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Original article

On the Construction of Mathematical Models of the Membrane Theory of Convex Shells

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Abstract

Introduction. The paper considers the issues of constructing mathematical models of the momentless equilibrium stress state of elastic convex shells using methods of the complex analysis. At the same time, shells with a piecewise smooth (ribbed) lateral surface were considered for the first time. The work objective was to find classes of shells for which it is possible to build meaningful mathematical models.

Materials and Methods. Using the methods of the theory of the discontinuous Riemann-Hilbert problem for generalized analytic functions, a criterion for the unconditional solvability of the corresponding static problem for the equilibrium equation of a convex shell with a ribbed lateral surface has been obtained. This criterion, combined with the methods of the theory of generalized analytical functions, is a tool for constructing mathematical models of the state of momentless stress equilibrium of elastic convex shells.

Results. A method has been developed for constructing mathematical models of the momentless equilibrium stress state of a convex shell under the action of a variable external load and the condition of stress concentration at the corner points of the median surface. The introduction of a vector parameter, as well as the concepts of “order of quasi-correctness” and “quasi-stability”, into the boundary condition provided both quantitative and qualitative comparison of mathematical models. Classes of shells have been found for which the description of mathematical models is given in terms of the geometry of the boundary in the vicinity of the corner points of the median surface. The obtained result, when applied to shallow convex shells, provides a geometric criterion of quasi-stability. It is established that for a shallow shell, which is not quasi-stable, the only adequate mathematical model is a probabilistic one.

Discussion and Conclusions. The proposed method for constructing a two-parameter family of problems with a modified boundary condition makes it possible to simulate the momentless equilibrium stress state for fairly wide classes of convex shells with a piecewise-smooth lateral surface under a sleeve connection. At the same time, the developed algorithm for calculating the boundary condition index allowed us to answer the question of the existence of an adequate mathematical model for a shell with a side surface of an arbitrary configuration, and for shells of a special type (specifically, shallow or shells of revolution), to formulate a geometric criterion for the existence of a mathematical model.

Keywords: thin elastic shell, generalized analytic function, Riemann-Hilbert problem, index of boundary value condition, mathematical model.

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Научная статья

К вопросу о построении математических моделей мембранной теории выпуклых оболочек

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Аннотация

Введение. В работе рассмотрены вопросы построения математических моделей безмоментного состояния напряженного равновесия упругих выпуклых оболочек с использованием методов комплексного анализа. При этом впервые рассмотрены оболочки с кусочно-гладкой (ребристой) боковой поверхностью. Целью работы являлось отыскание классов оболочек, для которых возможно построение содержательных математических моделей.

Материалы и методы. С помощью методов теории разрывной задачи Римана-Гильберта для обобщённых аналитических функций получен критерий безусловной разрешимости соответствующей статической задачи для уравнения равновесия выпуклой оболочки с ребристой боковой поверхностью. Этот критерий в сочетании с методами теории обобщённых аналитических функций представляет собой инструмент построения математических моделей состояния безмоментного напряжённого равновесия упругих выпуклых оболочек.

Результаты исследования. Разработан метод построения математических моделей безмоментного состояния напряжённого равновесия выпуклой оболочки при действии переменной внешней нагрузки и условии концентрации напряжений в угловых точках срединной поверхности. Введение в граничное условие векторного параметра, а также понятий «порядок квазикорректности» и «квазиустойчивость» позволяют провести как количественное, так и качественное сравнение математических моделей. Найдены классы оболочек, для которых описание математических моделей дается в терминах геометрии границы в окрестности угловых точек срединной поверхности. Полученный результат в применении к пологим выпуклым оболочкам позволяет дать геометрический критерий квазиустойчивости. Установлено, что для пологой оболочки, не являющейся квазиустойчивой, единственной адекватной математической моделью является вероятностная.

Обсуждение и заключения. Предлагаемый метод построения двухпараметрического семейства задач с модифицированным граничным условием позволяет моделировать состояние безмоментного напряженного равновесия для достаточно широких классов выпуклых оболочек с кусочно-гладкой боковой поверхностью при условии втулочной связи. При этом разработанный алгоритм вычисления индекса граничного условия позволяет ответить на вопрос о существовании адекватной математической модели для оболочки с боковой поверхностью произвольной конфигурации, а для оболочек специального вида (например, пологих или оболочек вращения) — сформулировать геометрический критерий существования математической модели.

Ключевые слова: тонкая упругая оболочка, обобщенная аналитическая функция, задача Римана-Гильберта, индекс граничного условия, математическая модель.

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Introduction. The issues considered in the paper were studied and described in the works of I. N. Vekua [1, 2] and A. L. Goldenveizer [3, 4], who initiated the application of methods of the theory of generalized analytical functions to the momentless (membrane) theory of thin elastic shells and the theory of surface bending. To date, the final results in this direction have been obtained for convex shells with a smooth edge (i.e., with a smooth boundary of its median surface). The most significant of them — the correctness and quasi-correctness of the key problem with a static boundary condition with simply connected and multiply connected median surfaces — are consequences of the fact that the index of the corresponding Riemann-Hilbert problem is an invariant of the connectivity of the surface. The author's application of I. N. Vekua's methods to the problems of the theory of convex shells with a piecewise smooth edge[1][5, 6] recognized a connection between the "geometry" of the median surface in the vicinity of its angular point and the picture of the solvability of the corresponding Riemann-Hilbert problems with a discontinuous coefficient of the boundary condition. The use of these methods in [7–9] allowed us to obtain an effective formula for the index under some additional geometric conditions on the angular points of the surface, and, as a consequence, the geometric criterion of quasi-correctness of the main boundary problem.

The purpose of this work is to construct mathematical models of the momentless equilibrium stress state of convex shells with ribbed side surfaces based on the geometric criterion of quasi-correctness of the main boundary problem.

Materials and Methods. Let S be a simply connected surface of a given class of regularity [7] with a piecewise smooth edge $L=\bigcup_{j=1}^{n} L_j$ and angulat points p_i $(i=1,...,n)$. Let us define on S along L, piecewise continuous vector field $\mathrm{r}=\{\alpha(s),\beta(s)\}$, which admits discontinuities of the first kind at points p_j, where $\alpha(s)$, $\beta(s)$ ($\alpha^2+\beta^2=1$, $\beta\geq 0$) — tangential and normal components, s — a natural parameter, and we denote by J the homeomorphism of surface S_0 to complex plane $z=x+iy$ given in [9]. Let area $D=J(S)$ be the image of the surface when mapped to plane z with boundary Γ and angular points $q_i=J(p_i)$. Consider the following problem (task R): to find in domain D, complex-valued solution $w(z)$ of the equation:

$$w_{\bar z}(z)-B(z)\overline{w}(z)=F(z),\quad z\in D, \tag{1}$$

satisfying the Riemann–Hilbert condition

$$\mathrm{Re}\{\lambda(\zeta),w(\zeta)\}=\gamma(\zeta),\quad \zeta\in\Gamma, \tag{2}$$

where

$$\lambda(\zeta)=s(\zeta)\left[\beta(\zeta)t(\zeta)-\alpha(\zeta)s(\zeta)\right], \tag{3}$$

$s(\zeta)=s_1(\zeta)+is_2(\zeta)$, $t(\zeta)=t_1(\zeta)+it_2(\zeta)$, $i^2=-1$, s_i, t_i $(i=1,2)$ – real-valued functions, complex-valued functions $\gamma(\zeta)$, $\lambda(\zeta)$ Hölder functions on each of the arcs $\Gamma_j=J(L_j)$, $w_{\bar z}=\frac{1}{2}(w_x+iw_y)$, $B(z)$, $F(z)$ — functions of class $L_r(D)$, $r>2$. Here, it is assumed that the solutions of class $W^{1,r}$ at the points of discontinuity q_i admit "integrable infinity", i.e., they admit an estimate $|w(z)|<\mathrm{const}\cdot|z-q_i|^{-\alpha_j}$, $0<\alpha_j<1$. Following [10], the class of such solutions is denoted by H^*.

As is known [11], the static boundary value problem of the momentless theory for an elastic convex shell with a ribbed side surface in the mathematical formulation is problem R, where $w(z)$ — a complex stress function, $F(z)$ — a complex-valued function of the external load. In this case, the condition $w\in H^*$ is equivalent to the stress concentration condition at the corner points of the median surface. We will construct a mathematical model of the equilibrium state of the shell based on the results on the solvability of problem R for a surface of a special type

[1] Tyurikov EV. Goldenveizer Generalized Boundary Value Problem for Momentless Spherical Domes. In: Proc. XIV Int. Conf. "Modern Problems of Continuum Mechanics". Rostov-on-Don. P. 290–293. (In Russ.)

(canonical dome [9]). To simplify the presentation, we assume that at each corner point, the direction of one of the arcs coincides with one of the main directions k_2 (k_1) and call the dome 2-canonical (1-canonical). Problem R for canonical dome K is called canonical if the direction of field r at each corner point p is the direction of the generalized tangent [7, p. 46]. Let us introduce the notation: δ_i^2 — ratio of the corresponding principal curvatures k_1, k_2 at point p_i $(0<\delta_i<1)$, $p(\nu_i)$ — corner point p_i with internal angle ν_i, $T(\nu_i)$ — the set (*sector*) of directions of the *generalized* tangent at this point, T — the set of continuous on L vector fields r, defining the direction of the generalized tangent at each corner point $p(\nu_j)$. As established in [7], canonical problem R is quasi-correct for any field $r\in T$, if $n\geq 2$. Problem R is family R^r of problems (1)–(3), each of which is given by the selection of vector field r. Following I. N. Vekua [1], we will call problem R^r s-quasi-correct in class H^*, if it is unconditionally solvable in this class, and its solution depends on s real arbitrary constants (s — *the order of quasi-correctness*).

Definition 1. Canonical problem R *is called quasi-stable with respect to the field of directions of a generalized tangent if problem* R^r *is* s*-quasi-correct for any field* $r\in T$.

Remark 1. Owing to the theorem on the solvability of the Riemann–Hilbert problem for generalized analytic functions [11], problem R is quasi-stable if and only if index κ of problem R is an invariant of field $r\in T$. Here, the index of the corresponding conjugation problem with coefficient $\Lambda(\zeta)=\bar{\lambda}(\zeta)\lambda^{-1}(\zeta)$ is called the index of the problem. In the case of $\kappa\geq -1$, the order of quasi-correctness is $s=\kappa+1$.

The technique [6, 8] of calculating the index of a boundary condition of form (2) is used below, as well as the concept [10] of *special node* p_i of problem (1), (2) or *special point* $q_i=J(p_i)$ of discontinuity of the boundary condition (2). Following [6], the direction of the generalized tangent at a corner point is called *special* if the corresponding point of the discontinuity of boundary condition (2), (3) is *a special node* of problem *R*.

Definition 2. Corner point $p(\nu)$ *is called an instability point of problem* R, *if sector* $T(\nu)$ *contains a special direction.*

We introduce the notation: ν, σ — one-sided limits at corner point $p(\nu)$ of the unit vector tangent to L, where vector σ sets main direction k_2 on the surface at point p, and inside corner ν is given by pair $(-\nu,\sigma)$.

Statement 1. If the direction of vector r *at point* $p(\nu)$ *coincides with the direction of vector* ν, *then point* $q=J(p)$ *is a special node of boundary condition (2) if and only if:*

$$\nu=\arccos\frac{1}{1+\delta}. \tag{4}$$

If vector v is replaced by vector σ, *then:*

$$\nu=\operatorname{arcctg}\sqrt{t}, \tag{5}$$

Where t — the only positive root of the equation:

$$2\sqrt{\frac{1+\delta^2 t}{\delta^2+t}}+\frac{1+\delta^2 t}{\delta^2+t}-4\sqrt{\frac{E}{K\left(1+t^2\right)+4Et}}=\frac{1}{t}\ . \tag{6}$$

Here $E=\left(\frac{k_1-k_2}{2}\right)^2$, $K=k_1k_2$ [12, p.164]. *At that,* $\arccos\frac{1}{1+\delta}<\operatorname{arcctg}\sqrt{t}$.

We denote

$$\theta=\arccos\frac{1}{1+\delta},\ \mu=\operatorname{arcctg}\sqrt{t}\ . \tag{7}$$

The consequence of statement 1 is statement 2.

Statement 2. Corner point $p(\nu)$ *is a point of instability of problem* R *if and only if:*

$$\theta\leq\nu\leq\mu\,. \tag{8}$$

Corner point $p(\nu)$ is a 1-type (2-type) point if condition $0<\nu<\theta$ $\left(\mu<\nu\le\frac{\pi}{2}\right)$ is met. As established in [8], k-type $(k=1,2)$ point is a point of stability. If $p(\nu)$ — is the point of instability, then the only *special* direction r_0 of the generalized tangent divides sector $T(\nu)$ into two connected sets $T^1(\nu)$ and $T^2(\nu)$.

Statement 3. The index of problem R in class H^* is calculated from the formula:

$$\kappa=-4+\sum_{i=1}^{n}(4-\kappa_i),\tag{9}$$

where n — number of corner points of the boundary, $\kappa_i=m$ for point of stability $p(\nu)$ of m-type $(m=1,2)$, and $\kappa_i=s$ in case $\mathrm{r}\in T^{(s)}(\nu)$ $(s=1,2)$ for the point of instability $p(\nu)$.

Remark 2. Formula (9) for the index of problem R in class H_0 of bounded solutions, according to [10], takes the form:

$$\kappa=-4+\sum_{i=1}^{n}(3-\kappa_i).\tag{10}$$

The condition that the solution to problem R belongs to class H^* is the boundedness condition of the integral of the shell stretching energy [13, p. 83] in the vicinity of the corner point.

From formulas (9), (10), statement 4 follows.

Statement 4. If boundary L contains a single 1-type corner point, then problem R is for sure solvable in class H^* and has a unique solution $\forall\mathrm{r}\in T$; if $n\ge 2$, $n=n_1+n_2$, where n_k — the number of corner points of k-type $(k=1,2)$, then, problem R is quasi-stable with respect to $\mathrm{r}\in T$ with the order of quasi-correctness $s=2n_1+n_2-3$.

If boundary L contains instability points, then according to (9), problem R in class H^* is not quasi-stable with respect to field $\mathrm{r}\in T$. However, it is possible to distinguish such classes of fields with respect to which problem R is quasi-stable with different orders of quasi-correctness. Classes of such fields can be set by choosing the direction of the generalized tangent in only one of the sectors $T^{(1)}(\nu)$, $T^{(2)}(\nu)$ at each point of instability $p(\nu)$.

Remark 3. As it is easy to see, formulas (9), (10) together with statement 4 are valid in the case:

$$\mu<\nu\le\frac{\pi}{2}+\omega^2,\tag{11}$$

where ω — a sufficiently small value given by surface S. At this, the condition of smallness ω is not mandatory. For example, for umbilical $(k_1=k_2)$ point $p(\nu)$, it is enough to put $\omega^2=\frac{\pi}{6}$.

Let us now consider 1-canonical dome K. In this case, the description of the special nodes of boundary condition (2) is also given by statement 2, with the only difference that in equality (4) and equation (6), value $\delta^2=\frac{k_2}{k_1}<1$ must be replaced by $\delta^2=\frac{k_1}{k_2}>1$. Here, for corner point $p(\nu)$ of instability of problem R, inequality (8) takes the form:

$$\mu\le\nu\le\arccos\frac{1}{1+\delta}.\tag{12}$$

Based on the obvious graphical analysis of equation (6), we conclude:

1° value $\mu=\operatorname{arcctg}\sqrt{t}$ is a function of two parameters, namely: $\mu=\mu(\delta,k_1)$, $k_1\in(0;+\infty)$, if $\delta^2=\frac{k_2}{k_1}<1$, and $\mu=\mu(\delta,k_2)$, $k_2\in(0;+\infty)$, if $\delta^2=\frac{k_1}{k_2}>1$;

2° $\lim\limits_{\delta\to1-0}\mu(\delta,k_1)=\frac{\pi}{3}$, $\lim\limits_{\delta\to0}\mu(\delta,k_1)=\frac{\pi}{2}$ $\forall k_1\in(0;+\infty)$, if $\delta^2=\frac{k_2}{k_1}$;

3° $\lim\limits_{\delta\to1+0}\mu(\delta,k_2)=\frac{\pi}{3}$, $\lim\limits_{\delta\to+\infty}\mu(\delta,k_2)=0$ $\forall k_2\in(0;+\infty)$, if $\delta^2=\frac{k_1}{k_2}$;

4° for any fixed δ from the right (left) semineighborhood of the unit, function μ as a function of argument k_2 (k_1) is a slowly changing function [14] in the sense that $\mu(\delta_0, k_j) \in \left(0, \frac{\pi}{2}\right)$ for any fixed $\delta = \delta_0$, $k_j \in (0; +\infty)$ $(j = 1,2)$.

Research Results. We submit for consideration a family of surfaces S_τ, $\tau \in [0, \varepsilon)$, where τ — a small parameter, each of which is the median surface of a thin elastic shell V_τ from some family $\{V_\tau\}$, where V_0 and S_0 coincide with shell V and surface S, respectively. We assume that for $\forall \tau \in [0, \varepsilon)$, the following conditions are met:

1) surface S_τ is a canonical dome of the regularity class $W^{3,r}$, $r > 2$, with internal angles of magnitude ν_j at corner points p_i $(i = 1, ..., n)$ of boundary L_τ, where surface S_0 is a 2-canonical (1-canonical) dome S with boundary L;

2) ratios of principal curvatures $k_1^{(\tau)}$, $k_2^{(\tau)}$ of surface S_τ at corner points, coincide with values δ^2 at the corner points of surface S;

3) $k_j^{(\tau)} = k_j + \varepsilon_j(\tau)$, where $\lim_{\tau \to 0} \varepsilon_j(\tau) = 0$, $k_j = k_j^{(0)}$ $(j = 1,2)$.

Let J be the mapping of surface S_τ onto complex plane $z = x + iy$, given above, $D_\tau = J(S_\tau)$ — a family of simply connected regions with corresponding $\Gamma_\tau = J(L_\tau)$ and corner points $q_i = J(p_i)$, bounded in plane z. Consider a family of problems R_τ, $\tau \in [0, \varepsilon)$,

$$w_{\bar{z}}(z) - B_\tau(z)\overline{w}(z) = F_\tau(z), \quad z \in D_\tau, \tag{13}$$

$$\operatorname{Re}\{\lambda_\tau(\zeta) w(\zeta)\} = \gamma_\tau(\zeta), \quad \zeta \in \Gamma_\tau, \tag{14}$$

where functions $B_\tau(z)$, $\lambda_\tau(\zeta)$, $\gamma_\tau(\zeta)$ are defined by the middle surface S_τ of shell V_τ according to [8], and $\lambda_\tau(\zeta)$ has form (3).

It is assumed that the lateral surfaces of shells V_τ have common edges containing points p_i $(i = 1, ..., n)$. Note that problem (13), (14) $\forall \tau \in [0, \varepsilon)$ can be considered as a family of problems $R_\tau^{(r)}$, each of which is given by selecting a vector field on S_τ along L_τ.

Consider 1-canonical dome S. Statement 4 is true.

Statement 4. If in each of the corner points $p(\nu)$ of dome S, the following condition is met:

$$\nu < \arccos \frac{1}{1+\delta}, \tag{15}$$

then $\forall \tau \in [0, \varepsilon)$ problem R_τ is quasi-stable in class H^* with respect to the field of directions of a generalized tangent with the order of quasi-correctness $s = 3n - 3$. If at each point $p(\nu)$, the following condition is met:

$$\operatorname{arcctg} \sqrt{t} + \omega_0^2 < \nu < \frac{\pi}{2} + \omega^2, \tag{16}$$

where ω^2 is determined by remark 3, value $\omega_0 = \omega_0(\varepsilon)$ is given by family S_τ, $\tau \in [0, \varepsilon)$, then problem R_τ is quasi-stable in class H^* with the order of quasi-correctness $s = 2n - 3$.

For proof, it is enough to use statement 3, conditions 1–3 and the known properties [1, p. 97] of mapping J. Statement 4 remains valid for 2-canonical dome S, if conditions (15), (16) are replaced by the conditions:

$$\nu < \operatorname{arcctg} \sqrt{t} - \omega_0^2, \tag{17}$$

$$\arccos \frac{1}{1+\delta} < \nu < \frac{\pi}{2} + \omega^2 \tag{18}$$

respectively.

Now let us refine the concept of a corner point of k-type $(k = 1,2)$ of surface S. We say that point $p(\nu)$ of boundary Γ is a 1-type (2-type) point relative to family S_τ, $\tau \in [0, \varepsilon)$, if conditions (15) and (17) are met for 1-canonical and 2-canonical points, respectively, (conditions (16) and (18) for 1-canonical and 2-canonical points, respectively). Consider canonical dome S, each corner point of which is a point of 1-type or 2-type relative to the specified family S_τ. Then, the consequence of statement 4 is:

Statement 5. If p *and* q — the number of points of 1-type and 2-type, respectively $(p+q=n)$, *then* $\forall\tau\in[0,\varepsilon)$ *problem* R_τ *is for sure solvable in class* H^* *and quasi-stable with respect to directions* r *of the generalized tangent with the order of quasi-correctness* $s=3p+2q-3$. *Specifically, if* $p=0$, *then* $q\geq 2$.

The statement remains valid if class H^* is replaced by class H_0 of bounded in D_τ solutions, and order $s=3p+2q-3$ by order $s=2p+q-3$ provided $2p+q\geq 3$. Thus, problem R_τ is not unconditionally solvable in the following cases: $p=0$, $q\leq 2$ and $p=1$, $q=1$.

The case of an umbilical $(\delta=1)$ corner point $p(\nu)$, in which any direction is the main one, requires special consideration. In this case, according to[2] $\mu=\theta=\frac{\pi}{3}$, any direction of the generalized tangent is a special direction of problem R. However, even in this case, statement 4 remains valid if, for family $\{S_\tau\}$, we consider $\tau\neq 0$, and values ν, μ are replaced by value $\frac{\pi}{3}$.

Discussion and Conclusions. The results obtained can be used to construct mathematical models of thin and shallow shells of the positive Gaussian curvature with ribbed side surfaces. The most complete and advanced results of both linear and nonlinear elastic shell theory are obtained for thin shallow shells. A detailed discussion of the concept of "shallow shell", as well as a description of various versions of the theory is given in [15, p. 29]. The linear theory of flat convex curves was developed by I.N. Vekua [2, 16]. Within the framework of this theory, the issue of the realization of the equilibrium stress state of a shallow shell with a ribbed side surface, when a static boundary condition of a general form is met, is reduced to problem R discussed above.

Let P be a shallow shell [2, p. 164] with a ribbed side surface, S — its median surface with a piecewise smooth edge. We assume that at each corner point of surface S, the condition of *strong shallowness* $k_1\approx k_2$ is satisfied, which is equivalent to the following:

$$\delta\approx 1, \tag{19}$$

where δ — any of the ratios $\frac{k_1}{k_2}$, $\frac{k_2}{k_1}$. Condition (19) means that any corner point $p(\nu)$ should be considered both a 1-canonical and a 2-canonical point. Consider corner point $p(\nu)$, at which $\nu\approx\frac{\pi}{3}$, and a family of surfaces S_τ, $\tau\in[0,\varepsilon)$, given by conditions (1)–(3). According to properties 1°–4° of values θ, μ and statement 2, the corresponding problem R_τ $\forall\tau\in(0,\varepsilon)$ is not quasi-stable in class H^*, i.e., point $p(\nu)$ at $\nu\approx\frac{\pi}{3}$ — the point of instability of problem R_τ. In this case, the natural presumption is that in the right part of formula (10), value κ_i $(1\leq i\leq n)$ corresponding to point $p(\nu)$ is a discrete random variable with possible values 1 and 2. Thus, if at each corner point $p(\nu)$ of the median surface, the condition $\nu\approx\frac{\pi}{3}$ is met, then, owing to formula (9), the order of quasi-correctness of problem R is a discrete random variable taking values m, $m+1,\ldots,$ $m+n$, where n — the number of corner points, $m=2n-3$ for class of solutions H^*, and $m=n-3$ for class H_0.

Mechanics

[2] Tyurikov EV. Goldenveizer Generalized Boundary Value Problem for Momentless Spherical Domes.

The method proposed above can be used to construct mathematical models of the theory of thin shallow shells with ribbed side surfaces of any configuration. To do this, it is enough to use the results[3] on the solvability of problem R for spherical domes with a piecewise smooth edge. Let us consider for definiteness, the median surface S under the assumption that all the corner points $p(\gamma)$ of the boundary are "outgoing", that is, $\gamma < \pi$. In this case, a corner point on a spherical surface is a special node of the boundary condition if and only if $\gamma = \frac{\pi k}{3}$ $(k = 1,...,5)$. It follows that the "outgoing" corner point of the median surface of the shallow shell is an instability point if one of the following conditions is met: $\gamma \approx \frac{\pi k}{3}$ $(k = 1,2)$. Thus, formula (9) for the index can serve as validation for the following hypothesis:

if problem R for a shallow convex shell is unconditionally solvable in a given class of solutions, then its quasi-correctness order is a discrete random variable taking integer values K, $K+1,...,$ $K+N$, where N — the number of instability points, K — the number given by a set of corner points and a selection of continuous vector parameter r.

In conclusion, we note that the same reasoning can serve as validation for this hypothesis, but carried out for regular convex surfaces satisfying the condition of *local symmetry* [17] at corner points.

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