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Assessment of Dynamic States of Railway Vehicles: Structural Mathematical Modeling

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Abstract

Introduction. The speed rise of railway transport and an increase in the loads on the axles of wheelsets necessitate the modernization of the existing fleet. Scientific studies in the field of rolling stock dynamics are aimed at taking into account the oscillatory processes that occur during the movement of railway vehicles in a traditional design. The attachment of supplementary elements was considered at the coupling level of two cars and the attachment of a third trolley in the center of gravity of the railway platform. The scientific literature has not paid enough attention to the construction of mathematical models that make it possible to assess the dynamic states of such constructive solutions. The objective of this study is to create a method for evaluating the dynamic conditions of a car. The situation is considered when an additional set of mass-inertial and elastic elements is introduced into its structure, and the general dynamic condition of the vehicle depends on the adjustment of their parameters.

Materials and Methods. The basic research tool is the structural mathematical modeling, which is based on an approach where the source design scheme is a mechanical oscillatory system in the form of a solid body on elastic supports with supplementary typical elements introduced into its structure. The dynamic analogue of the calculation scheme used is the block diagram of the automatic control system, the use of which provides detailing the connections between typical elastic and mass-inertia elements.

Results. A method for estimating the dynamic states of railway vehicles is proposed. It is based on the construction of mathematical models, taking into account the introduction of an additional structure of mass-inertia and elastic elements. The impact of additional parameters on the dynamic condition of the vehicle is investigated. Analytical relations have been obtained that provide reducing the dynamic loads on the major structural elastic elements when changing the corresponding parameters of a technical object. The transfer function of interpartial relations is given, which provides controlling the interaction between the coordinates of the vehicle movement under the action of two kinematic disturbances of the in-phase type.

Discussion and Conclusion. The generated mathematical model provides for assessment, monitoring and control of the dynamic state of the vehicle under conditions of kinematic disturbances. The research results can be used to modernize existing vehicles and create new ones with improved dynamics.

Keywords: vehicle dynamics, dynamic condition, structural mathematical modeling, block diagram

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Оригинальное эмпирическое исследование

Возможности оценки динамических состояний железнодорожных транспортных средств: структурное математическое моделирование

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Аннотация

Введение. Увеличение скоростей движения железнодорожного транспорта и повышение нагрузок на оси колесных пар обуславливают необходимость модернизации существующего парка. Научные исследования в области динамики подвижного состава направлены на учёт колебательных процессов, возникающих при движении железнодорожных транспортных средств в традиционном конструктивном исполнении. Присоединение дополнительных элементов рассматривалось на уровне сцепки двух вагонов и присоединении третьей тележки в центре тяжести железнодорожной платформы. Построению математических моделей, позволяющих оценить динамические состояния таких конструктивных решений, в научной литературе не уделено достаточно внимания. Цель данного исследования — создать метод оценки динамических состояний вагона. Рассматривается ситуация, когда в его структуру вводится дополнительная совокупность масс-инерционных и упругих элементов, причем от корректировки их параметров зависит общее динамическое состояние транспортного средства.

Материалы и методы. Базовым инструментом проведения исследований является структурное математическое моделирование, в основе которого лежит подход, когда исходная расчетная схема представляет собой механическую колебательную систему в виде твердого тела на упругих опорах с дополнительной введёнными в её структуру типовыми элементами. Динамическим аналогом используемой расчетной схемы является структурная схема системы автоматического управления, применение которой позволяет детализировать связи между типовыми упругими и масс-инерционными элементами.

Результаты исследования. Предложен метод оценки динамических состояний железнодорожных транспортных средств, основанный на построении математических моделей, с учетом введения дополнительной структуры масс-инерционных и упругих элементов. Исследовано влияние дополнительных параметров на динамическое состояние транспортного средства. Получены аналитические соотношения, позволяющие при изменении соответствующих параметров технического объекта снизить динамические нагрузки на основные конструктивные упругие элементы. Приведена передаточная функция межпарциальных связей, позволяющая контролировать взаимодействие между координатами движения транспортного средства при действии двух кинематических возмущений синфазного типа.

Обсуждение и заключение. Сформированная математическая модель позволяет оценить динамическое состояние железнодорожного транспортного средства в условиях действия кинематических возмущений. Результаты исследований могут быть применены при модернизации существующих и создании новых транспортных средств с улучшенной динамикой.

Ключевые слова: динамика транспортных средств, динамическое состояние, структурное математическое моделирование, структурная схема

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Introduction. The expansion of technical-and-economic ties at the interregional level, providing the growth of the country's industrial engineering potential, and maintaining the growing system of international trade and commercial relations largely depends and relies on rail transport [1]. The rail traffic volume is constantly increasing. This drives the need to take into account the perverse effects of increased dynamic loads [2], which directly affect the reliability of operation of both rolling stock and the track structure. Despite such negative factors, it is necessary to fulfill the planned indicators, which include an increase in the service speed, compliance with weight and length standards for trains, increasing axle load to 30 tons or more. This reflects the real demands of the development of the Russian economy and stimulates the creation of new more powerful locomotives, the renewal of the fleet of rolling stock, and the modernization of track facilities [3]. At the same time, the possibility of negative consequences of the intensification of transportation processes should also be considered. One of the major issues is the increase in the rate of wear of the track structure with the corresponding resulting difficulties [4].

Currently, particular attention is being paid to the development of a methodology for assessing the dynamic condition of rolling stock, the interaction of technical means and rail tracks, energy savings, and improving the reliability and safety of transportation processes. The mathematical modeling methodology is described, e.g., in [5]. At the same time, there are other possibilities for finding rational solutions [6]. It is important to pay attention to the modernization of the existing fleet of freight wagons, whose operation is no longer effective under increased loads [7]. One of the approaches that could be adopted for the development is the concept of installing an additional two-axle bogie for freight 4ax wagons [8]. In this case, we can expect a more uniform distribution of the load on the track structure, as well as the possibility of increasing the weight of the transported goods while maintaining the regulations for axial load values within 22 tons [9]. The unevenness of the wheel-rail contact parameters initiates the oscillatory movements of the car, which in turn forms the oscillatory movements of the car body. The oscillation process is also formed by the conditions of interaction of wagons inside the train [10]. In this case, dynamic restraint forces occur, superimposing on the static components of the overall reaction, which can significantly increase the level of dynamic interactions in the wheel-rail contact [11, 12]. However, the possibilities of structural mathematical modeling in assessing the dynamic states of railway vehicles with the introduction of additional links have not yet been given enough attention. Therefore, the objective of this study is to form a method for evaluating the dynamic states of a railway vehicle when introducing an additional set of mass-inertial and elastic elements into its structure, the correction of the parameters of which would affect the overall dynamic state of the vehicle.

Materials and Methods. The methodological basis of the research is the structural theory of vibration isolation systems, which provides for the accurate assessment of the dynamic properties of a railway vehicle in a linear formulation, taking into account concentrated parameters and small oscillations relative to the position of static equilibrium or a steady-state process. The design scheme is a mechanical vibratory system with dynamic equivalent in the form of a block diagram of an automatic control system. This makes it possible to detail the connections between the elements of the vehicle, as well as to use methods characteristic of the theory of automatic control (transfer functions, transformations of structural circuits, convolution and simplification, frequency characteristics) [13].

Standard design schemes of freight railway vehicles are described by well-known calculation schemes, and their dynamic features can be estimated using linear calculation schemes. The article considers a railway vehicle in the form of a four-axle freight wagon designed for the transportation of heavy goods such as coal, ore, sand, rolled metal, small-sized metal structures, etc. (Fig. 1).

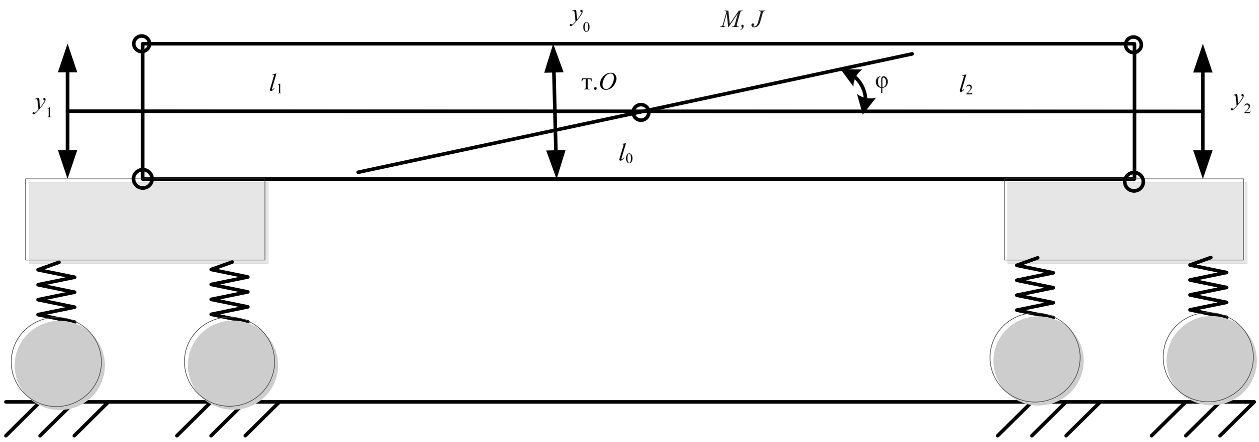

Fig. 1. Schematic diagram of a four-axle freight wagon

The structure of the presented railway vehicle contains a body with mass *M* and moment of inertia *J*. It is based on two four-wheel bogies, conventionally represented as a set of mass-inertial and elastic elements. The dynamic features of the system under consideration show the excessive impact on the structural elements of the bogies of the railway vehicle. In this regard, to improve its dynamic properties, a third two-axle truck is additionally introduced into the suspension structure, which is located in the center of the wagon (Fig. 2).

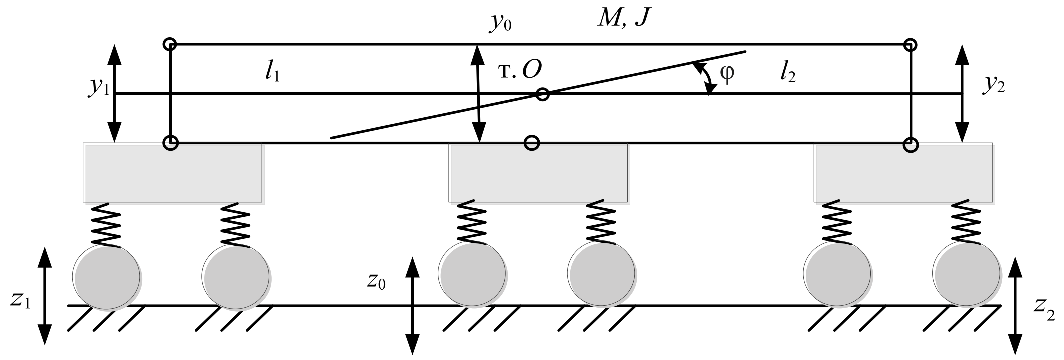

Fig. 2. Schematic diagram of a six-axle freight car

With a car's own weight of about 23 tons, and an axial load of 22 tons, this can provide an increase in the weight of transported goods by approximately 20 tons, i.e., significantly increase the efficiency of transportation processes without creating excessive dynamic loads on the track structure (TS). The practical implementation of the proposed approach requires the modernization of the design of a standard wagon. The modernization consists in creating an additional bogie attachment unit to reduce the load on the axle of a freight wagon by introducing an additional two-axle track. The upgrading of the attachment unit is carried out in the same structural and technical forms as the fastening using pins in two “standard” two-axle bogies, i.e., through the installation of over- and underpin bolsters with the corresponding standard-type pin assembly.

The proposed method of increasing the efficiency of using four-axle freight cars under conditions of additional loads is aimed at solving the problem of increasing the load on the axle of the wheelset to 30 tons, and speeding up the trains. This is achieved through upgrading a typical freight car by installing an additional two-axle bogie with an appropriate device that provides conditions for its dynamic interaction with the frame structure of the freight car.

The installation of an additional two-axle bogie by redistributing the load between a common set of wheelsets provides the possibility of transporting heavy loads while reducing the load on the axle. This maintains better operating conditions for the track and the superstructure while maintaining an acceptable length of the train.

It is assumed that the pin assembly has an elastic rubber gasket that provides cushioning for the interaction of the bogie and the frame of the wagon body. At the same time, it is not supposed to upgrade the wheels of the wheelsets. Only the form of wiring of the elements of the “regular” pneumatic braking system and the configuration of the supply of pneumatic pipelines are changed.

Research Results. The design scheme of the considered railway vehicle in the first approximation can be represented as a mechanical oscillatory system consisting of a solid body with mass m and moment of inertia J, based on three elastic elements with stiffness k_1, k_0, k_2. Kinematic effects are represented by in-phase harmonic functions of the same frequency (Fig. 3).

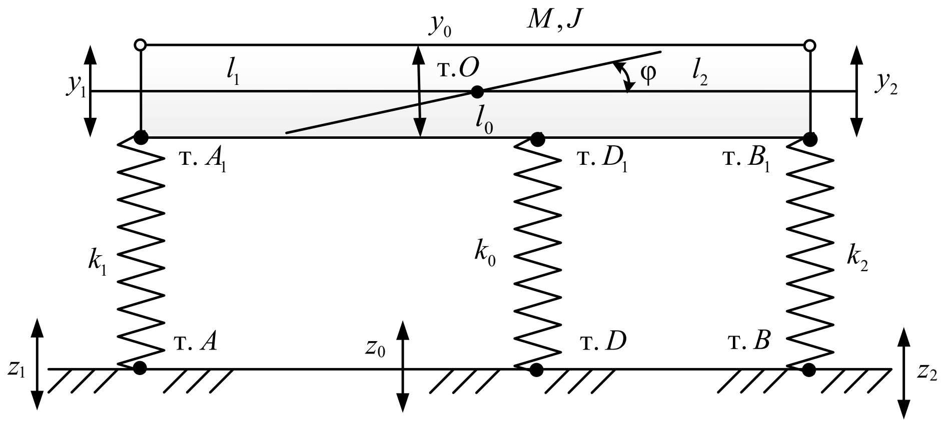

Fig. 3. Design scheme of a railway vehicle
under kinematic disturbance (z_1, z_2, z_0)

The center of mass of the system — point O is located at distances l_1 and l_2 from the ends of the solid (points A_1, B_1). The element with stiffness k_0 is fixed at points D and D_1; distance OD_1 is indicated as l_0. The movement of the system is considered in coordinates y_1, y_2 and y_0, φ, associated with a fixed basis. The calculations use the following ratios:

$$y_0 = ay_1 + by_2,\ \phi = c\left(y_2 - y_1\right),\ y_1 = y_0 - l_1\phi,\ y_2 = y_0 + l_2\phi,$$
$$y_D = y_0 + l_1\phi,\ a = \frac{l_2}{l_1 + l_2},\ b = \frac{l_1}{l_1 + l_2},\ c = \frac{1}{l_1 + l_2}. \tag{1}$$

Lagrange formalism is used to derive differential equations of motion [14], which requires the construction of expressions for kinetic and potential energies. In this case, we have:

$$T = \frac{1}{2}m\left(ay_1' + by_2'\right)^2 + Jc^2\left(y_2' - y_1'\right)^2, \tag{2}$$

$$\Pi = \frac{1}{2}k_1\left(y_1 - z_1\right)^2 + \frac{1}{2}k_0\left(y_D - z_0\right)^2 + \frac{1}{2}k_2\left(y_2 - z_2\right)^2. \tag{3}$$

Note that potential energy (3) can also be written as:

$$\Pi = \frac{1}{2}k_1\left(y_1 - z_1\right)^2 + \frac{1}{2}k_2\left(y_2 - z_2\right)^2 + \frac{1}{2}k_0\left[ay_1 + by_2 + l_0c\left(y_2 - y_1\right) - z_0\right]^2 =$$
$$\frac{1}{2}k_1\left(y_1 - z_1\right)^2 + \frac{1}{2}k_2\left(y_2 - z_2\right)^2 + \frac{1}{2}k_0\left[a_1y_1 + b_1y_2 - z_0\right]^2, \tag{4}$$

where $a_1 = a - l_0c$, $b_1 = b + l_0c$.

The system of equations of motion in coordinates y_1, y_2 in the time domain takes the form:

$$y_1''\left(ma^2 + Jc^2\right) + y_1\left(k_1 + k_0a_1^2\right) - y_2''\left(Jc^2 - mab\right) + y_2k_0a_1b_1 = k_1z_1 + k_0a_1z_0, \tag{5}$$

$$y_2''\left(mb^2 + Jc^2\right) + y_2\left(k_2 + k_0b_1^2\right) - y_1''\left(Jc^2 - mab\right) + y_1k_0a_1b_1 = k_2z_2 + k_0b_1z_0. \tag{6}$$

After applying the Laplace integral transformations under zero initial conditions [15], the system of equations (5), (6) can be represented in the operator form:

$$\bar{y}_1\left[\left(ma^2 + Jc^2\right)p^2 + k_1 + k_0a_1^2\right] - \bar{y}_2\left[\left(Jc^2 - mab\right)p^2 - k_0a_1b_1\right] = k_1\bar{z}_1 + k_0a_1\bar{z}_0, \tag{7}$$

$$\bar{y}_2\left[\left(mb^2 + Jc^2\right)p^2 + k_2 + k_0b_1^2\right] - \bar{y}_1\left[\left(Jc^2 - mab\right)p^2 - k_0a_1b_1\right] = k_2\bar{z}_2 + k_0b_1\bar{z}_0, \tag{8}$$

where $p = j\omega$ — complex variable ($j = \sqrt{-1}$), indicator <–> above the variable means its Laplace image [5].

On the basis of (7), (8), in accordance with the provisions of the method of structural mathematical modeling [5], a structural mathematical model in the form of a block diagram is constructed, which is dynamically equivalent to an automatic control system (Fig. 4).

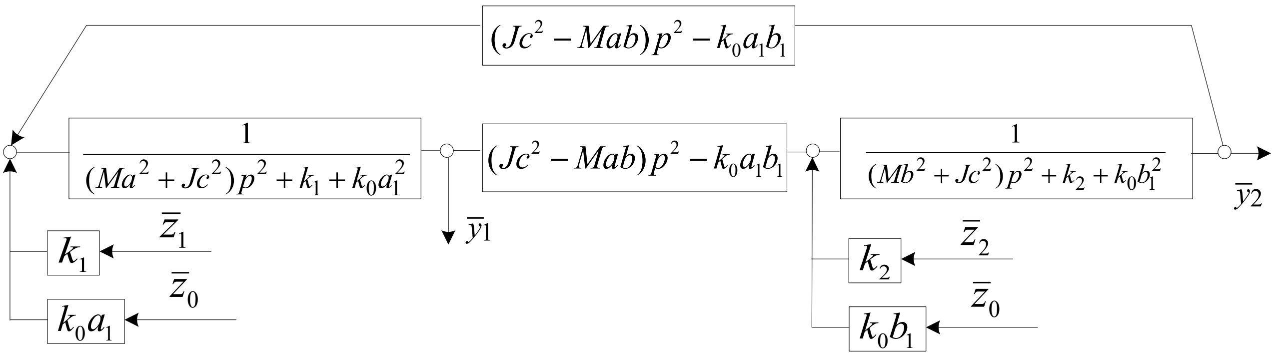

Fig. 4. Structural mathematical model of a railway vehicle

Discussion and Conclusion. The feature of the system is that it has two partial blocks, each of which determines the corresponding partial frequencies:

$$n_1^2 = \frac{k_1 + k_0a_1^2}{ma^2 + Jc^2}, \tag{9}$$

$$n_2^2 = \frac{k_2 + k_0b_1^2}{mb^2 + Jc^2}, \tag{10}$$

which, in turn, determine the boundaries of the location of the natural oscillation frequencies of the system as a whole:

$$\omega_{1\text{соб}}^2 < n_1^2 < n_2^2 < \omega_{2\text{соб}}^2, \tag{11}$$

where $\omega_{1\text{соб}}$, $\omega_{2\text{соб}}$ — natural oscillation frequencies of the system. When operating on them, resonant modes may occur.

Among the features of the system there are several simultaneously acting external disturbances. Assuming, for the sake of simplification, that $\bar{z}_1 = \bar{z}_2 = \bar{z}_0$ (quite acceptable at the stages of preliminary dynamic estimates), we consider that force factors act on the input of the first and second partial blocks:

$$\bar{Q}_1 = \bar{z}\left(k_1 + k_0a_1\right), \tag{12}$$

$$\bar{Q}_2 = \bar{z}\left(k_2 + k_0b_1\right). \tag{13}$$

Using the block diagram in Figure 4, we write down the expressions for the transfer functions, assuming that there is a relationship between the external perturbation factors formed by the ratio:

$$\bar{Q}_1 = \bar{Q}, \bar{Q}_2 = \alpha\bar{Q}, \tag{14}$$

$$W_1(p) = \frac{\bar{y}_1}{\bar{z}} = \frac{\left(k_1 + k_0a_1\right)\left[\left(mb^2 + Jc^2\right)p^2 + k_2 + k_0b_1^2\right] + \alpha\left(k_2 + k_0b_1\right)\left[\left(Jc^2 - mab\right)p^2 - k_0a_1b_1\right]}{A(p)}, \tag{15}$$

$$W_2(p) = \frac{\bar{y}_2}{\bar{z}} = \frac{\alpha\left(k_2 + k_0b_1\right)\left[\left(ma^2 + Jc^2\right)p^2 + k_1 + k_0a_1^2\right] + \left(k_1 + k_0a_1\right)\left[\left(Jc^2 - mab\right)p^2 - k_0a_1b_1\right]}{A(p)}, \tag{16}$$

where

$$A(p) = \left[\left(ma^2 + Jc^2\right)p^2 + k_1 + k_0a_1^2\right]\left[\left(mb^2 + Jc^2\right)p^2 + k_2 + k_0b_1^2\right] - \left[\left(Jc^2 - mab\right)p^2 - k_0a_1b_1\right] \tag{17}$$

is the frequency characteristic equation of the system.

The numerator of the transfer functions in expressions (15), (16) determines the modes of dynamic vibration damping, which can be detailed from the equations obtained by "zeroing" numerators (15), (16):

$$\omega^2_{1\text{дин}} = \frac{\left(k_1 + k_0 a_1\right)\left(k_2 + k_0 b_1^2\right) - \alpha\left(k_2 + k_0 b_1\right) k_0 a_1 b_1}{\left(k_1 + k_0 a_1\right)\left(mb^2 + Jc^2\right) + \alpha\left(k_2 + k_0 b_1\right)\left(Jc^2 - mab\right)}, \tag{18}$$

$$\omega^2_{2\text{дин}} = \frac{\left(k_2 + k_0 b_1\right)\left(k_1 + k_0 a_1^2\right) - \alpha\left(k_1 + k_0 a_1\right) k_0 a_1 b_1}{\alpha\left(k_1 k_2 + k_0 b_1\right)\left(ma^2 + Jc^2\right) + \left(k_1 k_2 + k_0 a_1\right)\left(Jc^2 - mab\right)}. \tag{19}$$

It follows from expressions (18), (19) that in a system with two degrees of freedom, in the presence of two interconnected disturbing factors, dynamic vibration damping modes at two frequencies may occur, whose parameters depend on the values of connectivity coefficient α. This coefficient can take negative, positive and zero values.

The analysis of the structural mathematical model (Fig. 4) also implies the possibility of a special dynamic mode at the frequency:

$$\omega^2_{\text{спец}} = \frac{k_0 a_1 b_1}{Jc^2 - mab}, \tag{20}$$

when the partial blocks get the possibility of disconnection. In this case, the system (Fig. 3) splits into two autonomous blocks, which do not create situations of interaction of partial structures.

The implementation of this mode can cause a significant difference in the values of deviations at points A_1 and B_1, and a "spread" of values of the dynamic reactions at points A_1, B_1 and D_1. The determination of dynamic reactions at points A, B and D can be implemented according to the methodology described in [14], in which the dynamic reaction is defined as the product of a dynamic displacement (e.g., points A_1, B_1 and D_1) by the value of the reduced dynamic stiffness.

For coordinate $\bar{y}_1$, the dynamic offset is determined from the expression for transfer function (15), and for coordinate $\bar{y}_2$ — from expression (16). The frequency characteristic equation is used to determine the reduced dynamic stiffness (17):

$$k_{\text{пр}1}(p) = k_1 + \frac{k_0 a^2 a_1}{\left(mb^2 + Jc^2\right)p^2 + k_2 + k_0 b_1^2} - \frac{\left[\left(Jc^2 - mab\right)p^2 - k_0 a_1 b_1\right]^2}{\left(mb^2 + Jc^2\right)p^2 + k_2 + k_0 b_1^2}. \tag{21}$$

Similarly, the value of the reduced dynamic stiffness in coordinate $\bar{y}_2$ can be found:

$$k_{\text{пр}2}(p) = k_2 + \frac{k_0 b^2 b_1}{\left(ma^2 + Jc^2\right)p^2 + k_1 + k_0 a_1^2} - \frac{\left[\left(Jc^2 - mab\right)p^2 - k_0 a_1 b_1\right]^2}{\left(ma^2 + Jc^2\right)p^2 + k_1 + k_0 a_1^2}. \tag{22}$$

To determine the specific values $\bar{k}_{\text{пр}1}(p)$ and $\bar{k}_{\text{пр}2}(p)$, it is necessary to find the value of frequency ω^2, which is determined from the condition that $\bar{y}_2 / \bar{y}_1 = 1$. More generally, it is assumed that:

$$\frac{\bar{y}_2}{\bar{y}_1} = \gamma, \tag{23}$$

where γ — coefficient of connectivity of the movement of elements by coordinates $\bar{y}_1$ and $\bar{y}_2$. Thus, this coefficient (for the case γ=1) can be written as:

$$\gamma = \frac{\bar{y}_2}{\bar{y}_1} = \frac{\alpha\left(k_2 + k_0 b_1\right)\left[\left(ma^2 + Jc^2\right)p^2 + k_1 + k_0 a_1^2\right] + \left(k_1 + k_0 a_1\right)\left[\left(Jc^2 - mab\right)p^2 - k_0 a_1 b_1\right]}{\left(k_1 + k_0 a_1\right)\left[\left(mb^2 + Jc^2\right)p^2 + k_2 + k_0 b_1^2\right] + \alpha\left(k_2 + k_0 b_1\right)\left[\left(Jc^2 - mab\right)p^2 - k_0 a_1 b_1\right]}. \tag{24}$$

Taking specific values γ, it is possible to find the oscillation frequencies for the motion of the object in question in coordinates $\bar{y}_1$ and $\bar{y}_2$. Specifically, at γ = 1, the frequency of translational vertical vibrations of a solid is determined from the expression:

$$\omega^2=\frac{\bar{y}_2}{\bar{y}_1}=\frac{\alpha\left(k_2+k_0b_1\right)\left(k_1+k_0a_1^2\right)-\left(k_1+k_0a_1\right)k_0a_1b_1+\left(k_1+k_0a_1\right)k_0a_1b_1-}{\alpha\left(k_2+k_0b_1\right)\left(ma^2+Jc^2\right)+\left(k_1+k_0a_1\right)\left(Jc^2-mab\right)-}\cdots$$
$$\cdots\frac{-\left(k_1+k_0a_1\right)\left(k_2+k_0b_1^2\right)-\left(k_2+k_0b_1\right)\left(k_1+k_0a_1^2\right)+\alpha\left(k_2+k_0b_1\right)k_0a_1b_1}{-\left(k_1+k_0a_1\right)\left(mb^2+Jc^2\right)-\alpha\left(k_2+k_0b_1\right)\left(Jc^2-mab\right)}. \quad (25)$$

Frequency ω of in-phase harmonic disturbances $z_1(t)$, $z_2(t)$ and $z_0(t)$ provides the motion of a solid body at $\varphi = 0$, i.e., $\bar{y}_1=\bar{y}_2=\bar{y}_D$. For other values γ (γ ≠ 1), $\bar{y}_1$ and $\bar{y}_2$ are determined, and on their basis, for geometric reasons, value $\bar{y}_D$ is determined. For a solid body with known $\bar{y}_1$ and $\bar{y}_2$, the position of the center of rotation (or oscillations) can be easily found [12].

The dynamic constraint reactions $\bar{R}_{A1}$, $\bar{R}_{B1}$ and $\bar{R}_{D1}$ can be found in the first approximation by the formulas:

$$\bar{R}_{A1}=k_1\bar{y}_1,\ \bar{R}_{B1}=k_2\bar{y}_2,\ \bar{R}_{D1}=k_0\bar{y}_D.$$

In general, dynamic reactions are determined using dynamic displacements $\bar{y}_1, \bar{y}_2, \bar{y}_D$, that are defined by expressions (15), (16). As for the dynamic displacement at point *D*, expression $\bar{y}_D=a_1\bar{y}_1+b_1\bar{y}_2$ is used, whose parameters are determined by the above values. In expressions (15), (16), data on the connectivity of force factors of influence (parameter α) can be entered. To obtain specific data on the values of dynamic reactions, frequency parameters are introduced at which the ratio of oscillation amplitudes $\bar{y}_2$ and $\bar{y}_1$ is implemented through the coefficient of coupling of oscillation amplitudes γ.

The overall restraint force at points *A*, *B* and *D* is determined by the sum of two components: static and dynamic. The static component can be found from the expression for the transfer functions of dynamic reactions at $p = 0$ and setting the parameters of the static load (weight of the wagon and the cargo being transported). With the intensive development of oscillatory processes, when fluctuations in coordinates $\bar{y}_1, \bar{y}_2, \bar{y}_D$ increase, the overall reaction can vary significantly and differ from the static restraint force. In the presence of a dynamic component, the overall reaction can take on various values, in particular, zero or negative.

The mathematical model formed within the framework of the proposed method is indicated by expression (25). It provides assessing the dynamic state of railway vehicles when additional connections are introduced into their structure to form a set of recommendations for obtaining stable modes of operation. The investigation of the features of the system using approaches typical of structural mathematical modeling allows us to consider in detail the connections between the elements. With regard to the technical object in question, this makes it possible to adjust the dynamic state of the technical object, based on varying the parameters of the set of additionally introduced elements, to reduce the load on the main parts of the suspension, as well as to establish the presence of natural frequencies and frequencies of dynamic vibration damping in the system.

In the future, it is planned to conduct research on the introduction of dampers and motion conversion devices into the structure of the vehicle to assess the possibilities of structural mathematical modeling. Another interesting area is the assessment of the possibilities of changing dynamic reactions depending on external actions, which will allow us to assess the efforts exerted on various elements of the vehicle suspension.

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