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Study on free oscillations of a micromechanical gyroscope taking into account the nonorthogonality of the torsion axes

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Introduction. The paper is devoted to the study on free oscillations of the sensing element of a micromechanical R-R-type gyroscope of frame construction developed by the Kuznetsov Research Institute of Applied Mechanics, taking into account the nonorthogonality of the torsion axes. The influence of the instrumental manufacturing error on the accuracy of a gyroscope on a movable base in the case of free oscillations is studied. The work objective was to improve the device accuracy through developing a mathematical model of an R-R type micromechanical gyroscope, taking into account the nonorthogonality of the torsion axes, and to study the influence of this error on the device accuracy. The urgency of the problem of increasing the accuracy of micromechanical gyroscopes is associated with improving the accuracy of inertial navigation systems based on micromechanical sensors.

Materials and Methods. A new mathematical model that describes the gyroscope dynamics, taking into account the instrumental error of manufacturing the device, and a formula for estimating the error of a gyroscope, are proposed. The dependences of the state variables obtained from the results of modeling and on the basis of the experiment are presented. Methods of theoretical mechanics and asymptotic methods, including the Lagrange formalism and the Krylov-Bogolyubov averaging method, were used in the research.

Results. A new mathematical model of the gyroscope dynamics, taking into account the nonorthogonality of the torsion axes, is developed. The solution to the equations of small oscillations of the gyroscope sensing element and the estimate of the precession angle for the case of a movable base are obtained. A comparative analysis of the developed model and the experimental data obtained in the case of free oscillations of the gyroscope sensing element with a fixed base is carried out. The analysis has confirmed the adequacy of the constructed mathematical model. Analytical expressions are formed. They demonstrate the fact that the nonorthogonality of the torsion axes causes a cross-influence of the amplitudes of the primary vibrations on the amplitudes of the secondary vibrations of the sensing element, and the appearance of an additional error in the angular velocity readings when the gyroscope is operating in free mode.

Discussion and Conclusions. The results obtained can be used to improve the device accuracy using the algorithm for analytical compensation of the gyroscope error and the method for identifying the mathematical model parameters.

Keywords: gyroscope R-R type, gyro precession, gyro error estimation, micromechanical gyroscope, free oscillations.

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Introduction. The development of high-precision micromechanical inertial sensors, including micromechanical gyroscopes (MMG), used to solve navigation problems and control the movement of aircraft and mobile robots, is an urgent task of instrument engineering [1]. The advantages of MMG include small weight and size, as well as low cost compared to gyroscopes based on other physical principles. However, the main disadvantages of MMG are the variability of its metrological characteristics and the low accuracy of measuring the parameters of the angular motion of the object (angular rate and angle of rotation). The principle of operation of vibrating gyroscopes is based on the property of the Foucault pendulum to keep the plane of small vibrations motionless in inertial space [2].

Fundamentals of the theory of gyroscopes of the generalized Foucault pendulum class, which include MMG, are given in [2–5]. They describe various design schemes for the construction of MMG, and investigate the influence of instrumental manufacturing errors and changing operating conditions on the dynamics of the gyroscope. The principal feature of the gyroscopes of the generalized Foucault pendulum class is nonlinearity due to the finite vibrations of the sensing elements (SE) or physical nonlinearity associated with the features of the vibration control system [2–6].

Studies on the MMG dynamics and design were also published in the works of foreign authors [6–9]. For example, in publications [6, 9], a formula for estimating gyro drifts was obtained, based on the use of a developed mathematical model of motion that describes a slow change in the toroidal coordinates of the SE vibrations. In [7, 8], the issues on manufacturing MMG are discussed, and the equations of its small vibrations are analyzed. In paper [7], the equations of the MMG motion with angular (R-R-type) and linear (L-L-type) oscillatory types of the SE movement are compiled. In the above paper, a comparative analysis of the dynamics of such devices is carried out within the framework of linear models, and recommendations are given on the selection of MMG parameters, based on the conditions for increasing sensitivity and ensuring the required bandwidth, as well as the requirements for the linearity of the scale factor.

When designing MMG, the developers tend to use the phenomenon of internal resonance in the system, due to the combination of the natural frequencies of the SE vibrations [3, 4]. However, it is noted in [7, 8] that errors in the manufacturing technology, unknown and unpredictable deviations of structural elements from the design positions cause additional errors in the device measurements.

To improve the accuracy of the measurement of the angular rate of the MMG, the objective is set: to study free oscillations (in the absence of control) of the R-R-type MMG SE, taking into account the effects arising from the nonorthogonality of the torsion axes. This defect appears due to the imperfection of the manufacturing technology of the device. The tasks of developing a new mathematical model of the MMG dynamics considering the nonorthogonality of the torsion axes, evaluating the device drift, and describing the effect of the nonorthogonality of the torsion axes on the dynamics of the MMG SE, are set.

Materials and Methods. A model design of an R-R-type vibrating MMG — a design with an intermediate frame in accordance with the classification from the source is considered [3]. The kinematic scheme of the gyroscope (Fig. 1) is implemented in the form of a two-degree gimbal of the SE.

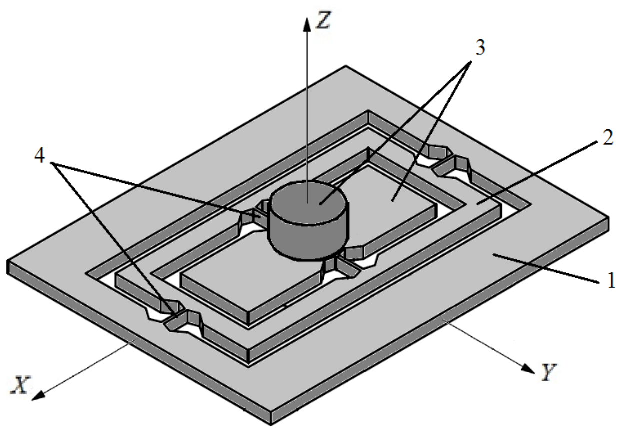

Fig. 1. Design diagram of the device: 1) base (body); 2) intermediate (external) frame; 3) sensing element consisting of balanced plate and inertial mass; 4) torsion bars

To describe the SE position, we introduce coordinate systems (Fig. 2) associated with: the device body — $OXYZ$; with the external frame of the elastic suspension of the gyroscope — $Ox_1y_1z_1$; with the balanced plate ——

$Oxyz$. Moreover, OZ is the axis of sensitivity of the gyroscope, and the coordinate system $Ox_2y_2z_2$ differs from the system $Ox_1y_1z_1$ by turning by a constant angle of nonorthogonality of all torsions. In the presented systems, the origin of coordinates corresponds to point O and is located in the geometric center of the balanced plate.

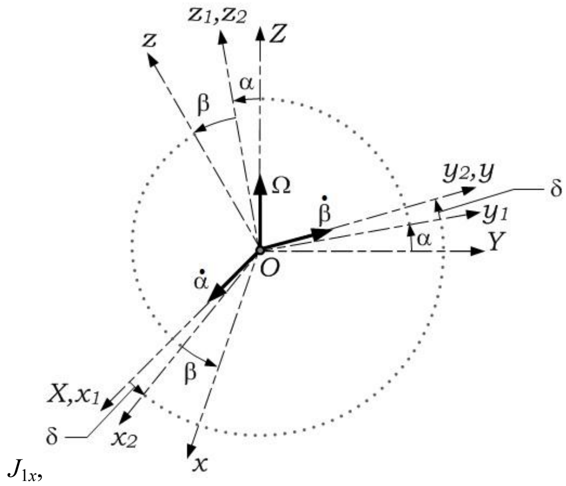

Fig. 2. Coordinate systems

In the system $Ox_1y_1z_1$ we will set the axial moments of inertia of the intermediate frame J_{1x}, J_{1y}, J_{1z}, and in the system $Oxyz$ — the axial moments of inertia of the SE J_{2x}, J_{2y}, J_{2z}. Note that in this paper, the axes of the coordinate systems $Ox_1y_1z_1$ and $Oxyz$ are considered the main central axes of inertia of the intermediate frame and SE, respectively.

When modeling the motion of the SE, the assumption is made that the torsion design provides infinite bending stiffness. The position of the SE relative to the base of the MMG is described by two generalized coordinates — angles α and β, as well as a small constant angle δ, that characterizes the nonorthogonality of the torsion axes (Fig. 2). The relative position of the coordinate systems is determined by a sequence of elementary rotations:

$$OXYZ \xrightarrow[x]{\alpha} Ox_1y_1z_1 \xrightarrow[z_1]{\delta} Ox_2y_2z_2 \xrightarrow[y_2]{\beta} Oxyz,$$

where under each arrow, the axis is indicated around which there is a counterclockwise rotation by the angle indicated above the corresponding arrow.

We will set up the equations of the dynamics of the MMG SE in the form of Lagrange equations of the 2nd kind [10, 11]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\alpha}}\right) - \frac{\partial L}{\partial \alpha} = -\frac{\partial \Phi}{\partial \dot{\alpha}}, \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\beta}}\right) - \frac{\partial L}{\partial \beta} = -\frac{\partial \Phi}{\partial \dot{\beta}}, \tag{1}$$

where $L = T - \Pi$ — the Lagrange function; T and Π — kinetic and potential energies of the system, respectively; Φ — dissipative function that characterizes the loss on internal friction. The expressions for these values have the form:

$$\begin{aligned} T &= \frac{1}{2}\left(J_{2x}\omega_x^2 + J_{2y}\omega_y^2 + J_{2z}\omega_z^2\right) + \frac{1}{2}\left(J_{1x}\dot{\alpha}^2 + \left(J_{1y}\sin^2\alpha + J_{1z}\cos^2\alpha\right)\Omega^2\right), \\ \Phi &= \frac{1}{2}d_\alpha\dot{\alpha}^2 + \frac{1}{2}d_\beta\dot{\beta}^2, \quad \Pi = \frac{1}{2}c_\alpha\alpha^2 + \frac{1}{2}c_\beta\beta^2, \end{aligned} \tag{2}$$

where d_α, d_β — friction coefficients; c_α, c_β — torsion stiffness coefficients.

The expressions for the projections ω_x, ω_y, ω_z of the SE angular rate on the movable axes x, y, z have the form:

$$\begin{aligned} \omega_x &= \dot{\alpha}\cos\beta\cos\delta - \Omega\cos\alpha\sin\beta + \Omega\sin\alpha\cos\beta\sin\delta, \\ \omega_y &= \dot{\beta} + \Omega\sin\alpha\cos\delta - \dot{\alpha}\sin\delta, \\ \omega_z &= \dot{\alpha}\sin\beta\cos\delta + \Omega\cos\alpha\cos\beta + \Omega\sin\alpha\sin\beta\sin\delta. \end{aligned} \tag{3}$$

Given the smallness of angles α, β and δ, the trigonometric functions in expressions (2) and (3) from these angles can be replaced by Taylor series expansions, limiting the summands to the first order of smallness. Then we obtain the equations of small vibrations from the equations of motion (1), written with account for the expressions (2) and (3).

Using the Lagrange formalism [10] in the case of a constant angular rate of the base, we obtain the equations of small oscillations of the SE, written up to the terms of the first order of smallness in the form:

$$\ddot{\alpha}+\omega_\alpha^2\alpha=j_1\Omega\dot{\beta}-\frac{\omega_\alpha}{Q_\alpha}\dot{\alpha}+\frac{j_1}{j_2}\delta\ddot{\beta},\qquad \ddot{\beta}+\omega_\beta^2\beta=-j_2\Omega\dot{\alpha}-\frac{\omega_\beta}{Q_\beta}\dot{\beta}+\delta\ddot{\alpha},\tag{4}$$

where the following notation is introduced (similar to how it is performed in paper [12]):

$$j_1=\frac{J_{2x}+J_{2y}-J_{2z}}{J_{1x}+J_{2x}},\quad j_2=\frac{J_{2x}+J_{2y}-J_{2z}}{J_{2y}},\quad \omega_\alpha=\sqrt{\frac{c_\alpha}{J_{1x}+J_{2x}}},$$

$$\omega_\beta=\sqrt{\frac{c_\beta}{J_{2y}}},\quad Q_\alpha^{-1}=\frac{d_\alpha}{\omega_a\left(J_{1x}+J_{2x}\right)},\quad Q_\beta^{-1}=\frac{d_\beta}{\omega_\beta J_{2y}}.$$

Here, j_1, j_2 — dimensionless moments of inertia of the elastic suspension; ω_α, ω_β and Q_α, Q_β — natural frequencies of vibrations and Q-values at corners α, β, respectively.

When deriving the oscillation equations (4), the angular rate of the gyroscope body Ω was considered small relative to the natural frequency ω_α, т.е. $|\Omega| << \omega_\alpha$, and angle δ was also assumed to be a small value, i.e., $\delta << 1$. Note that in equations (4), the terms are dropped due to the presence of geometric nonlinearity of the MMG. The influence of the nonlinearity of the geometry of the SE motion on the dynamics of the R-R-type MMG is described in monograph [5].

Taking into account that the right-hand sides of equations (4) are small perturbations, i.e., $\ddot{\alpha}+\omega_\alpha^2\alpha=O(\varepsilon)$, up to the terms of the first order of smallness, we can write: $\ddot{\alpha}=-\omega_\alpha^2\alpha+O(\varepsilon)$. Thus, the second derivatives of angles α and β are excluded from the right part of equations (4).

We consider the case of an isotropic elastic suspension, i.e., the equality of natural oscillation frequencies and equal Q-factors:

$$\omega_\alpha=\omega_\beta=\omega_0,\qquad Q_\alpha=Q_\beta=Q,$$

where ω_0 — the characteristic value of the natural frequency of vibrations; Q — the characteristic value of Q-factor.

It should be noted that the case of a difference in quality $\left(Q_\alpha\neq Q_\beta\right)$ and a small difference in frequency $\left(\omega_\alpha\neq\omega_\beta\right)$ under studying free oscillations of the MMG SE is considered in paper [12]. With the entered notation and the accepted assumptions, we write down the equations of the SE motion with an accuracy of the first-order terms of smallness in a dimensionless form:

$$\begin{aligned}&\ddot{\alpha}+\omega_0^2\alpha=j_1\Omega\dot{\beta}-Q^{-1}\omega_0\dot{\alpha}-\frac{j_1}{j_2}\delta\omega_0^2\beta,\\&\ddot{\beta}+\omega_0^2\beta=-j_2\Omega\dot{\alpha}-Q^{-1}\omega_0\dot{\beta}-\delta\omega_0^2\alpha.\end{aligned}\tag{5}$$

Note that the system of equations (5) is reduced to the standard form of writing a regularly perturbed system of differential equations with one fast angular variable [13, 14]. One of the most common ways to find solutions to regularly perturbed systems is to use asymptotic motion separation methods [13–16].

The solution to nonlinear equations (5) is obtained using the Krylov-Bogolyubov averaging method [14], and we will use Van der Pol variables as slowly changing variables [13] p_1, q_1, p_2, q_2:

$$\alpha=p_1\sin(\omega_0t)+q_1\cos(\omega_0t),\qquad \dot{\alpha}=\omega_0p_1\cos(\omega_0t)-\omega_0q_1\sin(\omega_0t),$$

$$\beta=p_2\sin(\omega_0t)+q_2\cos(\omega_0t),\qquad \dot{\beta}=\omega_0p_2\cos(\omega_0t)-\omega_0q_2\sin(\omega_0t).$$

Using the averaging procedure [15, 16] over an explicitly incoming time, we obtain an averaged system of differential equations solved with respect to the derivatives of slow Van der Pol variables:

$$\begin{aligned} p_1' &= -\frac{1}{2}Q^{-1}p_1 + \frac{j_1\Omega}{2\omega_0}p_2 - \frac{j_1\delta}{2j_2}q_2, \quad q_1' = -\frac{1}{2}Q^{-1}q_1 + \frac{j_1\Omega}{2\omega_0}q_2 + \frac{j_1\delta}{2j_2}p_2, \\ p_2' &= -\frac{1}{2}Q^{-1}p_2 - \frac{j_2\Omega}{2\omega_0}p_1 - \frac{\delta}{2}q_1, \quad q_2' = -\frac{1}{2}Q^{-1}q_2 - \frac{j_2\Omega}{2\omega_0}q_1 + \frac{\delta}{2}p_1. \end{aligned} \tag{6}$$

The dash in equations (6) indicates the differentiation in dimensionless time $\tau = \omega_0 t$.

The resulting model in the form of linear differential equations describes the free oscillations of a gyroscope SE on a movable base. The solution to the system of equations (6) can be written as:

$$\begin{aligned} p_1(\tau) &= \exp\left(-\frac{\tau}{2Q}\right)\left[p_{10}\cos\left(\sqrt{\nu^2+\gamma^2}\tau\right) + \sqrt{\frac{j_1}{j_2}}\cdot\frac{(\nu p_{20}-\gamma q_{20})}{\sqrt{\nu^2+\gamma^2}}\sin\left(\sqrt{\nu^2+\gamma^2}\tau\right)\right], \\ q_1(\tau) &= \exp\left(-\frac{\tau}{2Q}\right)\left[q_{10}\cos\left(\sqrt{\nu^2+\gamma^2}\tau\right) + \sqrt{\frac{j_1}{j_2}}\cdot\frac{(\gamma p_{20}+\nu q_{20})}{\sqrt{\nu^2+\gamma^2}}\sin\left(\sqrt{\nu^2+\gamma^2}\tau\right)\right], \\ p_2(\tau) &= \exp\left(-\frac{\tau}{2Q}\right)\left[p_{20}\cos\left(\sqrt{\nu^2+\gamma^2}\tau\right) - \sqrt{\frac{j_2}{j_1}}\cdot\frac{(\nu p_{10}+\gamma q_{10})}{\sqrt{\nu^2+\gamma^2}}\sin\left(\sqrt{\nu^2+\gamma^2}\tau\right)\right], \\ q_2(\tau) &= \exp\left(-\frac{\tau}{2Q}\right)\left[q_{20}\cos\left(\sqrt{\nu^2+\gamma^2}\tau\right) - \sqrt{\frac{j_2}{j_1}}\cdot\frac{(\nu q_{10}-\gamma p_{10})}{\sqrt{\nu^2+\gamma^2}}\sin\left(\sqrt{\nu^2+\gamma^2}\tau\right)\right] \end{aligned} \tag{7}$$

where $p_{10} = p_1(0), p_{20} = p_2(0), q_{10} = q_1(0), q_{20} = q_2(0)$ — initial conditions; dimensionless angular rate of the device base $\nu = \sqrt{j_1 j_2}\Omega/(2\omega_0)$; parameter characterizing the nonorthogonality of the torsion axes $\gamma = \sqrt{j_1}\delta/\left(2\sqrt{j_2}\right)$.

The second terms in formulas (7) characterize the cross-influence of primary vibrations on secondary vibrations and vice versa. Note that in the case of orthogonal torsion axes, when $\gamma = 0$, solution (7) coincides with the results of paper [5].

The obtained analytical solutions (7) of the oscillation equations are of interest for the development of methods for identifying parameters, as well as predicting the gyro drift and considering it when using the method of algorithmic error compensation.

Research Results. To validate the developed model, we compare the simulation results calculated from formulas (7) and experimental data. The measurement information was obtained using an observation system. As the measurement information of electrostatic sensors, we have Van der Pol variables p_1, q_1, p_2, q_2.

In the experiment, a sample device with the following parameters of a mathematical model with a fixed base ($\nu = 0$): $Q = 3856$, $j_1 = j_2 = 1$, $\gamma = 0.2\cdot10^{-5}$, as initial conditions for the Van der Pol variables, values equal to the measurements at the initial moment of time were selected:

$$p_{10} = 13.467\cdot10^{-3}, \quad q_{10} = 20.429\cdot10^{-3}, \quad p_{20} = 0.787\cdot10^{-3}, \quad q_{20} = 1.172\cdot10^{-3}.$$

Parameter γ value corresponds to the angle of nonorthogonality of torsion axes δ equal to one angular second. A graphical representation of the dependencies of Van der Pol variables $p_1(\tau)$, $q_1(\tau)$, $p_2(\tau)$, $q_2(\tau)$ that slowly change over a dimensionless time is shown in Fig. 3.

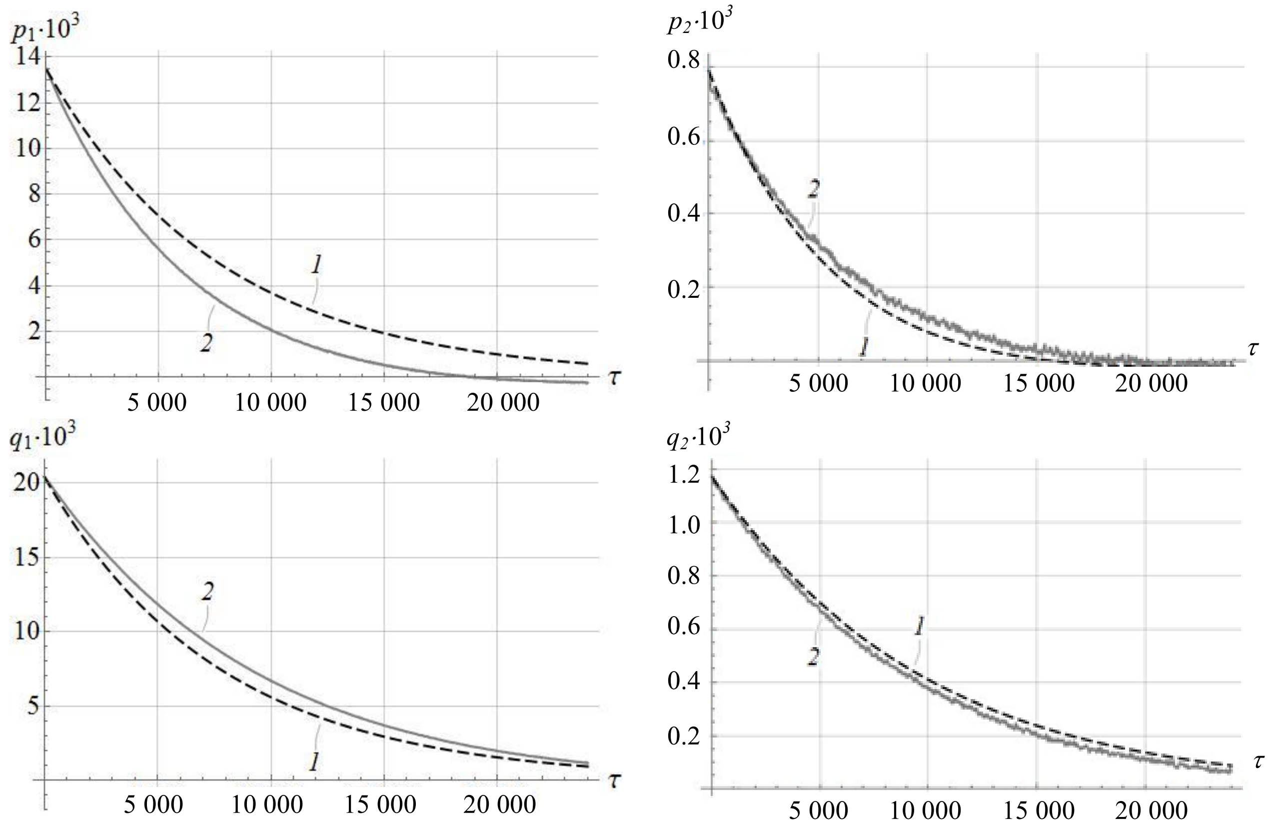

Fig. 3. Dependences of Van der Pol variables: 1 — simulation results; 2 — experimental data

The graphs (Fig. 3) show a significant coincidence of the dependencies for variables p_2, q_2, obtained from the simulation results with the experimental data. The dependences for variables p_1, q_1, obtained from the simulation results, are in qualitative agreement with the experimental data, and the observed small quantitative deviations may be due to nonlinear effects, such as the nonlinearity of the geometry of the SE motion [5], or the phenomena of variability of quality, variability of frequency, and the error of the inertial mass displacement [12]. Consideration of nonlinear effects affecting the dynamics of MMG in the construction of mathematical models of SE oscillations increases the accuracy of micromechanical sensors as part of the inertial navigation systems [2].

The gyro drift due to nonlinear effects and other instrumental errors will be estimated using the auxiliary functional I [5, 6, 9]:

$$I = \frac{2\sqrt{j_1 j_2}\left(q_1 q_2 + p_1 p_2\right)}{j_2\left(q_1^2 + p_1^2\right) - j_1\left(q_2^2 + p_2^2\right)}, \tag{8}$$

which is related to angle θ via the relation:

$$\theta = \frac{1}{2}\arctan\left(I\right).$$

Moreover, this parameter is proportional to the integral of the angular rate:

$$\theta = -\frac{\sqrt{j_1 j_2}}{2}\int_0^\tau \Omega\left(\tau_1\right)d\tau_1 .$$

Using formula (8), taking into account solution (7), it is possible to estimate the gyroscope drift associated with the nonorthogonality of the torsion axes, which arose due to the imperfection of the manufacturing technology. Figure 4 shows the dependences of functional I on the dimensionless time according to the results of the experiment and the calculation according to formula (8).

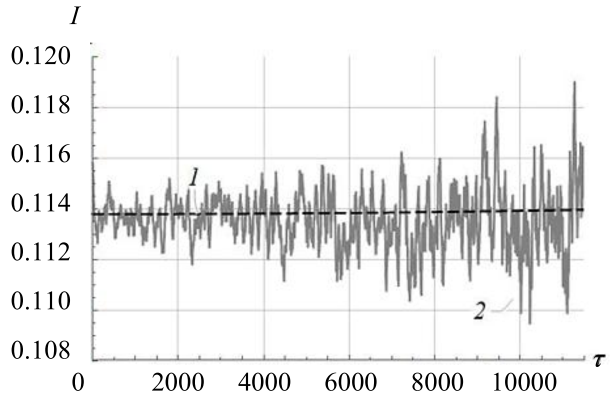

Fig. 4. Functional dependence $I(\tau)$: 1 — simulation results; 2 — experimental data

The proximity of the drift estimate (Fig. 4) and the Van der Pol variables (Fig. 3) obtained analytically with experimental data characterize good accuracy of the constructed model, especially if we take into account the fact that it neglected nonlinear effects, as well as the phenomena of variability of frequency and variability of quality. Despite these neglections, the model provides building methods for identifying parameters, with the help of which it is possible to clarify the dependences obtained under modeling. The application of methods for identifying the parameters of a mathematical model will cause an increase in the accuracy of the MMG in the forced oscillation mode, which is the operating mode of gyroscopes.

Discussion and Conclusions. A new mathematical model of the R-R-type MMG for the mode of free oscillations of the SE is constructed. The model takes into account the nonorthogonality of the torsion axes, which arises as a result of the technological impossibility to ensure high accuracy of the device manufacturing. The formula for estimating the precession angle with a movable base of the device is obtained. Through comparing the simulation results and the experimental data, the validation of the MMG mathematical model was carried out. It is shown that the nonorthogonality of the torsion axes causes a cross-influence of primary vibrations on the magnitude of secondary vibrations and vice versa. The research results can be used in the algorithm of analytical compensation of the gyroscope error to increase the accuracy of the MMG.

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Claimed contributorship

M. R. Saypulaev: basic concept formulation; research objectives and tasks; construction of a mathematical model and obtaining an analytical solution to the oscillation equations; conducting modeling; text preparation; formulation of conclusions. I. V. Merkuryev: academic advising; analysis of the research results; the text revision; correction of the conclusions. A. V. Solovyev: discussion of the research results; the text revision. A. N. Tarasov: verification of the comparative analysis of experimental data and simulation results; discussion of the research results.

All authors have read and approved the final manuscript.