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*Research article*

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## Coupled Axisymmetric Thermoelectroelasticity Problem for a Round Rigidly Fixed Plate

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**Abstract**

***Introduction.*** To describe the operation of temperature piezoceramic structures, the theory of thermoelectroelasticity is used, in which the mathematical model is formulated as a system of nonself-adjoint differential equations. The complexity of its integration in general leads to the study of problems in an unrelated formulation. This does not allow us to evaluate the effect of electroelastic fields on temperature. The literature does not present studies on these problems in a three-dimensional coupled formulation in which closed solutions would be constructed. At the same time, conducting such studies allows us to understand the interaction picture of mechanical, thermal and electric fields in a structure. To solve this problem, a new closed solution of a coupled problem for a piezoceramic round rigidly fixed plate has been constructed in this research. It provides for qualitative assessment of the cross impact of thermoelectroelastic fields in this electroelastic system.

***Materials and Methods.*** The object of the study is a piezoceramic plate. The case of unsteady temperature change on its upper front surface is considered, taking into account the convection heat exchange of the lower plane with the environment (boundary conditions of the 1st and 3rd kind). The electric field induced as a result of the thermal strain generation is fixed by connecting the electrodated surfaces to the measuring device. The thermoelectroelasticity problem includes the equations of equilibrium, electrostatics, and the unsteady hyperbolic heat equation. It is solved by the generalized method of finite biorthogonal transformation, which makes it possible to construct a closed solution of a nonself-adjoint system of equations.

***Results.*** A new closed solution of the coupled axisymmetric thermoelectroelasticity problem for a round plate made of piezoceramic material was constructed.

***Discussion and Conclusion.*** The obtained solution to the initial boundary value problem made it possible to determine the temperature, electric and elastic fields induced in a piezoceramic element under arbitrary temperature axisymmetric external action. The calculations performed provided determining the dimensions of solid electrodes, which made it possible to increase the functionality of piezoceramic transducers. Numerical analysis of the results enabled us to identify new connections between the nature of external temperature action, the deformation process, and the value of the electric field in a piezoceramic structure. This can validate a proper program of experiments under their designing and significantly reduce the volume of field studies.

**Keywords:** thermoelectroelasticity problem, coupled problem, round piezoceramic rigidly fixed plate, biorthogonal finite integral transformations

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*Научная статья*

# Связанная осесимметричная задача термоэлектроупругости для круглой жестко закрепленной пластины

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**Аннотация**

***Введение.*** Для описания работы температурных пьезокерамических конструкций используется теория термоэлектроупругости, в которой математическая модель сформулирована в виде системы несамосопряженных дифференциальных уравнений. Сложность ее интегрирования в общем виде приводит к исследованию задач в несвязанной постановке. Это не позволяет оценить эффект влияния электроупругих полей на температурное. В литературе не представлены исследования данных задач в трехмерной связанной постановке, в которых были бы построены замкнутые решения. При этом проведение именно таких исследований позволяет понять картину взаимодействия механических, тепловых и электрических полей в конструкции. Для решения данной проблемы в настоящей работе построено новое замкнутое решение связанной задачи для пьезокерамической круглой жестко закрепленной пластины, позволяющее качественно оценить взаимное влияние термоэлектроупругих полей в данной электроупругой системе.

***Материалы и методы.*** Объектом исследования является пьезокерамическая пластина. Рассматривается случай нестационарного изменения температуры на ее верхней лицевой поверхности при учете конвекционного теплообмена нижней плоскости с окружающей средой (граничные условия 1 и 3 рода). Индуцируемое в результате образования температурных деформаций электрическое поле фиксируется путем подключения электродированных поверхностей к измерительному прибору. Задача термоэлектроупругости включает уравнения равновесия, электростатики и нестационарное гиперболическое уравнение теплопроводности. Она решается обобщенным методом конечного биортогонального преобразования, позволяющего построить замкнутое решение несамосопряженной системы уравнений.

***Результаты исследования.*** Построено новое замкнутое решение связанной осесимметричной задачи термоэлектроупругости для круглой пластины, выполненной из пьезокерамического материала.

***Обсуждение и заключение.*** Полученное решение начально-краевой задачи позволяет определить температурное, электрическое и упругое поля, индуцируемые в пьезокерамическом элементе при произвольном температурном осесимметричном внешнем воздействии. Проведенные расчеты позволяют определить размеры сплошных электродов, которые дают возможность повысить функциональные возможности пьезокерамических преобразователей. Численный анализ результатов позволяет выявить новые связи между характером внешнего температурного воздействия, процессом деформирования и величиной электрического поля в пьезокерамической конструкции. Это дает возможность обосновать рациональную программу экспериментов при их проектировании и значительно сократить объем натурных исследований.

**Ключевые слова:** задача термоэлектроупругости, связанная осесимметричная задача, жестко закрепленная пластина, биортогональные конечные интегральные преобразования

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**Introduction.** Various mathematical models are used to improve the functionality of piezoceramic sensors [1–3] based on the interdependence of thermoelectroelastic fields. To more accurately account for the effect of coupling of these fields, it is needed to construct closed solutions. Some simplifications are used to solve systems of initial nonself-adjoint differential equations. Thus, the problems can be considered in an uncoupled formulation, or the problems consider and analyze elements that have a degenerate geometry. An uncoupled stationary problem for a long electroelastic cylinder is considered in [4, 5], and article [6] is devoted to the analysis of thermal stresses in a hollow sphere. Papers [7, 8] are related to the determination of the temperature field in a piezoceramic shell and a round plate in solving uncoupled problems. Coupled dynamic problems for a homogeneous piezoceramic layer, as well as dynamic problems in a coupled formulation for a gradient–inhomogeneous piezoceramic layer, were considered in [9 10]. In [11, 12], fields in an unbounded medium were analyzed. In [13, 14], a long hollow cylinder was considered, and thermoelectroelastic fields were analyzed.

Currently, the literature does not describe the results of constructing closed solutions to the mentioned non-stationary problems in a three-dimensional coupled formulation. Therefore, in this paper, we consider a round plate made of piezoceramic composition and having a rigid fixation, for which a new closed solution to the problem of thermoelectroelasticity is obtained. The use of a limit on the rate of temperature change on its front surface [10] makes it possible not to include the inertial characteristics of the system under study and apply the equilibrium equations in the calculated ratios.

**Materials and Methods.** In the process of solving, a generalized finite biorthogonal transformation was used, which provided the reduction of the dimension of a nonself-adjoint system of equations and the construction of a closed solution through significant simplifying research in the image space.

**Mathematical model.** Consider certain area $\Omega : \{0 \le r_* \le b, 0 \le \theta \le 2\pi, 0 \le z_* \le h^*\}$, which is occupied by a piezoceramic solid circular plate in the cylindrical coordinate system $(r_*, \theta, z_*)$ Arbitrary temperature boundary conditions can be used for the problem under study. However, for the certainty of the solution, on the upper $(z_* = 0)$ front surface, the temperature change $\omega_1^*(r_*, t_*)$ at a given ambient temperature $\vartheta^*$ on the lower $(z_* = h^*)$ plane ($t_*$ — time) is considered. The cylindrical thermally insulated surface is rigidly fixed: there is no radial component of the displacement vector and the angle of rotation, and its lower part is fixed in the vertical plane. The lower plane of the round plate in question is grounded. The front electrodated planes of the plate are connected to the measuring device. The design scheme of the plate is shown in Figure 1.

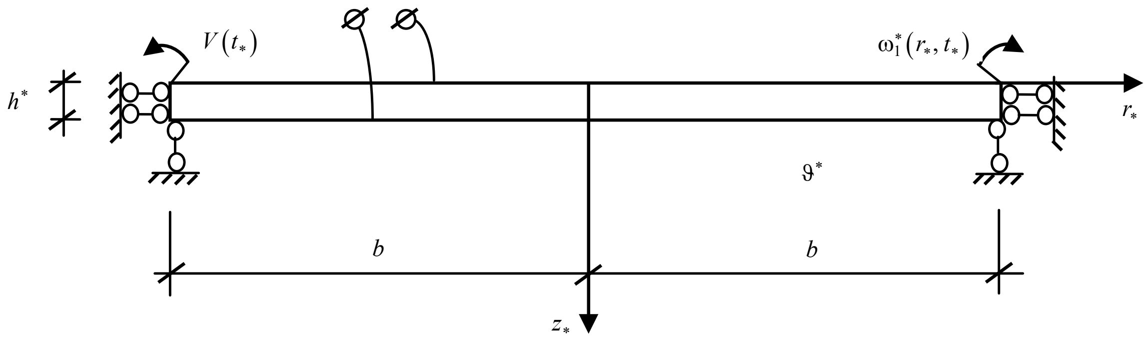



Fig. 1. Plate design diagram

The mathematical formulation of the problem under consideration in a dimensionless form for an axially polarized piezoceramic material with a hexagonal crystal lattice of 6 mm composition has the form:

$$\frac{\partial}{\partial r}\nabla U + a_1\frac{\partial^2 U}{\partial z^2} + a_2\frac{\partial^2 W}{\partial r\partial z} + a_3\frac{\partial^2 \phi}{\partial r\partial z} - \frac{\partial \Theta}{\partial r} = 0, \tag{1}$$

$$a_1\nabla\frac{\partial W}{\partial r} + a_4\frac{\partial^2 W}{\partial z^2} + a_2\nabla\frac{\partial U}{\partial z} + a_5\nabla\frac{\partial \phi}{\partial r} + a_6\frac{\partial^2 \phi}{\partial z^2} - a_7\frac{\partial \Theta}{\partial z} = 0,$$

$$-\nabla\frac{\partial \phi}{\partial r} - a_8\frac{\partial^2 \phi}{\partial z^2} + a_9\nabla\frac{\partial U}{\partial z} + a_{10}\nabla\frac{\partial W}{\partial r} + a_{11}\frac{\partial^2 W}{\partial z^2} + a_{12}\nabla\Theta + a_{13}\frac{\partial \Theta}{\partial z} = 0,$$

$$\nabla\frac{\partial \Theta}{\partial r} + \frac{\partial^2 \Theta}{\partial z^2} - \left(\frac{\partial}{\partial t} + \beta\frac{\partial^2}{\partial t^2}\right)\left[\Theta + a_{14}\left(\nabla U + \frac{\partial W}{\partial z}\right) - a_{15}\frac{\partial \phi}{\partial z}\right] = 0;$$

$$r = 0,1; \{U, W, \phi, \Theta\}_{|r=0} < \infty; \left\{U, \frac{\partial W}{\partial r}, \frac{\partial \Theta}{\partial r}\right\}_{|r=1} = 0, \tag{2}$$

$$D_{r|r=1} = 0\left\{\left[-\frac{\partial \phi}{\partial r} + a_{10}\left(\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z}\right) + a_{12}\Theta\right]_{|r=1} = 0\right\};$$

$$r = 0, 1; \{U, W, \phi, \Theta\}_{|r=0} < \infty; \left\{U, \frac{\partial W}{\partial r}, \frac{\partial \Theta}{\partial r}\right\}_{|r=1} = 0, \tag{2}$$

$$z = 0, h; \sigma_{zz} = 0\left\{a_{16}\nabla U + a_4\frac{\partial W}{\partial z} + a_6\frac{\partial \phi}{\partial z} - a_7\Theta = 0\right\}; \sigma_{rz} = 0\left\{\frac{\partial W}{\partial r} + \frac{\partial U}{\partial z} = 0\right\}, \tag{3}$$

$$\phi_{|z=0} = \phi_0,\, \phi_{|z=h} = 0,\, \Theta_{|z=0} = \omega_1,\, \left(\frac{\partial\Theta}{\partial z} + a_{17}\Theta\right)_{|z=h} = a_{17}\vartheta;$$

$$t = 0,\, \{U, W, \phi, \Theta\} = 0,\, \frac{\partial\{U, W, \phi\}}{\partial t}_{|t=0} = 0,\, \frac{\partial\Theta}{\partial t}_{|t=0} = \dot{\Theta}_0; \qquad (4)$$

where

$$\{U,W,r,z\} = \{U^*,W^*,r_*,z_*\}/b,\, \{\phi,\phi_0\} = \frac{e_{31}}{C_{11}b}\{\phi^*,\phi_0^*\},\, \{\phi,\phi_0\} = \frac{e_{31}}{C_{11}b}\{\phi^*,\phi_0^*\},$$

$$\{\Theta,\omega_1,\vartheta\} = \frac{\gamma_{11}}{C_{11}}\{\Theta^*,(\omega_1^* - T_0),(\vartheta^* - T_0)\},\, a_1 = \frac{C_{55}}{C_{11}},\, a_2 = \frac{C_{13}+C_{55}}{C_{11}},\, a_3 = \frac{e_{15}+e_{31}}{e_{31}},$$

$$a_4 = \frac{C_{33}}{C_{11}},\, a_5 = \frac{e_{15}}{e_{31}},\, a_6 = \frac{e_{33}}{e_{31}},\, a_7 = \frac{\gamma_{33}}{\gamma_{11}},\, a_8 = \frac{\varepsilon_{33}}{\varepsilon_{11}},\, a_9 = \frac{e_{31}(e_{15}+e_{31})}{C_{11}\varepsilon_{11}},\, a_{10} = \frac{e_{15}e_{31}}{C_{11}\varepsilon_{11}},$$

$$a_{11} = a_{10}\frac{e_{33}}{e_{15}},\, a_{12} = \frac{g_{11}e_{31}}{\gamma_{11}\varepsilon_{11}},\, a_{14} = T_0\frac{\gamma_{11}\gamma_{33}}{C_{11}k},\, a_{15} = T_0\frac{g_{33}\gamma_{11}}{e_{31}k},\, a_{16} = a_2 - a_1,$$

$a_{17} = \alpha \cdot b/\Lambda$, $\Theta^*(r_*, z_*, t_*)$ — temperature increment in dimensional form; $U^*(r_*, z_*, t_*), W^*(r_*, z_*, t_*)$ $\phi^*(r_*, z_*, t_*)$ — components of the displacement vector, electric field potential; $\sigma_{zz}(r, z, t)$, $\sigma_{rz}(r, z, t)$ — components of the mechanical stress tensor; $D_r(r, z, t)$ — radial component of the electric field induction vector; $\Lambda$, $k$, $\alpha_t$ — coefficients of thermal conductivity, volumetric heat capacity, and linear thermal expansion; $\phi_0^*(r, t)$ — electric potential induced on the upper front surface; $\gamma_{ii}$, $g_{ii}$ — components of the tensor of temperature stresses and pyroelectric coefficients ($i$ = 1,3, $\gamma_{ii} = C_{ii}\alpha_t$); $e_{15}$, $e_{31}$, $e_{33}$, $\varepsilon_{11}$, $\varepsilon_{33}$ — piezoelectric modules and permittivity coefficients; $\Theta^* = T - T$; $T$, $T_0$ — current temperature and temperature of the original state of the body; $\beta_{rel}$ — relaxation time; $\alpha$ — heat transfer coefficient, $\dot{\Theta}_0$ — known rate of temperature change; $\nabla = \frac{\partial}{\partial r} + \frac{1}{r}$.

To determine the potential of the electric field induced under deformation on the upper front surface $\phi_0^*(r, t)$, in the case of connecting electrodes to a measuring device with a large input resistance, an additional boundary condition is used:

$$\frac{\partial}{\partial t}\int_{(S)} D_{z|z=0}\, dS = 0 \qquad (5)$$

where $D_z(r, z, t)$ — axial component of the induction vector; $S$ — surface area.

**Construction of a general solution.** To fulfill the condition of fixing the cylindrical surface of the plate in the vertical plane, new functions $w(r, z, t)$, $W_1(t)$ are introduced:

$$W(r, z, t) = W_1(t) + w(r, z, t), \qquad (6)$$

this makes it possible to form a boundary value problem with respect to functions $U$, $w$, $\phi$, $\Theta$, which is investigated by the method of finite Fourier-Bessel transformations:

$$u_H(n,z,t) = \int_0^1 U(r,z,t)\, r J_1(j_n r)\, dr, \qquad (7)$$

$$\{w_H(n,z,t), \phi_H(n,z,t), N_H(n,z,t)\} = \int_0^1 \{w(r,z,t), \phi(r,z,t), \Theta(r,z,t)\}\, r J_0(j_n r)\, dr,$$

$$U(r,z,t) = 2\sum_{n=1}^{\infty} \frac{u_H(n,z,t)}{J_0(j_n)^2} J_1(j_n r) \qquad (8)$$

$$\{w(r,z,t), \phi(r,z,t), \Theta(r,z,t)\} = 2\sum_{n=0}^{\infty} \frac{\{w_H(n,z,t), \phi_H(n,z,t), N_H(n,z,t)\}}{J_0(j_n)^2} J_0(j_n r),$$

where $j_n$ — positive zeros of the function $J_1(j_n)\left(n = \overline{0,\infty};\ j_0 = 0\right)$, $J_v(\ldots)$ — Bessel functions.

It should be noted here that to satisfy the last boundary condition (2), it is necessary to assume that the original temperature of plate $T_0$ is equal to the ambient temperature $\vartheta^*$, and the temperature increment function on the upper front surface $\omega_1(1, t) = 0$. These assumptions, without much error, allow assuming that on the cylindrical surface of the plate, $\Theta(1, z, t) = 0$.

As a result of using the transformation algorithm in the image area, the following initial boundary value problem is obtained:

$$-j_n^2 u_H + a_1 \frac{\partial^2 u_H}{\partial z^2} - a_2 j_n \frac{\partial w_H}{\partial z} + a_3 j_n \frac{\partial \phi_H}{\partial z} + j_n N_H = 0, \tag{9}$$

$$-a_1 j_n^2 w_H + a_4 \frac{\partial^2 w_H}{\partial z^2} + a_2 j_n \frac{\partial u_H}{\partial z} - a_5 j_n^2 \phi_H + a_6 \frac{\partial^2 \phi_H}{\partial z^2} - a_7 \frac{\partial N_H}{\partial z} = 0,$$

$$j_n^2 \phi_H - a_8 \frac{\partial^2 \phi_H}{\partial z^2} + a_9 j_n \frac{\partial u_H}{\partial z} - a_{10} j_n^2 w_H + a_{11} \frac{\partial^2 w_H}{\partial z^2} + a_{12} j_n N_H + a_{13} \frac{\partial N_H}{\partial z} = 0,$$

$$-j_n^2 N_H + \frac{\partial^2 N_H}{\partial z^2} - \left(\frac{\partial}{\partial t} + \beta \frac{\partial^2}{\partial t^2}\right)\left[N_H + a_{14}\left(j_n u_H + \frac{\partial w_H}{\partial z}\right) - a_{15} \frac{\partial \phi_H}{\partial z}\right] = 0;$$

$$z = 0, h;\ a_{16} j_n u_H + a_4 \frac{\partial w_H}{\partial z} + a_6 \frac{\partial \phi_H}{\partial z} - a_7 N_H = 0;\ \frac{\partial u_H}{\partial z} - j_n w_H = 0, \tag{10}$$

$$\phi_{H|z=0} = \phi_{0H},\ \phi_{H|z=h} = 0,\ N_{H|z=0} = \omega_{1H},\ \left(\frac{\partial N_H}{\partial z} + a_{17} N_H\right)_{|z=h} = a_{17} \vartheta_H;$$

$$t = 0,\ \{u_H, \phi_H, N_H\} = 0,\ w_H = w_{0H},\ \frac{\partial \{u_H, \phi_H\}}{\partial t}_{|t=0} = 0, \tag{11}$$

$$\frac{\partial w_H}{\partial t}_{|t=0} = \dot{w}_{0H},\ \frac{\partial N_H}{\partial t}_{|t=0} = \dot{N}_{0H};$$

where $\left\{\phi_{0H}, \vartheta_H, w_{0H}, \dot{w}_{0H}, \dot{N}_{0H}\right\} = \int_0^1 \left\{\phi_0, \vartheta, -W_1(0), -\frac{dW_1(t)}{dt}_{|t=0}, \dot{\Theta}_0\right\} r J_0(j_n r)\, dr.$

At the next stage of the solution, the introduction of functions $U_H(n, z, t)$, $W_H(n, z, t)$, $\varphi_H(n, z, t)$, $Q_H(n, z, t)$ using the following relations:

$$u_H(n,z,t) = H_1(n,z,t) + U_H(n,z,t),\ w_H(n,z,t) = H_2(n,z,t) + W_H(n,z,t), \tag{12}$$

$$\phi_H(n,z,t) = H_3(n,z,t) + \varphi_H(n,z,t),\ N_H(n,z,t) = H_4(n,z,t) + Q_H(n,z,t),$$

allows the reduction of conditions (9) to homogeneous.

Here, $\{H_1 ... H_4\} = \{H_1^* ... H_4^*\} + \{f_9(z) ... f_{12}(z)\}\phi_{0H}(t),\ \{H_1^* ... H_4^*\} = \{f_1(z) ... f_4(z)\}\omega_{1H}(t) + \{f_5(z) ... f_8(z)\}\vartheta_H$, $f_1(z) ... f_{12}(z)$ — twice differentiable functions.

Substitution (12) in (9) – (11) when the conditions are satisfied:

$$z = 0,h;\ a_{16} j_n H_1 + a_4 \frac{\partial H_2}{\partial z} + a_6 \frac{\partial H_3}{\partial z} - a_7 H_4 = 0;\ j_n H_2 - \frac{\partial H_1}{\partial z} = 0, \tag{13}$$

$$H_{3|z=0} = \phi_{0H},\ H_{3|z=h} = 0,\ H_{4|z=0} = \omega_{1H},\ \left(\frac{\partial H_4}{\partial z} + a_{17} H_4\right)_{|z=h} = a_{17} \vartheta_H,$$

provides the formulation of the following task:

$$-j_n^2 U_H + a_1 \frac{\partial^2 U_H}{\partial z^2} - a_2 j_n \frac{\partial W_H}{\partial z} + a_3 j_n \frac{\partial \varphi_H}{\partial z} + j_n Q_H = F_1, \tag{14}$$

$$-a_1 j_n^2 W_H + a_4 \frac{\partial^2 W_H}{\partial z^2} + a_2 j_n \frac{\partial U_H}{\partial z} - a_5 j_n^2 \varphi_H + a_6 \frac{\partial^2 \varphi_H}{\partial z^2} - a_7 \frac{\partial Q_H}{\partial z} = F_2,$$

$$j_n^2 \varphi_H - a_8 \frac{\partial^2 \varphi_H}{\partial z^2} + a_9 j_n \frac{\partial U_H}{\partial z} - a_{10} j_n^2 W_H + a_{11} \frac{\partial^2 W_H}{\partial z^2} + a_{12} j_n Q_H + a_{13} \frac{\partial Q_H}{\partial z} = F_3,$$

$$-j_n^2 Q_H + \frac{\partial^2 Q_H}{\partial z^2} - \left(\frac{\partial}{\partial t} + \beta \frac{\partial^2}{\partial t^2}\right)\left[Q_H + a_{14}\left(j_n U_H + \frac{\partial W_H}{\partial z}\right) - a_{15} \frac{\partial \varphi_H}{\partial z}\right] = F_4,$$

$$z = 0, h;\ a_{16} j_n U_H + a_4 \frac{\partial W_H}{\partial z} + a_6 \frac{\partial \varphi_H}{\partial z} - a_7 Q_H = 0;\ \frac{\partial U_H}{\partial z} - j_n W_H = 0, \tag{15}$$

$$\varphi_{H|z=0} = \varphi_{H|z=h} = 0,\ Q_{H|z=0} = 0,\ \left(\frac{\partial Q_H}{\partial z} + a_{17} Q_H\right)_{|z=h} = 0;$$

$$t=0; U_H = U_{0H}; W_H = W_{0H}; \varphi_H = \varphi_{0H}; Q_H = Q_{0H}, \tag{16}$$

$$\left.\frac{\partial U_H}{\partial t}\right|_{t=0} = \dot{U}_{0H}, \left.\frac{\partial W_H}{\partial t}\right|_{t=0} = \dot{W}_{0H}, \left.\frac{\partial \varphi_H}{\partial t}\right|_{t=0} = \dot{\varphi}_{0H}, \left.\frac{\partial Q_H}{\partial t}\right|_{t=0} = \dot{Q}_{0H};$$

where

$$F_1 = j_n^2 H_1 - a_1 \frac{\partial^2 H_1}{\partial z^2} + a_2 j_n \frac{\partial H_2}{\partial z} - a_3 j_n \frac{\partial H_3}{\partial z} - j_n H_4,$$

$$F_2 = a_1 j_n^2 H_2 - a_4 \frac{\partial^2 H_2}{\partial z^2} - a_2 j_n \frac{\partial H_1}{\partial z} + a_5 j_n^2 H_3 - a_6 \frac{\partial^2 H_3}{\partial z^2} + a_7 \frac{\partial H_4}{\partial z},$$

$$F_3 = -j_n^2 H_3 + a_8 \frac{\partial^2 H_3}{\partial z^2} - a_9 j_n \frac{\partial H_1}{\partial z} + a_{10} j_n^2 H_2 - a_{11} \frac{\partial^2 H_2}{\partial z^2} - a_{12} j_n H_4 - a_{13} \frac{\partial H_4}{\partial z},$$

$$F_4 = j_n^2 H_4 - \frac{\partial^2 H_4}{\partial z^2} + \left(\frac{\partial}{\partial t} + \beta \frac{\partial^2}{\partial t^2}\right)\left[H_4 + a_{14}\left(j_n H_1 + \frac{\partial H_2}{\partial z}\right) - a_{15} \frac{\partial H_3}{\partial z}\right],$$

$$U_{0H} = -H_{1|t=0}, W_{0H} = w_{0H} - H_{2|t=0}, \varphi_{0H} = -H_{3|t=0}, Q_{0H} = -H_{4|t=0},$$

$$\dot{U}_{0H} = -\left.\frac{\partial H_1}{\partial t}\right|_{t=0}, \dot{W}_{0H} = \dot{w}_{0H} - \left.\frac{\partial H_2}{\partial t}\right|_{t=0}, \dot{\varphi}_{0H} = -\left.\frac{\partial H_3}{\partial t}\right|_{t=0}, \dot{Q}_{0H} = \dot{N}_{0H} - \left.\frac{\partial H_4}{\partial t}\right|_{t=0}.$$

Using the biorthogonal finite transformation (CMD) [15], we obtain a solution to problem (14) – (16). CMD with unknown components of vector functions of transformations is introduced on the segment $[0, h]$ $K_1(\lambda_{in}, z)\ldots K_4(\lambda_{in}, z)$, $N_1(\mu_{in}, z)\ldots N_1(\mu_{in}, z)$:

$$G(n,\lambda_{in},t) = \int_0^h \left[Q_H + a_{14}\left(j_n U_H + \frac{\partial W_H}{\partial z}\right) - a_{15}\frac{\partial \varphi_H}{\partial z}\right] K_4(\lambda_{in}, z) dz, \tag{17}$$

$$\{U_H, W_H, \varphi_H, Q_H\} = \sum_{i=1}^{\infty} G(n,\lambda_{in},t) \frac{\{N_1(\mu_{in},z), N_2(\mu_{in},z), N_3(\mu_{in},z), N_4(\mu_{in},z)\}}{\|K_{in}\|^2},$$

$$\|K_{in}\|^2 = \int_0^h K_4(\lambda_{in}, z) N_4(\mu_{in}, z) dz,$$

where $\lambda_{in}$, $\mu_{in}$ — the eigenvalues of the corresponding problems with respect to the components of the vector functions of the CMD ($k = 1\ldots4$).

In the course of transformations, we obtain a task for determining transformants $G(n, \lambda_{in}, t)$:

$$\left(\beta \frac{d^2}{dt^2} + \frac{d}{dt} + \lambda_{in}^2\right) G(n,\lambda_{in},t) = -F_H(n,\lambda_{in},t), \left(i = \overline{1,\infty} \qquad n = \overline{0,\infty}\right) \tag{18}$$

$$t = 0,\ G(\lambda_{in}, n, 0) = G_{0H}, \left.\frac{dG(\lambda_{in}, n, t)}{dt}\right|_{t=0} = G_0(\lambda_{in}, n), \tag{19}$$

whose solution has the following form:

$$G(n,\lambda_{in},t) = (m_{1in} - m_{2in})^{-1}\Big\{\left(\dot{G}_0 - G_0 m_{2in}\right) exp(m_{1in}t) - \left(\dot{G}_0 - G_0 m_{1in}\right) exp(m_{2in}t) +,$$

$$\left. + \beta^{-1}\int_0^t F_H(n,\lambda_{in},\tau)\left[exp\left(m_{2in}(t-\tau)\right) - exp\left(m_{1in}(t-\tau)\right)\right] d\tau \right\}, \tag{20}$$

in addition, two homogeneous problems with respect to components $K_1(\lambda_{in}, z)\ldots K_4(\lambda_{in}, z)$,

$$-j_n^2 K_{1in} + a_1 \frac{d^2 K_{1in}}{dz^2} - a_2 j_n \frac{dK_{2in}}{dz} - a_9 j_n \frac{dK_{3in}}{dz} + \lambda_{in}^2 a_{14} j_n K_{4in} = 0, \tag{21}$$

$$-a_1 j_n^2 K_{2in} + a_4 \frac{d^2 K_{2in}}{dz^2} + a_2 j_n \frac{dK_{1in}}{dz} - a_{10} j_n^2 K_{3in} + a_{11} \frac{d^2 K_{3in}}{dz^2} - \lambda_{in}^2 a_{14} \frac{dK_{4in}}{dz} = 0,$$

$$j_n^2 K_{3in} - a_8 \frac{d^2 K_{3in}}{dz^2} - a_3 j_n \frac{dK_{1in}}{dz} - a_5 j_n^2 K_{2in} + a_6 \frac{d^2 K_{2in}}{dz^2} + \lambda_{in}^2 a_{15} \frac{dK_{4in}}{dz} = 0,$$

$$\left(\lambda_{in}^2 - j_n^2\right) K_{4in} + \frac{d^2 K_{4in}}{dz^2} + j_n K_{1in} + a_7 \frac{dK_{2in}}{dz} + a_{12} j_n K_{3in} - a_{13} \frac{dK_{3in}}{dz} = 0;$$

$$z=0,h,\ a_{16}j_nK_{1in}+a_4\frac{dK_{2in}}{dz}+a_{11}\frac{dK_{3in}}{dz}-\lambda_{in}^2a_{15}K_{4in}=0,\ K_{3in|z=0}=K_{3in|z=h}=0, \tag{22}$$

$$\frac{dK_{1in}}{dz}-j_nK_{2in}-\frac{a_{10}}{a_1}j_nK_{3in}=0,\ K_{4in|z=0}=0,\left(\frac{dK_{4in}}{dz}+a_{17}K_{4in}\right)_{|z=h}=0;$$

and $N_1(\mu_{in}, z)\ldots N_4(\mu_{in}, z)$:

$$-j_n^2N_{1in}+a_1\frac{d^2N_{1in}}{dz^2}-a_2j_n\frac{dN_{2in}}{dz}+a_3j_n\frac{dN_{3in}}{dz}+j_nN_{4in}=0, \tag{23}$$

$$-a_1j_n^2N_{2in}+a_4\frac{d^2N_{2in}}{dz^2}+a_2j_n\frac{dN_{1in}}{dz}-a_5j_n^2N_{3in}+a_6\frac{d^2N_{3in}}{dz^2}-a_7\frac{dN_{4in}}{dz}=0,$$

$$j_n^2N_{3in}-a_8\frac{d^2N_{3in}}{dz^2}+a_9j_n\frac{dN_{1in}}{dz}-a_{10}j_n^2N_{2in}+a_{11}\frac{d^2N_{2in}}{dz^2}+a_{12}j_nN_{4in}+a_{13}\frac{dN_{4in}}{dz}=0,$$

$$-j_n^2N_{4in}+\frac{d^2N_{4in}}{dz^2}+\mu_{in}^2\left(N_{4in}+a_{14}j_nN_{1in}+a_{14}\frac{dN_{2in}}{dz}-a_{15}\frac{dN_{3in}}{dz}\right)=0;$$

$$z=0,h,\ a_{16}j_nN_{1in}+a_4\frac{dN_{2in}}{dz}+a_6\frac{dN_{3in}}{dz}-a_7N_{4in}=0,\ \frac{\partial N_{1in}}{\partial z}-j_nN_{2in}=0, \tag{24}$$

$$N_{3in|z=0}=N_{3in|z=h}=0,\ N_{4in|z=0}=0,\left(\frac{\partial N_{4in}}{\partial z}+a_{17}N_{4in}\right)_{|z=h}=0;$$

where

$$F_H\left(n,\lambda_{in},t\right)=\int_0^h\left(F_1K_{1in}+F_2K_{2in}+F_3K_{3in}+F_4K_{4in}\right)dz,$$

$$\left\{G_{0H},\dot G_{0H}\right\}=\int_0^h\left[\left\{Q_{0H},\dot Q_{0H}\right\}+a_{14}\left(j_n\left\{U_{0H},\dot U_{0H}\right\}+\frac{d\left\{W_{0H},\dot W_{0H}\right\}}{dz}\right)-a_{15}\frac{d\left\{\varphi_{0H},\dot\varphi_{0H}\right\}}{dz}\right]K_{4in}dz,$$

$m_{1in}$, $m_{2in}$ — roots of the characteristic equation: $\beta m_{in}^2+m_{in}+\lambda_{in}^2=0$.

Constructed homogeneous problem (23), (24) with respect to functions $N_1(\mu_{in}, z)\ldots N_4(\mu_{in}, z)$ is invariant to the initial calculated relations (14), (15).

Systems (21), (23) are reduced to the following equations with respect to $K_2(\lambda_{in}, z)$, $N_2(\mu_{in}, z)$:

$$\left(\frac{d^8}{dz^8}+e_{1in}\frac{d^6}{dz^6}+e_{2in}\frac{d^4}{dz^4}+e_{3in}\frac{d^2}{dz^2}+e_{4in}\right)\left\{K_{2in},N_{2in}\right\}=0. \tag{25}$$

In the paper, coefficients $e_{1in}...e_{4in}$ are not given due to the limitation of its volume.

In equation (25), the left part is decomposed into commutative factors, presented below:

$$\left(\frac{d^2}{dz^2}-A_{1in}^2\right)\left(\frac{d^2}{dz^2}+A_{2in}^2\right)\left(\frac{d^4}{dz^4}+m_{3in}^2\frac{d^2}{dz^2}+m_{4in}^2\right)\left\{K_{2in},N_{2in}\right\}=0, \tag{26}$$

where $A_{1in}=\sqrt{B_{1in}}$, $A_{2in}=\sqrt{S_{1in}}$, $m_{3in}^2=e_{1in}+B_{1in}+S_{1in}$, $m_{4in}^2=\dfrac{e_{4in}}{B_{1in}S_{2in}}$, $B_{1in},S_{1in}$ — real positive roots of the following characteristic equations:

$$B_{in}^4+e_{1in}B_{in}^3+e_{2in}B_{in}^2+e_{3in}B_{in}+e_{4in}=0,$$

$$S_{in}^3-\left(e_{1in}+B_{1in}\right)S_{in}^2+\left(e_{1in}B_{1in}+B_{1in}^2+e_{2in}\right)S_{in}-\frac{e_{4in}}{B_{1in}}=0.$$

When examining a round rigidly fixed piezoceramic plate, the general integral of equations (26) has the following form:

$$\left\{K_{2in},N_{2in}\right\}=\left\{D_{1in},E_{1in}\right\}exp\left(A_{1in}z\right)+\left\{D_{2in},E_{2in}\right\}exp\left(-A_{1in}z\right)+\left\{D_{3in},E_{3in}\right\}sin\left(A_{2in}z\right)+ \tag{27}$$

$$+\left\{D_{4in},E_{4in}\right\}cos\left(A_{2in}z\right)+\left\{D_{5in},E_{5in}\right\}sin\left(A_{3in}z\right)+\left\{D_{6in},E_{6in}\right\}cos\left(A_{3in}z\right)+$$

$$+\left\{D_{7in},E_{7in}\right\}sin\left(A_{4in}z\right)+\left\{D_{8in},E_{8in}\right\}cos\left(A_{4in}z\right),$$

where

$$A_{3in}=\left(\frac{m_{3in}^2+\sqrt{m_{3in}^4-4m_{4in}^2}}{2}\right)^{0.5},\ A_{4in}=\left(\frac{m_{3in}^2-\sqrt{m_{3in}^4-4m_{4in}^2}}{2}\right)^{0.5}.$$

It should be noted here that the condition of the actual positive values of coefficients $B_{1in}$, $S_{1in}$, $A_{1in}\ldots A_{4in}$ is fulfilled for most structures made of piezoceramic material. Otherwise, the formula structure (26), (27) simply changes.

Considering that the connections were previously obtained as a result of reducing (21), (23) to (25), we get expressions for functions $K_1(\lambda_{in}, z)$, $K_3(\lambda_{in}, z)$, $K_4(\lambda_{in}, z)$, $N_1(\lambda_{in}, z)$, $N_3(\lambda_{in}, z)$, $N_4(\lambda_{in}, z)$.

Substituting $K_1(\lambda_{in}, z)\ldots K_4(\lambda_{in}, z)$, $N_1(\mu_{in}, z)\ldots N_1(\mu_{in}, z)$ in conditions (22), (24) provides determining constants $D_{1in}\ldots D_{8in}$, $E_{1in}\ldots E_{8in}$ and eigenvalues $\lambda_{in}$, $\mu_{in}$.

The final expressions of functions $U(n, z, t)$, $W(n, z, t)$, $\phi(n, z, t)$, $\Theta(n, z, t)$ are obtained by applying the inversion formulas (17), (8). Then, taking into account (6), (12), we have:

$$U(r,z,t)=2\sum_{n=1}^{\infty}\frac{J_1(j_n r)}{J_0(j_n)^2}\left[H_1(n,z,t)+\sum_{i=1}^{\infty}G(n,\lambda_{in},t)N_1(\mu_{in},z)\left\|K_{in}\right\|^{-2}\right], \quad (28)$$

$$W(r,z,t)=W_1(t)+2\sum_{n=0}^{\infty}\frac{J_0(j_n r)}{J_0(j_n)^2}\left[H_2(n,z,t)+\sum_{i=1}^{\infty}G(n,\lambda_{in},t)N_2(\mu_{in},z)\left\|K_{in}\right\|^{-2}\right],$$

$$\phi(r,z,t)=2\sum_{n=0}^{\infty}\frac{J_0(j_n r)}{J_0(j_n)^2}\left[H_3(n,z,t)+\sum_{i=1}^{\infty}G(n,\lambda_{in},t)N_3(\mu_{in},z)\left\|K_{in}\right\|^{-2}\right],$$

$$\Theta(r,z,t)=2\sum_{n=0}^{\infty}\frac{J_0(j_n r)}{J_0(j_n)^2}\left[H_4(n,z,t)+\sum_{i=1}^{\infty}G(n,\lambda_{in},t)N_4(\mu_{in},z)\left\|K_{in}\right\|^{-2}\right].$$

Functions $f_1(z)\ldots f_{12}(z)$ are calculated from the simplification condition $F_1\ldots F_4$ when conditions (13) are satisfied:

$$j_n^2 H_1-a_1\frac{\partial^2 H_1}{\partial z^2}+a_2 j_n\frac{\partial H_2}{\partial z}-a_3 j_n\frac{\partial H_3}{\partial z}-j_n H_4=0,$$

$$a_1 j_n^2 H_2-a_4\frac{\partial^2 H_2}{\partial z^2}-a_2 j_n\frac{\partial H_1}{\partial z}+a_5 j_n^2 H_3-a_6\frac{\partial^2 H_3}{\partial z^2}+a_7\frac{\partial H_4}{\partial z}=0,$$

$$-j_n^2 H_3+a_8\frac{\partial^2 H_3}{\partial z^2}-a_9 j_n\frac{\partial H_1}{\partial z}+a_{10} j_n^2 H_2-a_{11}\frac{\partial^2 H_2}{\partial z^2}-a_{12} j_n H_4-a_{13}\frac{\partial H_4}{\partial z}=0,$$

$$j_n^2 H_4-\frac{\partial^2 H_4}{\partial z^2}=0.$$

Function $W_1(t)$ is determined from condition $W(1, h, t) = 0$:

$$W_1(t)=-2\sum_{n=0}^{\infty}\left[H_2(n,h,t)+\sum_{i=1}^{\infty}G(n,\lambda_{in},t)N_2(\mu_{in},h)\left\|K_i\right\|^{-2}\right]J_0(j_n)^{-1}.$$

For a qualitative assessment of the induced electric pulse on its upper front surface, it is required to form two electrodes with a radius of separation $R$ and connect them to a measuring device. In this case, potential $\phi_0(r, t)$, induced on two equipotential surfaces is represented as:

$$\phi_0(r,t)=\phi_{01}(t)H(R-r)+\phi_{02}(t)H(r-R), \quad (29)$$

where $H(\ldots)$ — the Heaviside step function.

Substituting (29) into (5) makes it possible to define expressions for determining potentials $\phi_0(t)$, $\phi_{02}(t)$:

$$\int_0^R D_{z|z=0}r\,dr=\int_R^1 D_{z|z=0}r\,dr=0. \quad (30)$$

As a result of solution (30), functions $\phi_0(t)$, $\phi_{02}(t)$ are defined as follows:

$$\phi_{01}(t)=Q_{01}^{-1}\left[Q_1(t)+Q_2(t)+Q_3(t)\right],\ \phi_{02}(t)=Q_{02}^{-1}\left[-Q_1(t)+\frac{1-R^2}{R^2}Q_2(t)+Q_4(t)\right],$$

where

$$Q_1(t)=R\sum_{n=1}^{\infty}\frac{J_1(j_n R)}{J_0(j_n)^2}\left\{\frac{a_{10}}{a_5}H_1^*(n,0,t)+\frac{a_{11}}{j_n}\frac{\partial H_2^*(n,z,t)}{\partial z}_{|z=0}-\frac{a_8}{j_n}\frac{\partial H_3^*(n,z,t)}{\partial z}_{|z=0}+\right.$$

$$+\sum_{i=1}^{\infty}\frac{G(\lambda_{in},0,t)}{\|K_{in}\|^2}\left[\frac{a_{10}}{a_5}K_1(\lambda_{in},0)+\frac{a_{11}}{j_n}\frac{dK_2(\lambda_{in},z)}{dz}_{|z=0}-\frac{a_8}{j_n}\frac{dK_3(\lambda_{in},z)}{dz}_{|z=0}\right]\Bigg\},$$

$$Q_2(t)=R^2\left[a_{11}\frac{\partial H_2^*(0,z,t)}{\partial z}_{|z=0}-a_8\frac{\partial H_3^*(0,z,t)}{\partial z}_{|z=0}+\sum_{i=1}^{\infty}\frac{G(\lambda_{i0},0,t)}{\|K_{i0}\|^2}\left[a_{11}\frac{dK_2(\lambda_{i0},z)}{dz}_{|z=0}-a_8\frac{dK_3(\lambda_{i0},z)}{dz}_{|z=0}\right]\right],$$

$$Q_3(t)=a_{13}\int_0^R\omega_1(r,t)rdr,\ Q_4(t)=a_{13}\int_R^1\omega_1(r,t)rdr,$$

$$Q_{01}=-\left\{\frac{R^4}{2}\left[a_{11}\frac{df_{10}(z)}{dz}_{|z=0}-a_8\frac{df_{11}(z)}{dz}_{|z=0}\right]+2R^2\sum_{n=1}^{\infty}\left[\frac{J_1(j_nR)}{J_0(j_n)}\right]^2\left[\frac{a_{10}}{a_5}f_9(z)+\frac{a_{11}}{j_n}\frac{df_{10}(z)}{dz}_{|z=0}-\frac{a_8}{j_n}\frac{df_{11}(z)}{dz}_{|z=0}\right]\right\},$$

$$Q_{02}=-\left\{\frac{(1-R^2)^2}{2}\left[a_{11}\frac{df_{10}(z)}{dz}_{|z=0}-a_8\frac{df_{11}(z)}{dz}_{|z=0}\right]+2R^2\sum_{n=1}^{\infty}\left[\frac{J_1(j_nR)}{J_0(j_n)}\right]^2\left[\frac{a_{10}}{a_5}f_9(z)+\frac{a_{11}}{j_n}\frac{df_{10}(z)}{dz}_{|z=0}-\frac{a_8}{j_n}\frac{df_{11}(z)}{dz}_{|z=0}\right]\right\}.$$

In this case, the potential difference $V(t)$ is determined by the equality:

$$V(t)=\phi_{01}(t)-\phi_{02}(t). \tag{31}$$

**Research Results.** Numerical results are presented for a plate made of piezoceramics of the composition PZT–4 [4, 11, 16]:

$\{C_{11}, C_{12}, C_{13}, C_{33}, C_{55}\} = \{13.9, 7.78, 7.3, 11.5, 2.26\}\times10^{10}$ Pa, $\{\varepsilon_{11}, \varepsilon_{33}\} = \{6.46, 5.62\} \times 10^{-9}$ F/m,
$\{e_{15}, e_{31}, e_{33}\} = \{12.7, -5.2, 15.1\}$ C/m$^2$, $\Lambda = 1{,}6$ W/(m·K), $\alpha_t = 0.4\times10^{-5}$ K$^{-1}$,
$k = 3\times10^6$ J/ (m$^3$·K), $g_{11} = g_{33} = -0.6\times10^{-4}$ C/(m$^2$·K), $\beta_{rel} = 10^{-4}$ s, $\alpha = 5.6$ W/(m$^2$·K).

The following case of temperature change $\omega_1^*(r_*,t_*)$ is investigated:

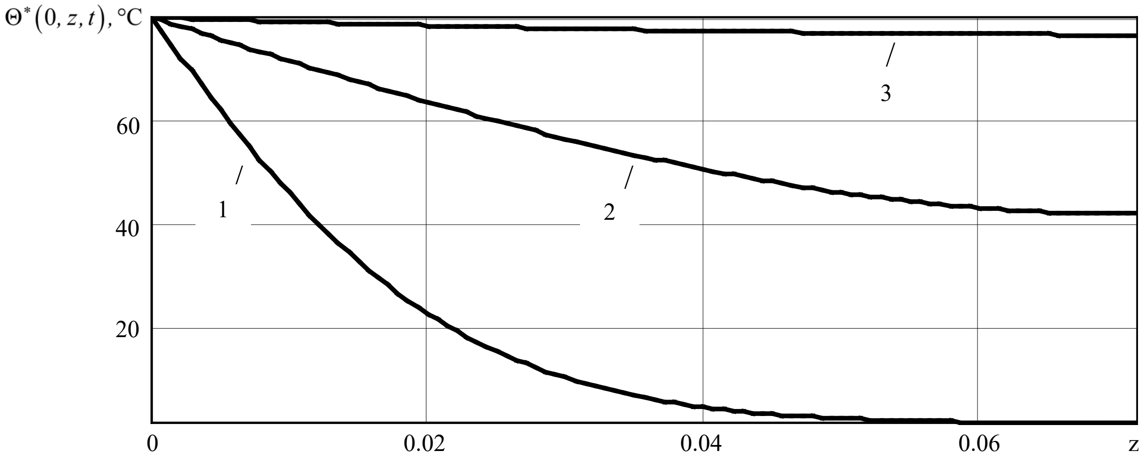
$$\omega_1^*(r_*,t_*)=\left(1-\frac{r_*}{b}\right)T_{max}^*\left[sin\left(\frac{\pi}{2t_{max}^*}t_*\right)H\left(t_{max}^*-t_*\right)+H\left(t_*-t_{max}^*\right)\right],$$

where $T_{max}^*, t_{max}^*$ — maximum temperature value and the corresponding time $\left(T_{max}^*=100°\text{C}, T_0=20°\text{C}\right)$.

Figure 2 shows graphs reflecting at various points in time $\left(t_{max}^*=0.1\text{ s}\right)$ the change in temperature $\Theta^*(0, z, t)$ in the thickness of the plate ($b = 14\times10^{-3}$ m, $h^* = 1\times10^{-3}$ m).

According to the calculation result, it is observed that due to the high coefficient of thermal conductivity and the small thickness of the piezoceramic plate, the steady-state temperature regime is formed quite quickly $\left(t_{max}^*=10\text{ s}\right)$ when it reaches $\Theta^*(0, z, t)$ on the lower front surface ($z = h$) 78°C (Fig. 2).

Figure 3 shows the change $\Theta^*\left(0,h/2,t\right)$ in time $\left(t_{max}^*=3\times10^{-5}\text{ s}\right)$ taking into account (represented by a solid line) and without account for (represented by dotted line, $\beta = 0$) the relaxation of the heat flux ($b = 14\times10^{-5}$ m, $h^* = 1\times10^{-5}$ m). It should be emphasized that the application of the hyperbolic Lord-Shulman heat conduction equation is needed only in the study of a piezoceramic micro-dimensional structure with a very rapid change $\omega_1^*(r_*,t_*)$.




Mechanics

Fig. 2. Diagrams $\Theta^*\left(0,z,t\right)-z$: $1 — t=t^*_{max}$, $2 — t=10t^*_{max}$, $3 — t=100t^*_{max}$

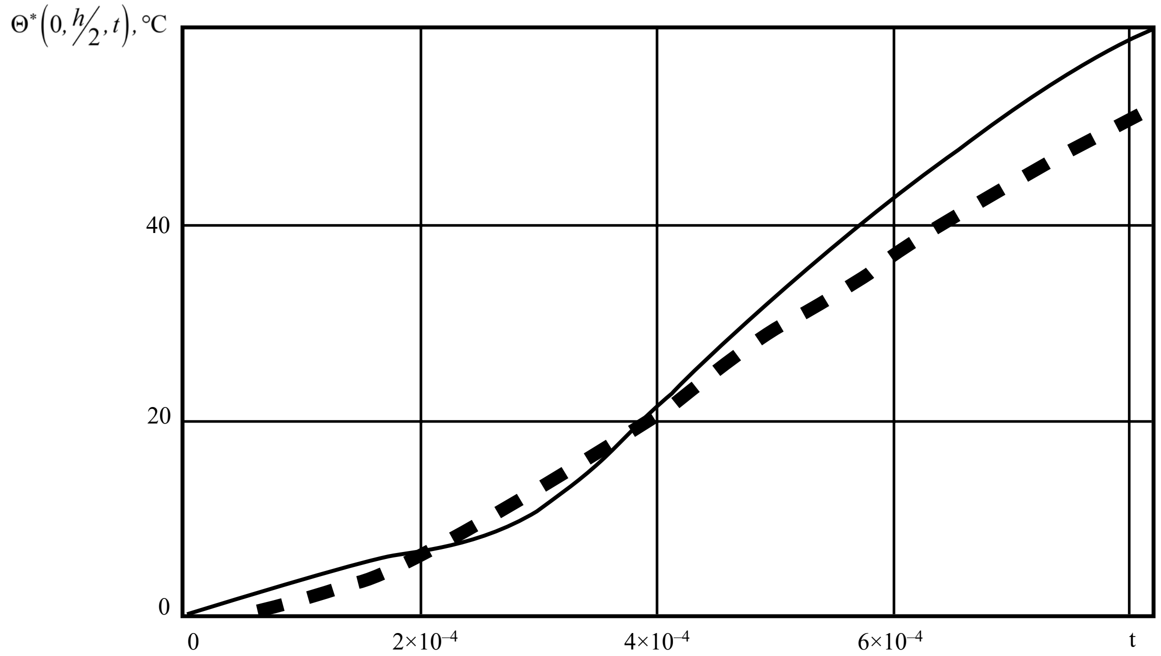



Fig. 3. Diagrams $\Theta^*\left(0,h/2,t\right)-t\left(t=\frac{\Lambda}{kb^2}t_*\right)$:

solid line — $\beta=10^{-5}\left(\text{s}\right)$, dotted line — $\beta=0$

The numerical results of determining function $\Theta^*(r, z, t)$ show that when conducting a study of a structure made of piezoceramic material, it is possible to neglect the impact of the rate of change of body volume and tension on the temperature field, i.e., to use only the equation of thermal conductivity in calculations.

Figure 4 shows a diagram of movements $W^*(0, z, t)$ over time $t$, and Figure 5 shows the dependence of the change in the radial component of normal stresses $\sigma_{rr}(r, z, t)$ along coordinate $r$ at different points in time: $1-t=t_{max}$, $2-t=10t_{max}$ $\left(t^*_{max}=1\text{ s}\right)$, solid line — $z = 0$, dotted line — $z = h$.

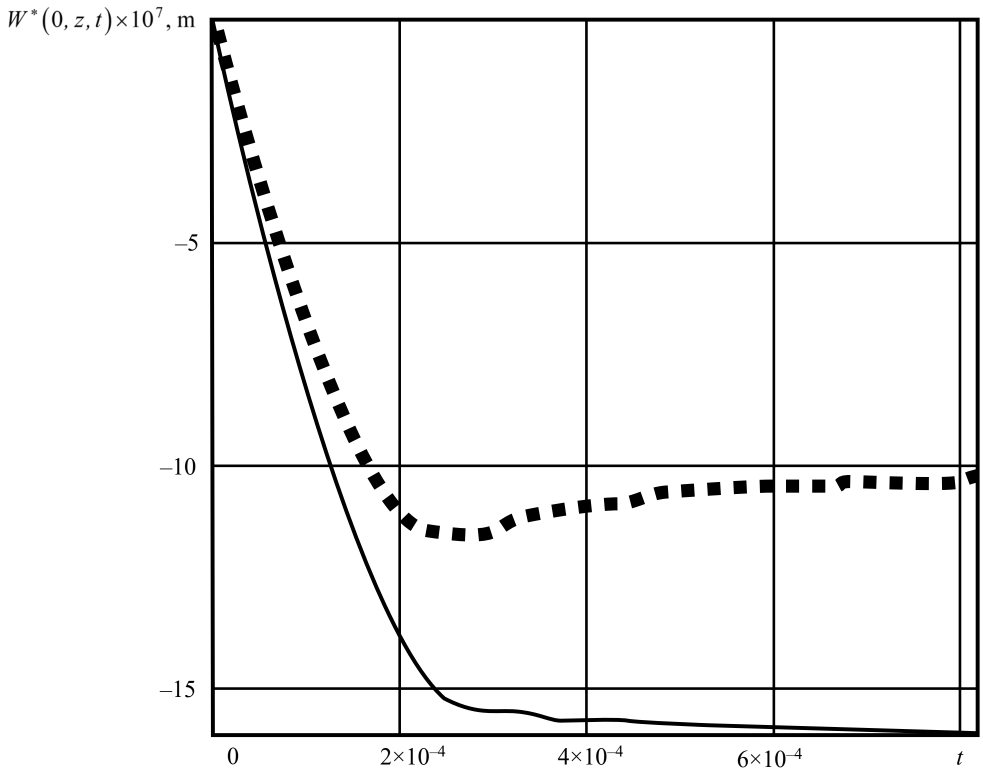


Fig. 4. Diagrams $W^*(0, z, t) - t$

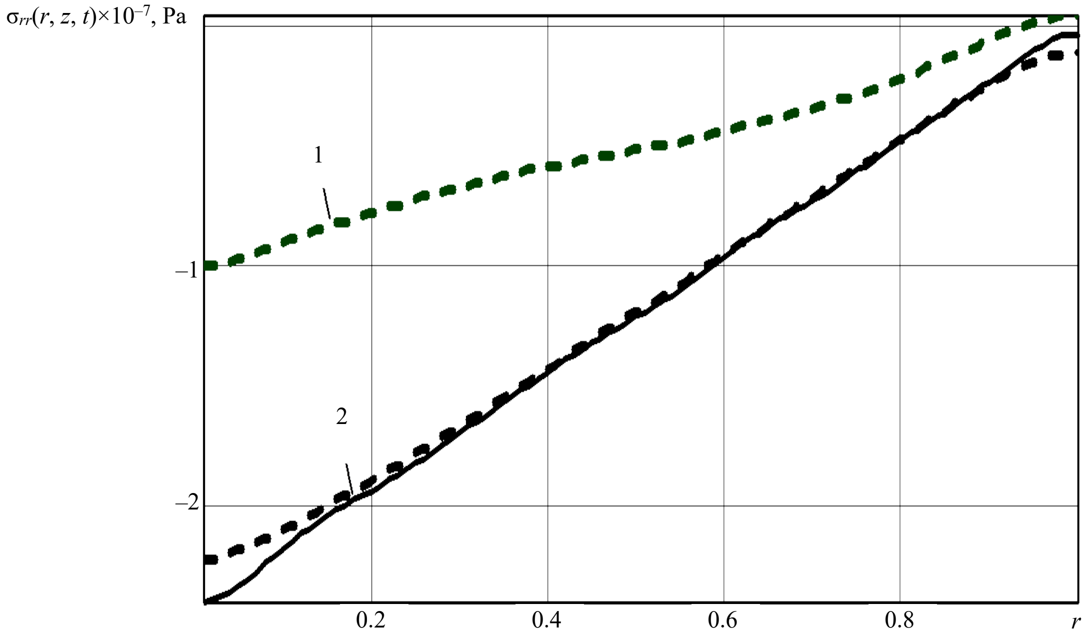



Fig. 5. Diagrams $\sigma_{rr}(r, z, t) - r$: 1 — $t = t_{max}$, 2 — $t = 10t_{max}$

It should be noted that under heating, the plate bends with increasing thickness; due to fixation, compressive normal stresses $\sigma_{rr}(r, z, t)$ are formed at all points. In the case of complete heating of the structure ($t = 10t_{max}$), the value of normal stresses $\sigma_{rr}(r, z, 10t_{max})$ in the height of the section practically coincide (Fig. 5, Diagram 2, solid and dotted lines). At this, $\sigma_{rr}(r, 0, t)$ remains constant over the entire time interval $t \geq t_{max}$ (Fig. 5, solid line), and on the lower plane at the initial moment of time $\sigma_{rr}(r, h, t)$, it is significantly less (Fig. 5, Diagram 1, dotted line).

For a qualitative assessment of the induced electric pulse in the form of a potential difference $V(t)$ (31), two electrodes with a radius of separation $R = 0.7$ and connected to a measuring device (Fig. 6, solid line) must be formed on the upper front surface of the element in question. At this, determination of $V(t)$ by connecting the upper and lower (grounded) solid electrodated surfaces of the plate to the voltmeter (Fig. 6, dotted line) is ineffective.

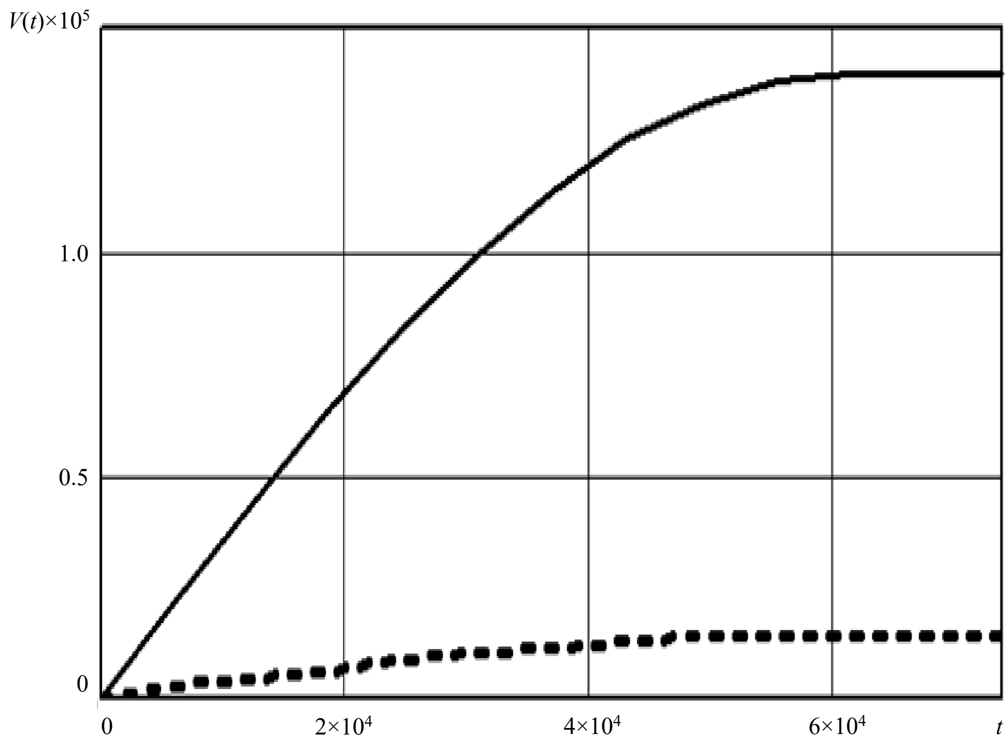



Fig. 6. Diagrams $V(t) - t$

**Discussion and Conclusion.** The developed closed solution of the coupled axisymmetric thermoelectroelasticity problem for a round plate made of piezoceramic material is more accurate than the solution that was developed when solving problems in an uncoupled formulation. This is due to the fact that the calculated ratios obtained make it possible to determine how the non-stationary temperature field affects the stress-strain state and the electric field of the element in question, which makes it possible to describe the behavior of a round piezoceramic plate under the influence of thermal and electrical loads with greater accuracy. In addition, it becomes possible to scientifically establish the dimensions of two uncoupled electrodes, which provides measuring the induced electric pulse most effectively.

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