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Analysis of the speed and curvature of the trajectory in the problem of pursuing a set of targets

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Introduction. A kinematic model of group pursuit of a set of targets on a plane is considered. Pursuers use a technique similar to parallel approach method to achieve goals. Unlike the parallel approach method, the speed vectors of pursuers and targets are directed arbitrarily. In the parallel approach method, the instantaneous directions of movement of the pursuer and the target intersect at a point belonging to the circle of Apollonius. In the group model of pursuing multiple goals, the pursuers try to adhere to a network of predictable trajectories.

Materials and Methods. The model sets the task of achieving goals by pursuers at designated points in time. This problem is solved by the methods of multidimensional descriptive geometry using the Radishchev diagram. The predicted trajectory is a composite line that moves parallel to itself when the target moves. On the projection plane "Radius of curvature — speed value", the permissible speed range of the pursuer is displayed in the form of level lines (these are straight lines parallel to one of the projection planes). Images of speed level lines are displayed on the projection plane "Radius of curvature — time to reach the goal". The search for points of intersection of the speed line images and the appointed time level line is being conducted. Along the communication lines, the values of the intersection points are lowered to the plane "Radius of curvature — speed value". Using the obtained points, we construct an approximating curve and look for the intersection point with the line of the assigned speed. As a result, we get values of the radius of the circle at the predicted line of the trajectory of the pursuer.

Results. Based on the results of the conducted research, test programs have been created, and animated images have been made in the computer mathematics system.

Discussion and Conclusions. This method of constructing trajectories of pursuers to achieve a variety of goals at a given time values can be in demand by developers of autonomous unmanned aerial vehicles.

Keywords: multidimensional analysis, Radishchev plot, target, pursuer, trajectory, radius of curvature.

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Introduction. We consider a model for calculating the trajectory of the pursuer on a plane, where at each moment of time, a predicted trajectory from the pursuer to the target is built, and the pursuer will try to stick to it (Fig. 1).

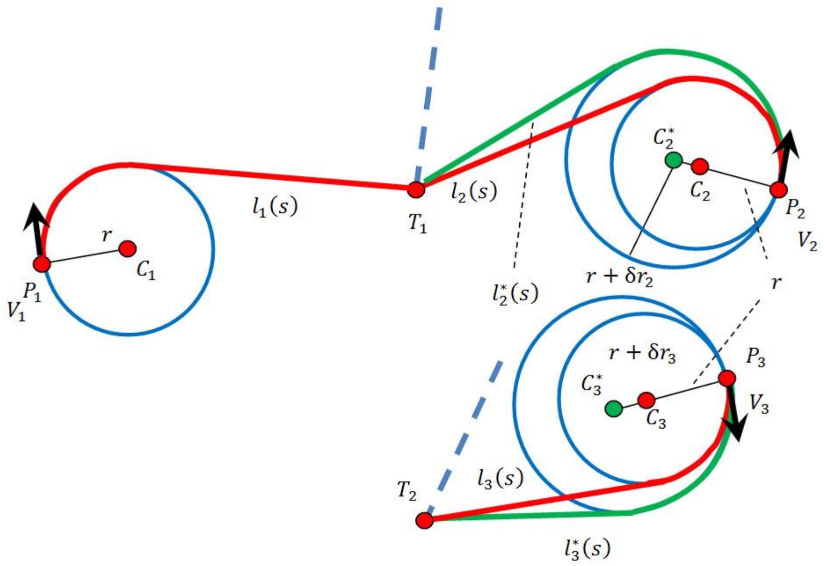

Fig. 1. Group pursuit of multiple goals

Curves $l_1(s)$, $l_2(s)$, $l_3(s)$ consist of a circular arc segment and a straight-line segment. In our model, the radius of the circles is the curvature constraint of the predicted trajectories of the pursuers[1,2,3].

The research task in this paper can be described as follows: pursuers, moving along a one-parameter network of predicted trajectories, must achieve their goals at designated times, including simultaneously. Methods of multidimensional descriptive geometry using the Radishchev diagram are selected for the solution. The one-parameter network (Fig. 2) consists of congruent lines of parallel transport. Each line is an analog of the line of sight (it is a straight line connecting the pursuer and the target).

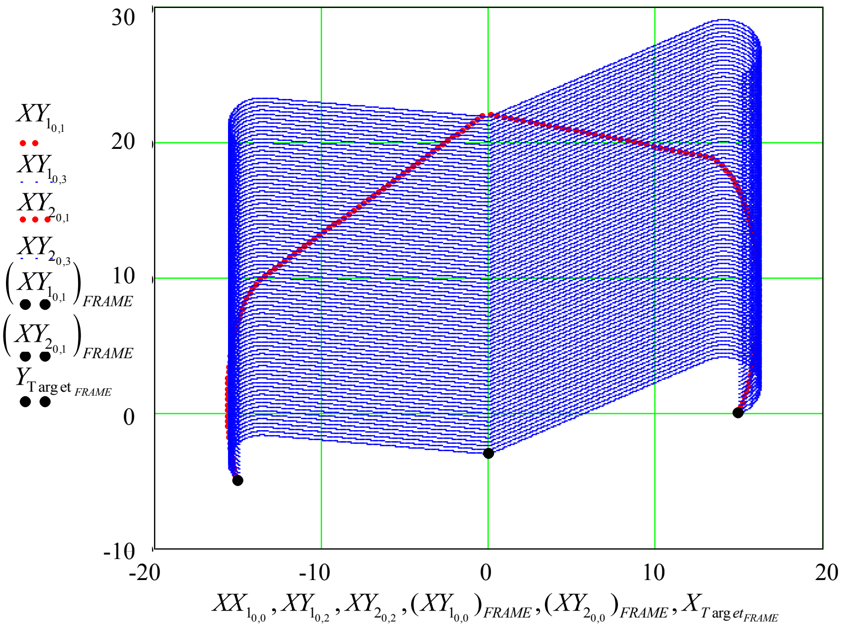

Fig. 2. One-parameter networks of predicted trajectory of pursuers

A combination of a circle and a straight line is selected for the model. There are many options for providing curvature constraints (Cornu spiral, a cubic parabola conjugating with a straight line, etc.). To achieve the goals, the pursuers use a method similar to the parallel approximation method [1–4]. However, in our case, the speed of the

[1]Bannikov AS. A nonstationary group pursuit problem. Lobachevskie chteniya: Proc. 5th Youth Sci. School-Conf. Trudy matematicheskogo tsentra im. N. I. Lobachevskogo. Kazan: Izd-vo Kazanskogo matematicheskogo obshchestva; 2006. P. 26–28. (In Russ.)

[2]Izmestyev IV, Ukhobotov VI. Pursuit problem of low-maneuverable objects with a ring-shape terminal set. In: Proc. Int. Conf. Ryazan: Publ. House of RSU named for S. Yesenin; 2016. P. 17–18. (In Russ.)

[3]Borie R, Tovey C, Koenig S. Algorithms and Complexity Results for Pursuit-Evasion Problems. In: Proc. Int. Joint Conf. on Artificial Intelligence (IJCAI), 2009. P. 59–66.

pursuers is directed arbitrarily, and in the method of parallel approximation, the speed lines of the pursuer and the target intersect at a point on the Apollonian circle.

In the test program, written using the materials of the paper, objects move around a square $[-30:30] \times [-30:30]$. The calculation is made in meters. The studies were carried out for speeds of 20 m/s. The initial radius of the circles with the predicted trajectories was 2 m. If at the moment of the beginning of the pursuit, the pursuer was at point P_i with speed vector V_i, then the center of circle C_i of radius r_i will be at point:

$$C_i = P_i \pm r_i \cdot \frac{\begin{bmatrix} -V_{i_y} \\ V_{i_x} \end{bmatrix}}{|V_i|}.$$

Then, from the point of target position T_i , a tangent to circle (C_i, r_i) is constructed. The combination of the tangent and the circle is the baseline of the predicted trajectory of pursuer $l_i(s)$. Note that in the equation of the baseline from a one-parameter set of predicted trajectories, parameterization is performed from the arc length.

At the new position of target T_i, line $l_i(s)$ shifts while remaining parallel to itself (Fig. 3).

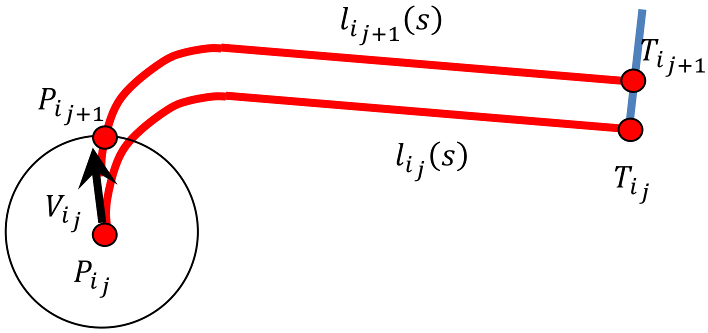

Fig. 3. Iterative process of calculating the trajectory of the pursuer

Assume that the *i*-th pursuer at the moment t_j is at point P_{i_j}, while having a predictable trajectory $l_{i_j}(s)$ between the current position of target T_{i_j}. In this case, the next point of the pursuer trajectory will be $P_{i_{j+1}}$.

$P_{i_{j+1}}$ — point of intersection of line $l_{i_{j+1}}(s)$, which corresponds to the position of target $T_{i_{j+1}}$ at the next moment in time t_{j+1} and the circle with center P_{i_j} and radius $\left|V_{i_j}\right| \cdot \Delta t$, $\Delta t = t_{j+1} - t_j$. This is the model for constructing the trajectories of the pursuer [4, 5].

Consider the task of group pursuit, when a group of pursuers catches up with a group of targets. We assume that each pursuer P_i seeks to achieve its goal T_i, although some pursuers may have the same goals (Fig. 1, 2).

Moreover, pursuer P_i reaches goal T_i in a certain time t_i, moving at a certain speed V_i. To achieve the goals simultaneously, it is required that all t_i are equal to a certain value.

Figure 1 shows that to change the length of the baseline, you can change the radius of the tangent circle. The tangent is introduced so that the pursuer can smoothly switch to a straight trajectory. If this were the case, then the task would be reduced to the pursuit by the parallel approximation method.

The initial speed of the pursuer is directed arbitrarily, which provides using the parallel approximation method with respect to curvature constraints (Fig. 2). For this, a composite baseline is used, which, when the target moves, remains parallel to itself; there is a smooth transition to the parallel approximation method with respect to curvature constraints (Fig. 3). Figure 2 is supplemented with a link to an animated image where you can see a smooth transition to parallel approximation[6].

The purpose of this paper is to describe the method by which the pursuer reaches the goal at the appointed time from the acceptable values. We can also consider the simultaneous achievement of goals by a group of pursuers [5–9].

[4]Dubanov AS, Seveen A-K. Kinematic model of the parallel approximation method: state registration certificate of computer software. Banzarov Buryat State University. RU 2020665641. No. 2020664886, 2020. (In Russ.)

[5]Dubanov AS, Seveen A-K. Modeling of trajectory of the pursuer on the surface using the parallel approximation method: state registration certificate of computer software. Banzarov Buryat State University. RU 2020666553. No. RU 2020666553, 2020. (In Russ.)

[6] Dubanov A. Dognat' odnovremenno. Ploskost' 1. URL: https://www.youtube.com/watch?v=7VNHNwCbWrg (accessed: 22.05.2021). (In Russ.)

Materials and Methods. Based on the research results, a test program for simultaneous achievement of goals by pursuers has been developed, which can be viewed at the author's resource. The proposed algorithm implements an iterative scheme for calculating the trajectory of the pursuer (Fig. 3).

The model assumes a dependence for pursuer P, who reaches goal T in time t:

$$t = F(P_s, T_s, n_P, n_T, V_P, V_T, R).$$

Here, P_s, T_s — coordinates of the position points of the pursuer and the target at the moment of the beginning of the pursuit; n_P, n_T — unit vectors of the direction of movement of the pursuer and the target at the time of the beginning of the pursuit; V_P, V_T — speed modules of the pursuer and the target during the pursuit; R — radius of the circle, whose meaning is shown in Fig. 1, 3.

In fact, the model calculates the number of steps for which the pursuer reaches the goal. With a known discrete time interval, the number of steps can be compared to real time.

If the target moves rectilinearly and uniformly, then the time-to-target dependence in the already started iterative process can be considered a function of two variables — the speed modulus of the pursuer and the radius of curvature of the circle:

$$t = F(V_P, R).$$

In the model, it is assumed that the pursuer moves at constant speed V_P, but nothing prevents us from changing the values of the speed modulus, as well as the radius of curvature. Let us assume that the speed module takes discrete values from the series $V_{P_i}, i \in [1\colon N]$, and the radii of the circles in Fig. 1, 3 take the values $R_j, j \in [1\colon M]$.

For further research, the Radishchev diagram is used, where coordinate planes (R, V) and (R, t) are used (Fig. 4).

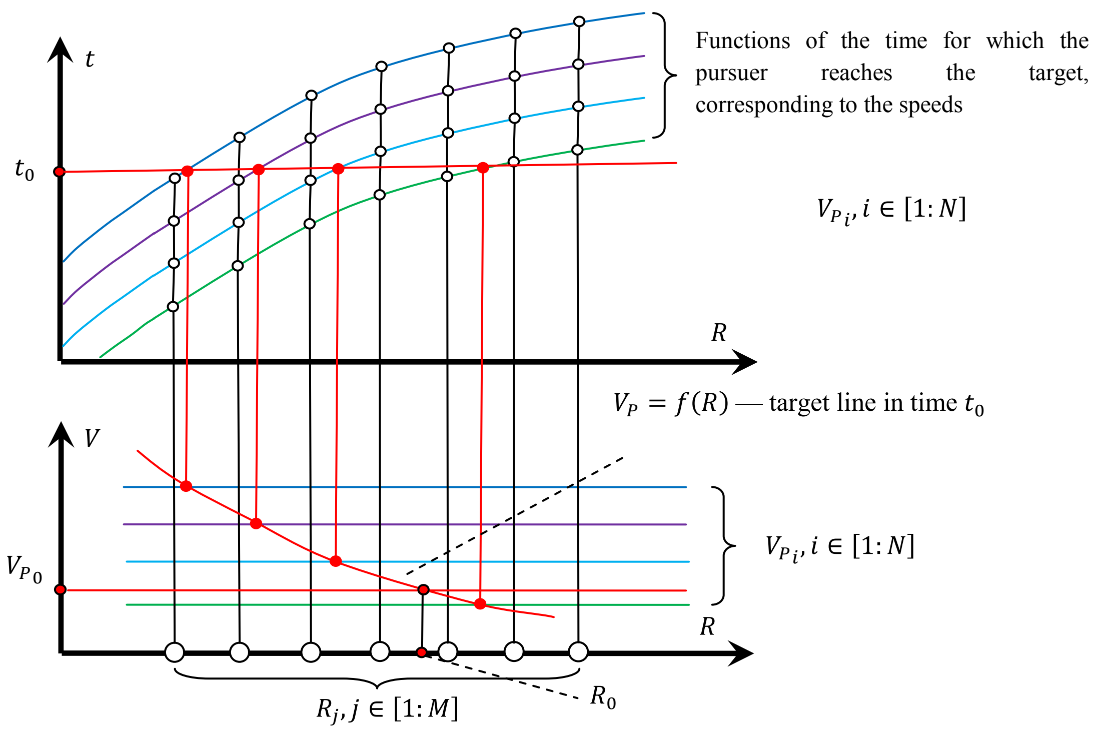

Fig. 4. Determination of the radius of the circle on the Radishchev diagram

Figure 4 shows the experimental construction of time dependences $t_{i,j} = F\left(V_{P_i}, R_j\right)$. Graphs on the plane$(R, t)$ show how the time to reach the target depends on the radius of the circle R at fixed speed value V_P.

As one of the optimizing factors [10–11] on the plane (R, t), equality $t = t_0$ is selected, where t_0 — the required time to achieve the target. Next, to solve our problem on plane (R, V), equality $V_P = V_{P_0}$ is selected as the second optimizing factor, where V_{P_0} — the constant speed of the pursuer.

The problem statement says that the speed module of the pursuer is unchanged. Nevertheless, the constructed series of speed values is required for calculating the radius of the circle R_0 on the projection plane(R, V).

Along the communication lines on the projection plane (R, V), there are corresponding points of intersection with the lines of the speed level ${V_P}_i$(Fig. 4). According to the obtained points, a polynomial regression is performed in the test program, and as a result, we get a function of the dependence of the pursuer speed on the radius of the circle at which the target is achieved in time t_0.

Then we seek the intersection point of function $V_P = f(R)$ with the line of the level $V_P = {V_P}_0$. Abscissa of the intersection point R_0 is the desired radius of the circle, at which pursuer P reaches target T during time t_0 at speed ${V_P}_0$.

The calculation is carried out under the condition that the target moves uniformly and rectilinearly. If the target changes direction or speed, then a new radius of the circle of the composite baseline is calculated (analogous to the line of sight of the parallel approximation method), a new time of reaching is set at the same speed of the pursuer.

With a uniform and rectilinear movement of the target, the lowest limit of the time of achievement is fixed when the speed of the pursuer is directed to point K on the Apollonius circle (Fig. 5). This position was considered in the works of R. Aizeks [12], L. S. Pontryagin [13], L. A. Petrosyan [14-16], N. N. Krasovsky and A. I. Subbotin [17].

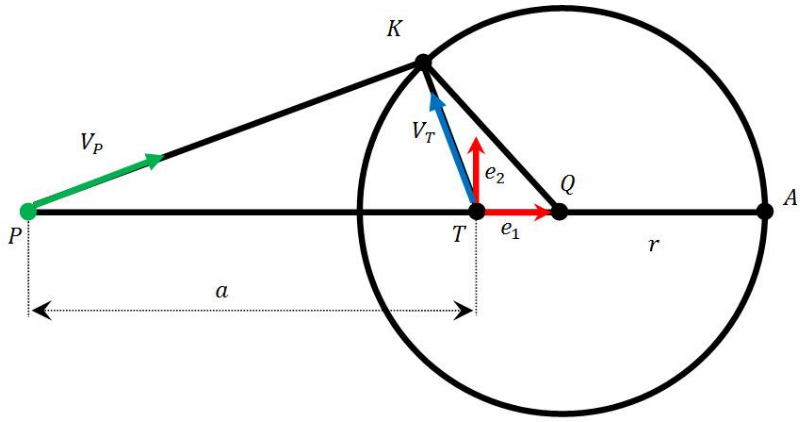

Fig. 5. The circle of Apollonius

The Apollonius circle is a geometric place of points, the ratio of the distances from which to two given points is a constant value, other than unity: $|PK|/|TK| = |V_P|/|V_T|$ (Fig. 5).

When considering the multiple pursuit of a group of targets in the test program, a preliminary calculation of the trajectories of the pursuers is made with the given initial parameters. From the times of achieving targets, the largest time is selected for calculating simultaneous achievement, and it will be the criterion for calculating the trajectories of the other pursuers[7, 8]. This moment is illustrated in an animated image, where three pursuers achieve two targets simultaneously[9].

Figure 6 shows how a shorter time was set for one of the pursuers to reach the target. Figure 6 is also supplemented with a link to an animated image where you can see the achievement of targets at different designated times[10, 11, 12].

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[8] Dubanov AS, Seveen A-K. Parallel approach model on plane of group of pursuers with simultaneous achievement of the goal: state registration certificate of computer software. RU 2021618920. No. 2021614416, 2021. (In Russ.)
[9] Dubanov A. NM 1. URL: https://www.youtube.com/watch?v=tdbgoNoby3A (accessed: 22.05.2021).
[10] Dubanov A. NM 3. URL: https://www.youtube.com/watch?app=desktop&v=F6MTsWZL2BY&feature=youtu.be (accessed: 22.05.2021).
[11] Dubanov A. NM 2. URL: https://www.youtube.com/watch?v=NNJDJOJT34I (accessed: 17.08.2021).
[12] Dubanov A. NM 1. Ibid. (accessed: 17.08.2021).

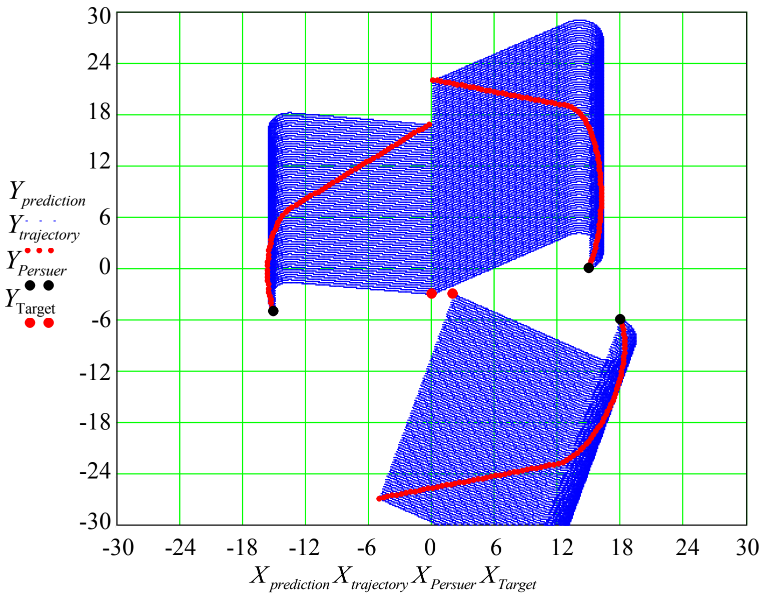

Fig. 6. Achieving targets at different designated times

Research Results. Figure 7 shows some results of multivariate analysis in the problem of simultaneous achievement of the target by two pursuers.

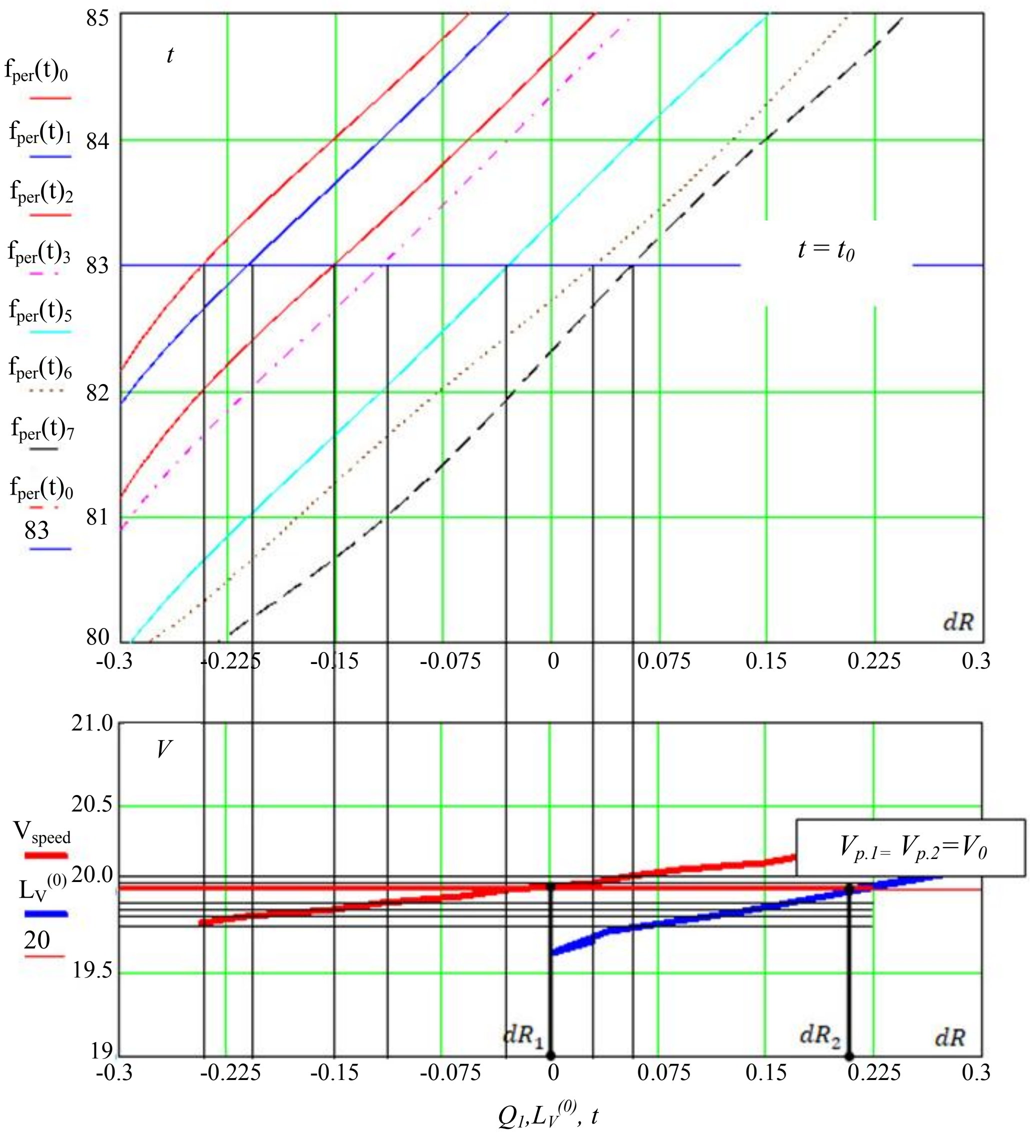

Fig. 7. Multivariate analysis results in the problem on simultaneous target achievement by two pursuers

The target moves rectilinearly and uniformly. A number of permissible speeds was built for each pursuer. The permissible values of the radius of the circle are varied using the discrete variable dR (scale in Fig. 7).

We construct a one-parameter network of lines on the projection plane(dR, t) . Each line corresponds to a certain speed value and expresses the dependence of the time-to-target on the increment of the radius of the circle. Figure 7 shows a one-parameter network of speed lines of one of the pursuers. A similar network was built for the second one in the test program of the multivariate analysis.

For each pursuer, the first optimizing factor is selected [10–11], which is responsible for simultaneous achievement:

$$t = t_0.$$

Here, t_0 — the longest of the times of reaching the target if the pursuers independently caught up with the target under the same initial conditions.

On the plane (dR, t) , we seek the intersection points of the lines of the level $t = t_0$ with the speed lines of the one-parameter network. The intersection points are found using the built-in procedures for solving equations. In the MathCAD computer mathematics system, this can be *root* procedure. Values dR and V on the projection plane (dR, V) correspond to the found intersection points.

The built-in polynomial regression procedure is applied to the obtained points on the projection plane, and the characteristic curve of the speed dependence on the radius of the circle of the composite baseline is found (Fig. 1).

On the projection plane (dR, V), Figure 7 shows the same characteristic line of speed dependence for another pursuer. Then, the second optimizing factor $V_1 = V_2 = V_0$ is applied. In the test program, objects move at the same speeds. The built-in means of computer mathematics find the points of intersection with the line of the level $V = V_0$. Values dR_1 and dR_2 correspond to these points.

When starting the iterative process, the simultaneous achievement of the target by two pursuers was recorded (the animation to Fig. 2). At the same time, the values of the increments dR_1 and dR_2 to the initial radius of the circle were found and set:

— value of the time to reach the target t_0,

— speed modules V_0, s.

The paper describes a method for achieving multiple goals by a group of pursuers with the ability to set the time to achieve them. Simultaneous achievement of goals is a particular result of this approach, which the method of parallel approximation develops. When implementing the method in space, it should be ensured that the vectors of the pursuer and the target are in the same plane[13].

Let us consider the case of pursuit in three-dimensional space under the following conditions: we want to reduce the problem to the method of parallel approximation, but the speed of the pursuer is directed arbitrarily. In this case, the baseline of the predicted trajectories of the pursuer movement should be built in the plane formed by the line of sight and the speed of the pursuer.

The next step of the pursuer is the intersection point of a sphere with a radius equal to the step of the pursuer, and the baseline, moved in parallel so that one end of it is aligned with the point of the target position.

Let us turn to the question of finding the circle of Apollonius and point K in three-dimensional space. The circle itself will be in the plane formed by the line of sight and the speed of the target. We define such parameters of the Apollonian circle as the center of the circle (point Q), the radius of the circle r, the Apollonian point (A) and point K. This end, the target speed vector, the speed module of the pursuer, the positions of the pursuer, and the target, are taken into account. There is an analytical solution to this problem in a flat coordinate system (see Fig. 5). The coordinate center is located at the point of the target position. The abscissa vector will be a unit vector along the line of sight connecting the positions of the pursuer and the target. The ordinate vector will be perpendicular to the abscissa vector, but in the plane formed by the line of sight and the target speed vector.

[13] Dubanov A. Simultaneous achievement of the goal on a plane. Geometrical modeling in MathCAD. URL: http://dubanov.exponenta.ru (accessed: 22.05.2021). (In Russ.)

Discussion and Conclusions. The multidimensional descriptive geometry methods used in this work are based on the variation of the speed modules and the radii of curvature of circles. At the same time, according to the conditions of the problem, the speed modules of the pursuers are unchanged.

The paper takes into account the results achieved in works [18, 19, 20].

The proposed approach makes it possible to analyze the modules of speeds and directions of the initial movement.

The model of four-dimensional space presented in the papers of V. P. Bolotov (Bolotov's hypergraph)[14] should be used to analyze:

— speed modules,

— radii of circles adjacent to the pursuers,

— initial directions of movement of the pursuers.

The research results presented in the paper can be in demand by developers of unmanned aerial vehicles that perform group coordinated tasks. The role of the guidance operator can be reduced to specifying goals and monitoring the performance of tasks.

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The author has read and approved the final manuscript.