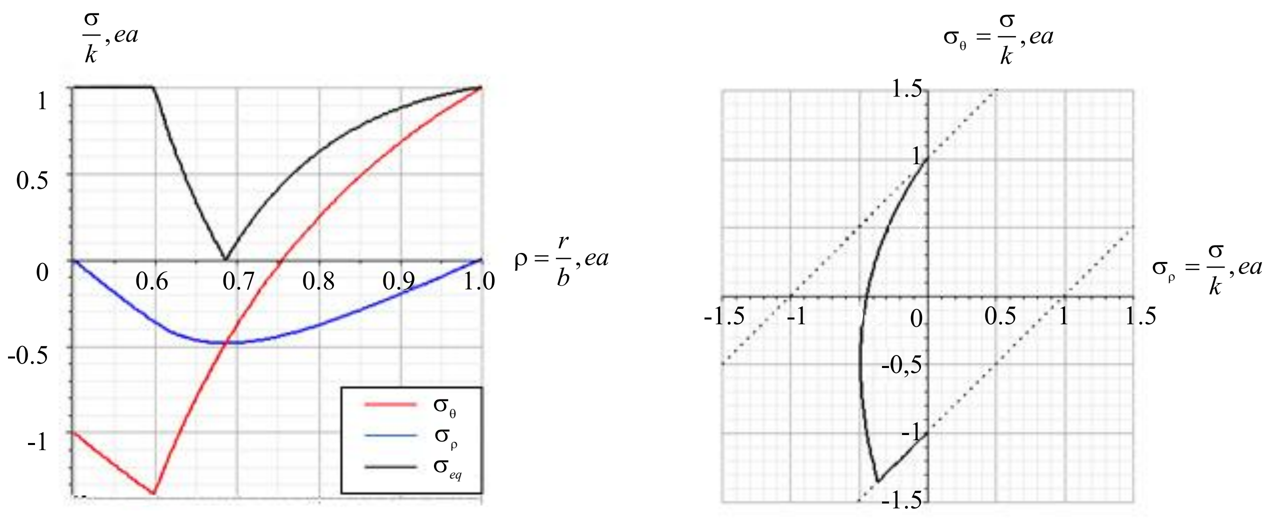
$$\sigma_\rho = \frac{2k_0}{\alpha}\left((1-M_\chi)\ln\frac{\rho}{a} + \left(\frac{1}{\rho} - \frac{1}{a}\right)N_\chi\right) - p_a,$$

$$\sigma_\theta = \frac{k_0}{\alpha}\left((1-M_\chi)(1+\ln\frac{\rho}{a}) + N_\chi\left(\frac{1}{\rho} - \frac{2}{a}\right)\right) - p_a.$$

During the loading process, the plastic zone originates at the boundary $\rho = a$, when the following condition is met:

$$\Delta T = k_0(1-\chi T_b) \Big/ \left(k_0\chi - \frac{(2\varsigma + \kappa - \eta)(a+2b)bE\alpha}{2(1-\nu)(a^2+ab+b^2)}\right).$$

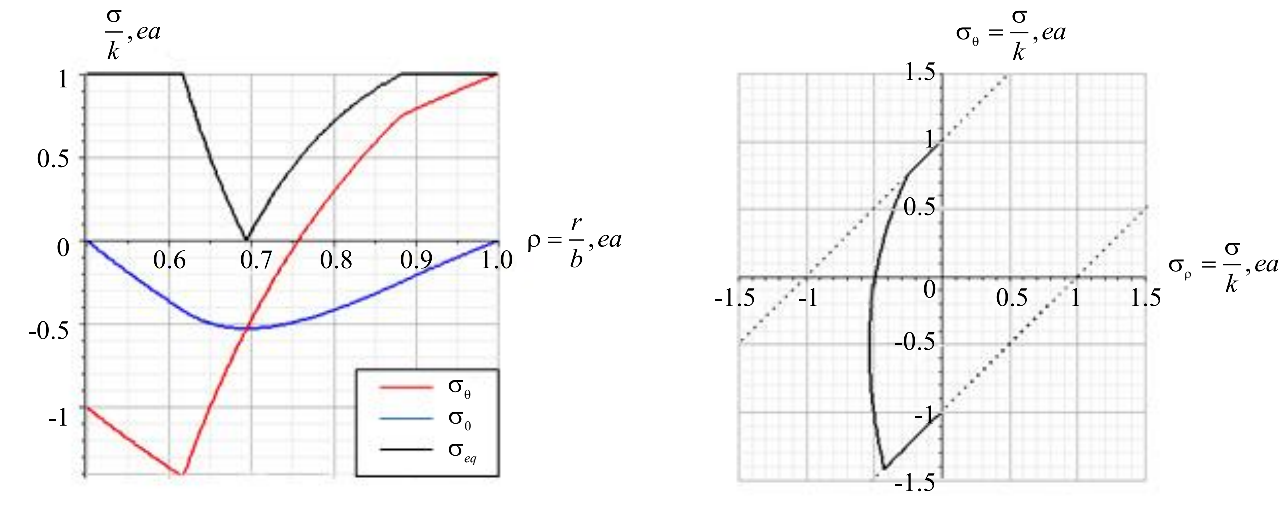
Research Results. Fig. 2 shows stress graphs and stress vector hodographs, when the sphere region corresponding to the condition $a \le \rho \le c_1$, is in a plastic state, and the sphere region corresponding to the condition $c_1 \le \rho \le b$, is in an elastic state.



a) *b*)

Fig. 2. Stress graphs (*a*) and stress vector hodographs (*b*)
for parameter values:: $k = 1$; $\nu = 0.3$; $a = 0.5$; $b = 1$; $\Delta T = 170$; $c_1 = 0.57$

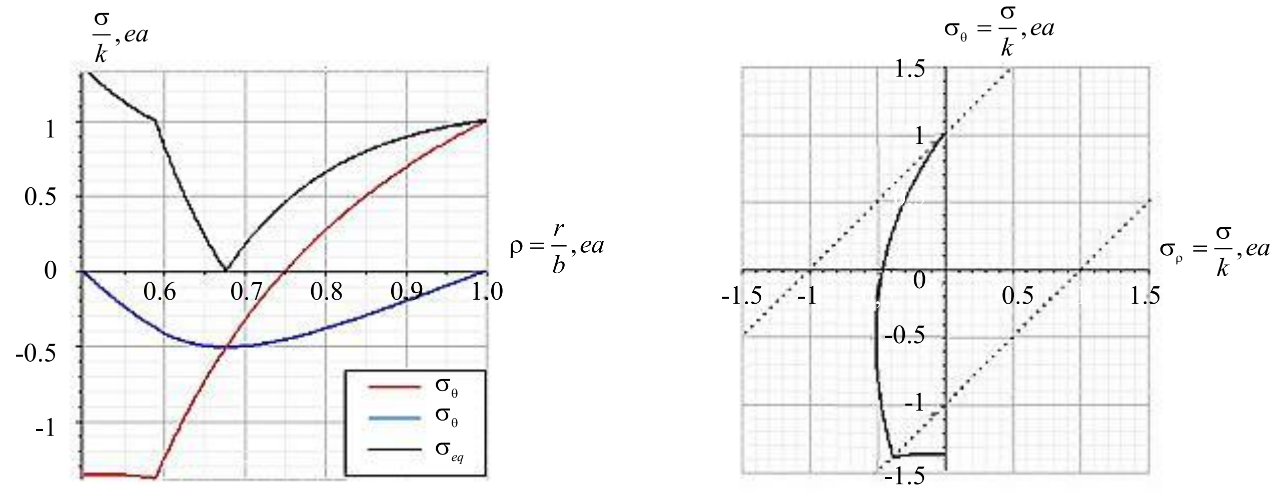
Fig. 3 shows stress graphs and stress vector hodographs, when the sphere regions corresponding to the conditions $a \le \rho \le c_1$ and $c_2 \le \rho \le b$, are in a plastic state, and the sphere region corresponding to the condition $c_1 \le \rho \le c_2$, is in an elastic state.



a) *b*)

Fig. 3. Stress graphs (*a*) and stress vector hodographs (*b*) for parameter values:
$\nu = 0.3;\ a = 0.5;\ \Delta T = 270;\ c_1 = 0.62;\ c_2 = 0.88$

Fig. 4 shows stress graphs and stress vector hodographs, when the sphere region corresponding to the condition $a \le \rho \le c_1$, is in a plastic state, and the sphere region corresponding to the condition $c_1 \le \rho \le b$, is in an elastic state.



a) *b*)

Fig. 4. Stress graphs (*a*) and stress vector hodographs (*b*) for parameter values:
$\nu = 0.3;\ a = 0.5;\ \Delta T = 215;\ c_1 = 0.58$

Fig. 5 shows stress graphs and stress vector hodographs, when the sphere regions corresponding to the conditions $a \le \rho \le c_1$ and $c_2 \le \rho \le b$, are in a plastic state, and the sphere region corresponding to the condition $c_1 \le \rho \le c_2$, is in an elastic state.

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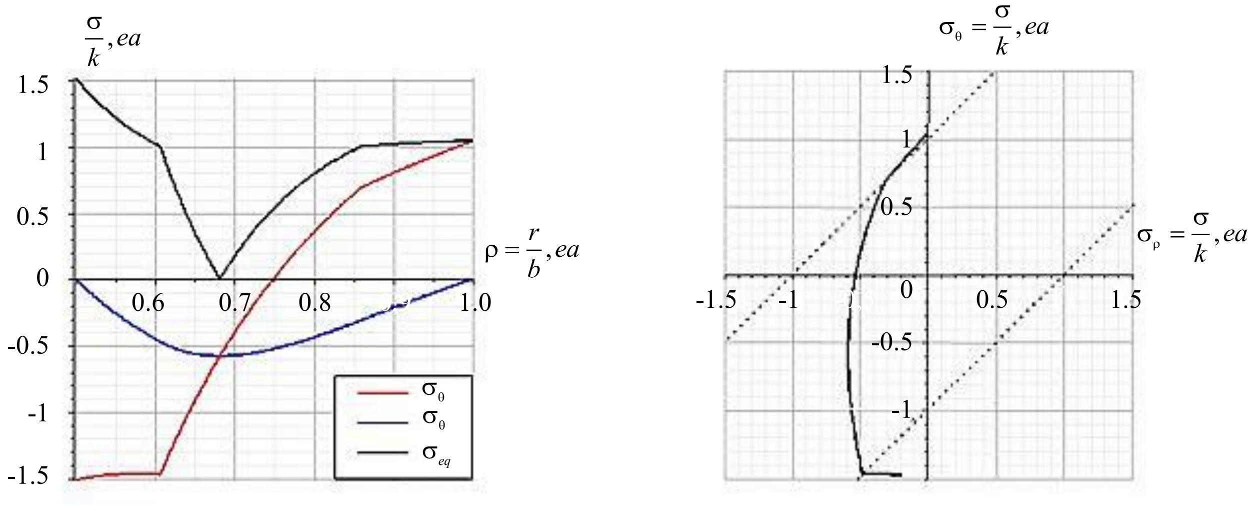
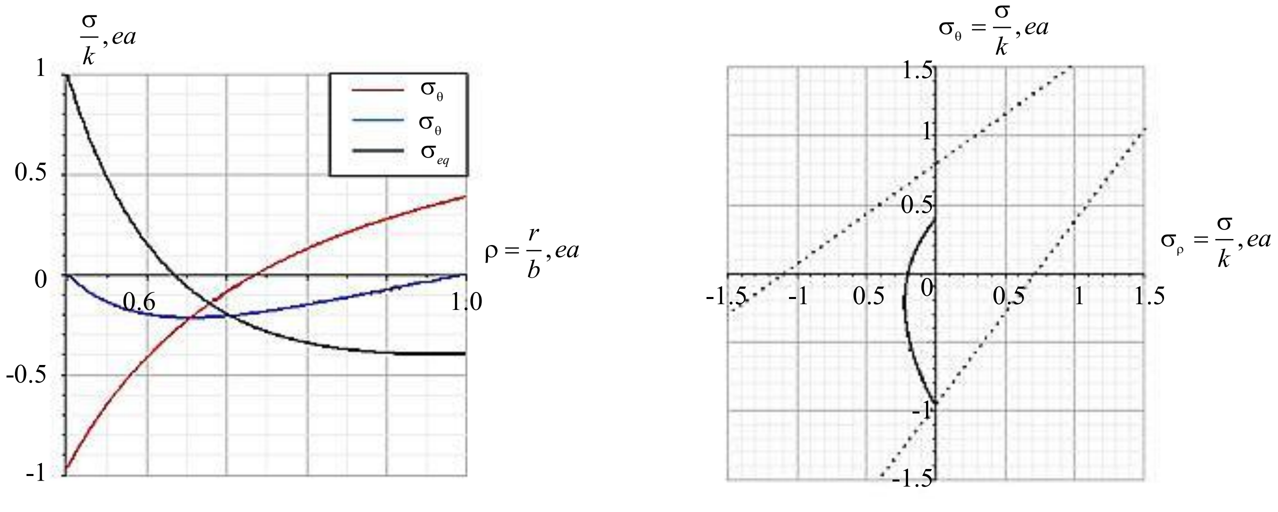


a) b)

Fig. 5. Stress graphs (a) and stress vector hodographs (b) for parameter values:

$$\nu = 0.3;\ a = 0.5;\ \Delta T = 270;\ c_1 = 0.61;\ c_2 = 0.86$$

Fig. 6 shows stress graphs and stress vector hodographs when the sphere regions corresponding to the conditions $a \le \rho \le c_1$ and $c_2 \le \rho \le b$, are in a plastic state, and the sphere region corresponding to the condition $c_1 \le \rho \le c_2$, is in an elastic state.



a) b)

Fig. 6. Stress graphs (a) and stress vector hodographs (b) for parameter values:

$$\nu = 0.3;\ a = 0.5;\ \Delta T = 79;\ \varsigma = 0.1;\ \eta = 0.1;\ E\alpha = 0.012;\ \chi = 0.0017$$

Discussion and Conclusions. The calculation results show that in this formulation of the problem, when there is only thermal action, the hollow sphere does not completely go into the plastic state (Fig. 2-6). Hardening causes an increase in the equivalent stress in the plastic region and a decrease in the radius of the elastoplastic boundary (Fig. 4, 5). The elastic region cannot completely disappear under loading. Plastic compressibility and the dependence of the plastic limit on temperature have a significant effect on the stress state of the hollow sphere (Fig. 6).

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