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Power analysis of chain transmission with gear chain and involute sprockets

S. B. Berezhnoy, G. V. Kurapov
Kuban State Technological University (Krasnodar, Russian Federation)

Introduction. The power analysis of a chain transmission with a toothed chain and involute sprockets considers the centrifugal forces and the friction coefficients between the plate and the sprocket tooth. The work objectives are to determine all meshing forces, and to calculate the coupling coefficient of the gear chain with the involute sprocket in the drive gears.
Materials and Methods. When evaluating the traction capacity of a chain transmission, such parameters as the power analysis and the coupling coefficient of the gear chain with the sprocket are important (it shows what fold the pressure on a given tooth is greater than the pressure perceived by the tooth in front). In the presented paper, the following diagrams are visualized: the arrangement of the plates in gearing with the involute sprocket teeth and the meshing forces. The factors that affect the involute profile of the sprocket tooth are considered. This includes the weight of the chain plate package and the force: centrifugal, friction, normal pressure and tension. At the same time, changes in the coupling coefficient for the subsequent teeth involved in traction are taken into account. The balance of the links i and $i + 1$ of the gear chain is studied in the coordinate system XOY with the center on the axis of rotation of the involute sprocket. The method enables to determine all the desired forces through the geometric calculation of the values of the angular transmission parameters. Using the equations obtained, the following parameters are specified: the coupling coefficient B_i, the tension of the driving branch S_1 and the slack branch S_2.
Results. A patented transmission stand with a gear chain and involute sprockets is presented. The tests carried out on it validated the study results of a chain transmission with a toothed gear and involute sprockets with the specified parameters. The correctness of the power analysis of the transmission with account for the centrifugal forces and the friction coefficients of the plates and the sprocket teeth was proved.
Discussion and Conclusions. It is noted that the centrifugal forces and the friction coefficients during engagement affect significantly the traction capacity of a transmission with a toothed chain and involute sprockets. The data obtained can be used to accurately estimate the traction capacity of such gears.

Keywords: chain transmission, toothed chain, involute sprocket, chain plate, traction capacity, coupling coefficients, centrifugal forces, friction forces, friction coefficients, joint angular velocity, driving side, slack side.

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Introduction. Chain transmissions with toothed chain are not sufficiently studied. Their release is limited, but they are increasingly used in production. Specifically, such gears are used to complete drive mechanisms in machines, precision machine tools, and other equipment. This paper considers the power analysis and calculation of the traction capacity of gears with a toothed chain and involute sprockets.

The traction capacity determines the operability of the involute gear chain transmission, which depends on a number of factors[1]. Among them there are:

[1] Kurapov GV. Performance study of a chain transmission with a gear chain and involute sprockets: Cand.Sci. (Eng.) diss. Krasnodar, 2019. 173 p. (In Russ.)

— transmission geometry,

— limiting contact stress of the weakest element of the pair “chain plate – sprocket tooth”,

— coupling coefficient of the toothed chain with the sprocket teeth,

— slipping of the toothed chain along the sprocket teeth,

— permissible specific pressure inside the joint,

— impact resistance of gear chain elements[1] [1-3].

Materials and Methods. The coupling coefficient is one of the main factors of the traction capacity of a chain transmission equipped with a toothed chain. During the operation of such a transmission, engagement occurs and forces act that move the chain plates along the working and occipital profiles of the sprocket teeth [4-7]. On the arc of the involute sprocket with a toothed chain, any of its links (link i, $i + 1$, $i + 2$, etc.) experience:

— friction force $N_i f$ of the chain on the sprocket tooth,

— tensile strength Q_i of adjacent links,

— normal pressure force N_i of the tooth sprocket profile (Fig. 1).

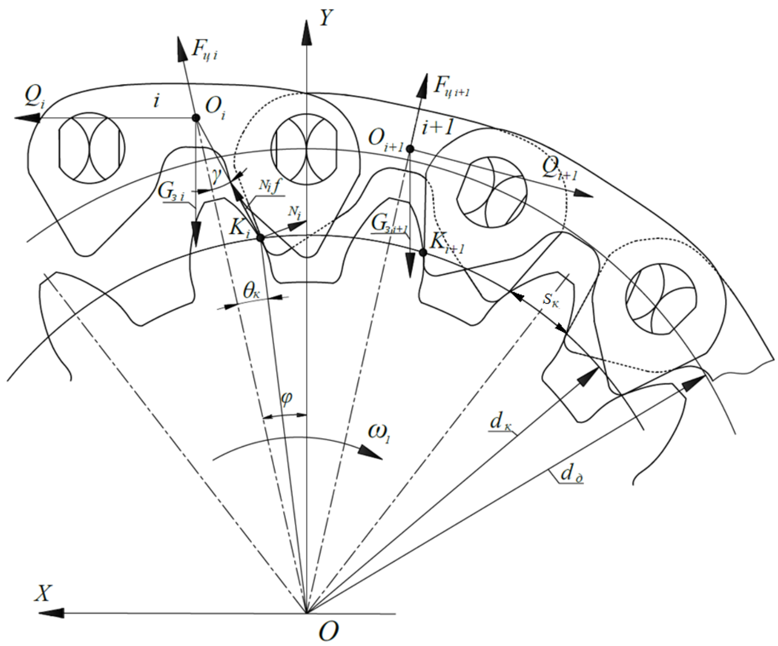

Fig. 1. Arrangement of plates in engagement with the involute sprocket teeth

In the literature, the coupling coefficient is calculated from the ratio of the tensile forces Q_i and Q_{i+1} in the adjacent chain links:

$$B_i = \frac{Q_i}{Q_{i+1}}. \tag{1}$$

This coefficient is not constant, so the plates of the gear chain move along the profiles of the sprocket teeth throughout the entire arc of contact. Fig. 2 shows the engagement of the first package of plates (link) of the chain with the sprocket tooth.

[1] Semenov VS. Availability assurance of roller chain drives during their operation. In: Proc. All-Russian Sci.-Pract. Conf. of postgraduates, doctoral students, and young scientists. Maykop, 2017. P. 121–125. (In Russ.)

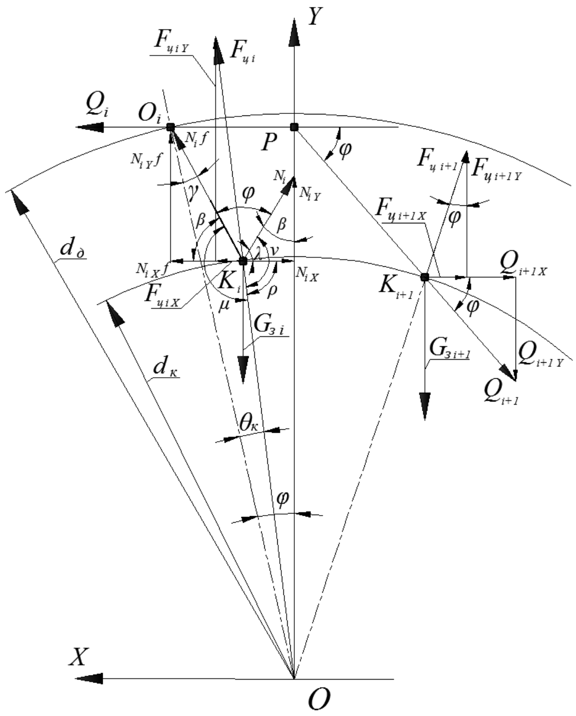

Fig. 2. Diagram of the forces acting in the engagement of the gear chain plate with the sprocket

The involute profile of the sprocket tooth is affected by the following factors [8-14].

1. Friction force $N_i f$, which prevents the movement of the plate pack to the top of the sprocket tooth. Here, f — the friction coefficient.

2. Weight of the chain plate package $G_з = m_i g$. Here, g — the acceleration of gravity, m_i — chain link weight depending on the pitch[1].

3. Normal pressure force N_i of the sprocket tooth profile concentrated at the point of engagement K_i.

4. Tensile forces Q_i and Q_{i+1} of the adjacent links of the gear chain.

5. Centrifugal force with account for the mass of the chain link $F_ц = \frac{m_i \omega^2}{R}$. Here, ω — angular speed of rotation of the sprocket. R — radius of the central part of the link (represented as a circle that changes position when the sprocket rotates) on the involute profile of the sprocket tooth.

However, when the chain moves from the driving branch to the driven branch, the tensile forces Q_i and Q_{i+1} change by the value of each value of the coupling coefficient B_i; so, it needs to be determined more precisely.

Let us build the XOY coordinate system. It passes through the center of the axis of rotation of the involute sprocket. The vector of the X-axis coincides with the vector of the linear velocity of the gear chain in the transmission.

Let us analyze the equilibrium of the first link i and the second $i + 1$.

Balance of forces on the axis X ($\sum X = 0$) will be:

$$-Q_i + N_{iX} - N_{iX} f - F_{цiX} + F_{цi+1X} + Q_{i+1X} = 0 . \quad (2)$$

Balance of forces on the axis Y ($\sum Y = 0$) will be:

$$G_{зi} + G_{зi+1} - F_{цiY} - N_{iY} - N_{iY} f + Q_{i+1Y} - F_{цi+1Y} = 0 . \quad (3)$$

At $\sum X = 0$ and $\sum Y = 0$, the gear chain plates will be stationary. If $\sum X = 0$ and $\sum Y \neq 0$, then the plates will move along the tooth profile.

[1] GOST 13552–81. USSR State Standard. Driving toothed chains. Specifications. USSR State Standards Committee. Moscow: Standartinform; 1987. 12 p. (In Russ.)

Let us determine the forces acting on the chain links. Fig. 2 shows: P — the center of mass of the gear chain link; $d_к$ — the diameter of the circle of the contact point K of the package of plates with the profile of the sprocket tooth, $d_к = \frac{d_b}{cos\theta}$; d_o — the diameter of the dividing circle of the sprocket, $d_o = \frac{t}{sin\frac{180}{z}}$.

Along X-axis:	Along Y-axis:
$N_{iX} = N_i \cdot \sin\beta$,	$N_{iY} = N_i \cdot \cos\beta$,
$N_{iX} f = N_i f \cdot \cos\beta$,	$N_{iY} f = N_i f \cdot \sin\beta$,
$F_{цiX} = F_{цi} \cdot \sin\varphi$,	$F_{цiY} = F_{цi} \cdot \cos\varphi$,
$F_{цi+1X} = F_{цi+1} \cdot \sin\varphi$,	$F_{цi+1X} = F_{цi+1} \cdot \cos\varphi$,
$Q_{i+1X} = Q_{i+1} \cdot \cos\varphi$,	$Q_{i+1Y} = Q_{i+1} \cdot \sin\varphi$.

Here, β — the angle of the tooth profile of the involute sprocket; $\varphi = \frac{\pi}{z}$ — half of the tooth pitch angle of the involute sprocket.

Values of angular parameters:

$$\lambda = \varphi + \gamma, \tag{4}$$

$$\mu = \pi - \gamma - \theta_к, \tag{5}$$

$$\nu = \frac{\pi}{2} + \gamma + \theta_к, \tag{6}$$

$$\rho = \frac{\pi}{2} - \varphi + \theta_к, \tag{7}$$

$$\beta = \frac{\pi}{2} - \varphi - \gamma. \tag{8}$$

Here, γ — half of the tooth wedge angle of the involute sprocket.

$$\gamma = \alpha_к - \theta_k. \tag{9}$$

$$\alpha_к = \arccos\frac{d_b}{d_к}. \tag{10}$$

$\theta_к$ — the angle between the axis OO_{i+1} of the tooth and the line connecting point K_1 to the center O of the sprocket (see Fig. 2).

$$\theta_k = \frac{S_k}{d_к}. \tag{11}$$

S_k — the thickness of the involute sprocket tooth at the point of contact K and the gear chain (see Fig. 1). $d_b = mzcos\alpha$ — the diameter of the main circle of the involute sprocket, where m — the sprocket module for the toothed chain transmission, z — the number of sprocket teeth.

Substituting all the values included in the equations (2) and (3), we obtain:

$$2G_з - F_{цi} \cdot \cos\varphi - N_i f \cdot \sin\beta - N_i \cdot \cos\beta + Q_{i+1} \cdot \sin\varphi - F_{цi+1} \cdot \cos\varphi = 0, \tag{12}$$

$$Q_{i+1} = \frac{(F_{цi} \cdot \cos\varphi + N_i \cdot \cos\beta + N_i f \cdot \sin\beta + F_{цi+1} \cdot \cos\varphi - 2G_з)}{\sin\varphi}, \tag{13}$$

$$Q_i = N_i \cdot \cos\beta - N_i f \cdot \cos\beta - F_{цi} \cdot \sin\varphi + F_{цi+1} \cdot \sin\varphi + Q_{i+1} \cdot \cos\varphi = 0. \tag{14}$$

Transforming the equations (13) and (14), we obtain the coupling coefficient B_i of the toothed chain with a given involute sprocket tooth:

$$B_i = \frac{Q_i}{Q_{i+1}} = \sin\varphi\left(\frac{N_i \cdot \cos\beta - N_i f \cdot \cos\beta - F_{ц\,i} \cdot \sin\varphi + F_{ц\,i+1} \cdot \sin\varphi + Q_{i+1} \cdot \cos\varphi}{F_{ц\,i} \cdot \cos\varphi + N_i \cdot \cos\beta + N_i f \cdot \sin\beta + F_{ц\,i+1} \cdot \cos\varphi - 2G_з}\right). \qquad (15)$$

If the tension value of the driving branch of the chain transmission S_1 is known, then the tension of the driven branch S_2 is determined with account for the overall coefficient of the chain – sprocket tooth coupling:

$$B_z = \frac{S_1}{S_2}, \qquad (16)$$

$$S_2 = \frac{S_1}{B_z}, \qquad (17)$$

$$B_z = B_0 \cdot B_1 \cdot B_2 \ldots B_n. \qquad (18)$$

Here, B_z — the overall coupling coefficient of the chain and the sprocket; n — the number of teeth of the driving involute sprocket.

Research Results. To verify the calculations, a stand[1] protected by a patent of the Russian Federation was designed and manufactured (Fig. 3). Bench tests of the chain transmission under study were carried out. The parameters:

1) electric motor power $N_{эл}$ = 12 kW,

2) toothed chain pitch $t_ц$ = 19.05 mm,

3) the number of teeth of the driving and driven sprocket $z_1 = z_2 = 23$,

4) speed of the driving and driven sprocket $n_1 = n_2 = 1000$ rpm.

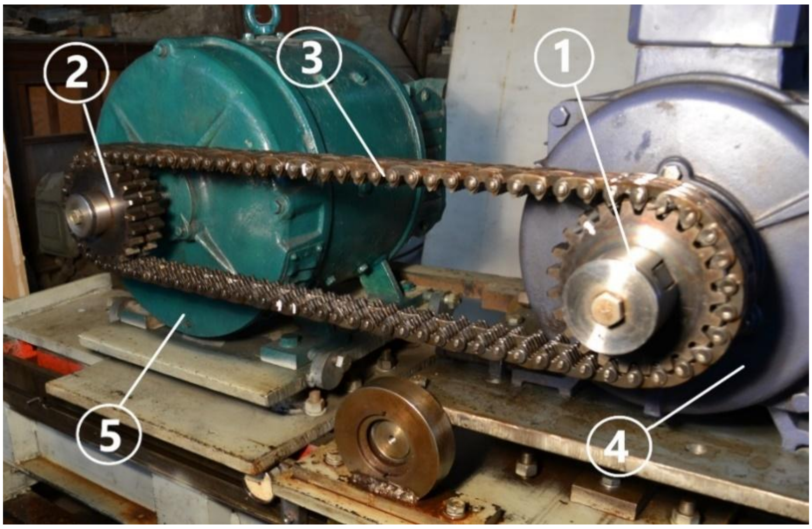

Fig. 3. Transmission test bench with toothed chain and involute sprockets:
1— driving sprocket, 2 — driven sprocket, 3 — toothed chain, 4 — electric motor, 5 — generator

For the bench tests, calculations were performed with account for the reference data[2, 3, 4, 5]. The results are presented in Table 1.

[1] Berezhnoy SS, et al. Test bench for chain gears. RF Patent no. 147242, 2014. (In Russ.)
[2] Chernavsky SA, et al. Design of mechanical gears: study guide for technical colleges. Moscow: Al'yans; 2008. 590 p. (In Russ.).
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Table 1

The obtained data for determining the traction capacity of a toothed chain transmission

Parameter	Notation and dimension	Numerical value
Tension of the working transmission branch	S_1, Н	4662
Tension of the idle transmission branch	S_2, Н	270
Total coupling coefficient of the chain and the leading involute sprocket	B_1	17.2
Centrifugal force of the chain	$F_ц$, Н	29.82

The proposed approach to determining the coupling coefficient has confirmed the correctness of the force calculation with account for the centrifugal forces and the friction coefficients of the plate package and the sprocket tooth.

Discussion and Conclusions. In this paper, all the forces acting in the engagement of the chain plate package and the sprocket tooth are determined. The procedure of power calculation of a toothed chain transmission is presented. The dependences for determining the impact of centrifugal forces and friction coefficients on the final calculation of the coupling coefficient are obtained. To confirm the theory, the chain transmission was studied on a test bench.

So, the centrifugal forces and the meshing friction coefficients significantly affect the traction capacity of the transmission with a toothed chain and involute sprockets.

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About the Authors:

Berezhnoy, Sergey B., professor of the Land Transport and Mechanics Department, Kuban State Technological University, (2, Moskovskaya St., Krasnodar, 350072, RF), Academician of the Engineering Academy of the Russian Federation, Dr.Sci. (Eng.), professor, Scopus ID: 25227504200, ORCID: https://orcid.org/0000-0001-5013-7345, beregnoy@kubstu.ru

Kurapov, Georgy V., associate professor of the of the Land Transport and Mechanics Department, Kuban State Technological University, (2, Moskovskaya St., Krasnodar, 350072, RF), Cand.Sci. (Eng.), ResearcherID: AAO-8482-2020, Scopus ID: 57219551108, ORCID: https://orcid.org/0000-0001-8101-265X, kurapov_georgii@mail.ru

Claimed contributorship

S. B. Berezhnoy: academic advising; the research results analysis of the power transmission calculation with account for new variables; formulation of conclusions. G. V. Kurapov: performing calculations according to the proposed method; conducting experiments on the test bench; processing of the results; creating the final version of the text.

All authors have read and approved the final manuscript.