

MECHANICS МЕХАНИКА








UDC 534.1

Original Theoretical Research

<https://doi.org/10.23947/2687-1653-2024-24-2-148-158>

On a Method for Calculating Bending and Shear Vibrations of a Porous Piezoelement in the Low-Frequency Region

Arkadiy N. Soloviev^{1,2,3}  , Valery A. Chebanenko^{2,3,4} , Pavel A. Oganesyan² ,
Elizaveta I. Fomenko² 



¹ Crimean Engineering and Pedagogical University named after Fevzi Yakubov, Simferopol, Republic of Crime EDN: VPUERG

² Southern Federal University, Rostov-on-Don, Russian Federation

³ Don State Technical University, Rostov-on-Don, Russian Federation

⁴ Southern Research Center, Russian Academy of Sciences, Rostov-on-Don, Russian Federation

 solovievare@gmail.com

Abstract

Introduction. Devices for collecting and storing energy from the external environment are low-power sources of electric energy that are actively used. The autonomous devices for monitoring the damaged condition of various structures include them as well. The working element of these devices is a piezoelectric generator (PEG) — a converter of mechanical energy into electrical energy. The design of PEG is associated with the preliminary construction of their mathematical and computer models, with the help of which the calculation and optimization of structures is carried out. One of the ways to model and calculate PEG is to develop approximate calculation methods based on applied theories. The applied theories for calculating bending vibrations of multilayer piezoelectric plates are known and previously developed in the literature. However, in the scientific literature there is not enough information about bending and shear vibrations as a tool for improving the efficiency of engineering calculations of the described structures. The objective of this work was to develop an applied method for calculating bending and shear vibrations of piezoceramic plates, including porous ones.

Materials and Methods. Piezoceramics PZT-4, including porous ones, were used as the piezoelectric material of the plate. When using porous ceramics, the rigidity of the structure decreased to a greater extent than the piezoelectric modules, which made it possible to obtain a more effective PEG under mechanical action. The mathematical formulation was carried out within the framework of the linear theory of electroelasticity with plate polarization in thickness. The sides of the plate were electroded, the right side was fixed, and a smooth contact in the vertical wall was set on the left side. Steady-state vibrations of the plate were caused by pressure on the front surfaces of the plate or the difference in electrical potentials at the electrodes. To calculate the characteristics of PEG, the authors proposed an applied theory based on hypotheses about the distribution of characteristics of the stress-strain state and the electric field.

Results. Transverse vibrations of a piezoceramic plate in the low-frequency region (below the first bending-shear resonance) were studied. Due to the fact that the mathematical formulation was considered within the framework of the linear theory of elasticity, the problem was divided into the sum of two. The first one took into account the mechanical effect: a distributed load and a transverse force at the left end acted on the front surfaces of the plate, and the potentials at the electrodes were zero. In the second task, there were no mechanical loads, but the potential difference was set at the electrodes. Based on hypotheses about the distribution of deformations, mechanical stresses and electric potential, both problems were reduced to a system of ordinary differential equations and boundary conditions. Comparison with the results of calculations by the finite element method in the ACELAN package showed the adequacy of the proposed applied theory in the low-frequency region.

Discussion and Conclusion. Since the formulation of the problem was considered in the linear theory of electroelasticity, and the low-frequency region was studied, the work succeeded in dividing the problem of bending-shear vibrations of a porous piezoceramic plate into two: bending — with mechanical action at zero potentials, and shear — when setting the potential difference and zero mechanical action. The corresponding hypotheses about bending and shear were used. Two systems of ordinary differential equations and boundary conditions, which were solved analytically without the use

of “heavy” finite element packages, were constructed. To compare the results and confirm the adequacy of the proposed method, the finite element modeling of such tasks was carried out in a specialized ACELAN package. The comparison showed that the error in determining displacements and electric potential when using this approach, in the case of setting mechanical loads and potential differences, did not exceed 6%. The method developed in the paper can be applied in the design of piezoelectric generators for energy storage in the low-frequency region.

Keywords: energy collection device, piezoelectric generator, porous ceramics, plate bending, plate shear, applied theory

Acknowledgements. The authors would like to thank the Editorial board of the journal and the reviewers for their attentive attitude to the article.

Funding Information. The research was done at the Southern Federal University with the financial support from the Russian Science Foundation (grant no. 22–11–00302). <https://rscf.ru/project/22-11-00302/>

For Citation. Soloviev AN, Chebanenko VA, Oganessian PA, Fomenko EI. On a Method for Calculating Bending and Shear Vibrations of a Porous Piezoelement in the Low-Frequency Region. *Advanced Engineering Research (Rostov-on-Don)*. 2024;24(2):148–158. <https://doi.org/10.23947/2687-1653-2024-24-2-148-158>

Оригинальное теоретическое исследование

Об одном методе расчета изгибных и сдвиговых колебаний пористого пьезоэлемента в низкочастотной области

А.Н. Соловьев^{1,2,3}  , В.А. Чебаненко^{2,3,4} , П.А. Оганесян² , Е.И. Фоменко² 

¹ Крымский инженерно-педагогический университет имени Февзи Якубова, г. Симферополь, Республика Крым, Российская Федерация

² Южный Федеральный Университет, г. Ростов-на-Дону, Российская Федерация

³ Донской государственный технический университет, г. Ростов-на-Дону, Российская Федерация

⁴ Южный Научный центр РАН, г. Ростов-на-Дону, Российская Федерация

 solovievare@gmail.com

Аннотация

Введение. Устройства сбора и накопления энергии из внешней среды представляют собой маломощные источники электрической энергии, которые активно используются, в том числе в автономных приборах мониторинга поврежденного состояния различных конструкций. Рабочим элементом этих устройств является пьезоэлектрический генератор (ПЭГ) — преобразователь механической энергии в электрическую. Конструирование ПЭГ связано с предварительным построением их математических и компьютерных моделей, с помощью которых производится расчет и оптимизация конструкций. Одним из способов моделирования и расчета ПЭГ является разработка приближенных методов расчета на основе прикладных теорий. В литературе известны и ранее разработаны прикладные теории расчета изгибных колебаний многослойных пьезоактивных пластин. Однако информации об изгибно-сдвиговых колебаниях, как инструменте повышения эффективности инженерных расчетов описанных конструкций, в научной литературе недостаточно. Целью настоящей работы являлась разработка прикладного метода расчета изгибных и сдвиговых колебаний пьезокерамических пластин, в том числе пористых.

Материалы и методы. В качестве пьезоактивного материала пластины используется пьезокерамика PZT-4, в том числе пористая. При использовании пористой керамики жесткость конструкции уменьшается в большей степени, чем пьезомодули, что позволяет получить более эффективный ПЭГ при механическом воздействии. Математическая постановка осуществлена в рамках линейной теории электроупругости при поляризации пластины по толщине. Боковые стороны пластины электродированы, правая сторона закреплена, а на левой задан гладкий контакт в вертикальной стенке. Установившиеся колебания пластины вызываются давлением на лицевые поверхности пластины или разностью электрических потенциалов на электродах. Для расчета характеристик ПЭГ в работе предлагается прикладная теория, основанная на гипотезах о распределении характеристик напряженно-деформированного состояния и электрического поля.

Результаты исследования. Рассмотрены поперечные колебания пьезокерамической пластины в низкочастотной области (ниже первого изгибно-сдвигового резонанса). В силу того, что математическая постановка рассмотрена в рамках линейной теории упругости, задача разделилась на сумму двух. В первой учитывалось механическое воздействие: на лицевые поверхности пластины действует распределенная нагрузка и поперечная сила на левом конце, а потенциалы на электродах равны нулю. Во второй задаче механические нагрузки отсутствовали, но задавалась разность потенциалов на электродах. На основе гипотез о распределении деформаций, механических

напряжений и электрического потенциала обе задачи были сведены к системе обыкновенных дифференциальных уравнений и граничных условий. Сравнение с результатами расчетов методом конечных элементов в пакете ACELAN показали адекватность предложенной прикладной теории в низкочастотной области.

Обсуждение и заключение. Поскольку постановка задачи рассматривалась в линейной теории электроупругости и изучалась низкочастотная область, в работе удалось задачу об изгибных и сдвиговых колебаниях пластины из пористой пьезокерамики разделить на две: изгибную — с механическим воздействием при нулевых потенциалах и сдвиговую — при задании разности потенциалов и нулевом механическом воздействии. Использованы соответствующие гипотезы об изгибе и сдвиге, построены две системы обыкновенных дифференциальных уравнений и граничных условий, которые решаются аналитически без использования «тяжелых» конечно-элементных пакетов. Для сравнения результатов и подтверждения адекватности предложенного метода проведено конечно-элементное моделирование таких задач в специализированном пакете ACELAN. Это сравнение показало, что ошибка в определении смещений и электрического потенциала при использовании этого подхода, в случае задания механических нагрузок и разности потенциалов, не превышает 6 %. Разработанный в статье метод может быть применен при проектировании пьезоэлектрических генераторов накопления энергии в низкочастотной области.

Ключевые слова: устройство сбора энергии, пьезоэлектрический генератор, пористая керамика, изгиб пластины, сдвиг пластины, прикладная теория

Благодарности. Авторы выражают благодарность редакции журнала и рецензентам за внимательное отношение к статье.

Финансирование. Исследование выполнено в Южном федеральном университете при финансовой поддержке Российского научного фонда (грант № 22–11–00302), <https://rscf.ru/project/22-11-00302/>

Для цитирования. Соловьев А.Н., Чебаненко В.А., Оганесян П.А., Фоменко Е.И. Об одном методе расчета изгибных и сдвиговых колебаний пористого пьезоэлемента в низкочастотной области. *Advanced Engineering Research (Rostov-on-Don)*. 2024;24(2):148–158. <https://doi.org/10.23947/2687-1653-2024-24-2-148-158>

Introduction. Piezoelectric generators (PEG) are used to convert mechanical energy into electrical energy, followed by its accumulation. One of the application areas of PEG is the creation of low-power autonomous renewable sources of electric energy. The working element of the PEG is a piezoceramic element of a certain shape. The shape and type of deformation of this element determine the piezomodule, which characterizes the conversion of mechanical deformation energy into electrical energy. Thus, piezomodule d_{33} is associated with tension-and-compression along the axis of polarization, d_{31} — with the same deformation in the transverse direction to this axis, d_{15} — with shear. The use of porous ceramics makes it possible to create more efficient PEG. This is due to the fact that the elastic modules of porous ceramics decrease significantly stronger with increasing porosity than piezomodules. Thus, under the same mechanical load, the deformation amplitude of porous ceramics is greater; therefore, the output electric potential is also greater.

PEG calculation can be performed by the finite element method implemented in ANSYS, ACELAN, COMSOL, and others packages. For piezoelectric elements, one or two sizes of which are significantly smaller than others (plates, rods), applied calculation theories can be constructed based on hypotheses about the distribution of mechanical and electric fields. Without the use of “heavy” finite element packages, applied theories make it possible to model various devices based on piezoactive materials. Piezoelectric, piezomagnetic and composite piezomagnetolectric materials are considered as such materials. The construction of these theories is based on the acceptance of hypotheses about the distribution of mechanical, electric and magnetic fields. These hypotheses are related to the vibration mode of elastic and piezoactive elements of PEG. The most common designs are active and semipassive bimorphs based on multilayer plates, polarized in thickness with electrodes on the front faces, performing transverse bending vibrations. A number of papers are devoted to the study of devices with shear deformation of piezoelectric elements. An electric model with piezoelectric defining equations of mode d_{15} and a single-degree-of-freedom model were combined to describe the energy collection characteristics of a piezoelectric cantilever in a shear mode in operation [1]. The proposed model is used to simulate the frequency dependence of the output peak voltage and power. The results show a good agreement with the experiment and the finite element calculation in ANSYS. In [2], a piezoelectric shear mode energy converter was developed to use the energy of a pressurized water flow. It converts the energy of the flow into electrical energy through piezoelectric conversion with vibration of the piezoelectric film. A finite element model has been developed to estimate the generated voltage of a piezoelectric film, which is in good agreement with the conducted field experiment. A one-dimensional fully coupled beam vibration model based on Timoshenko-type hypotheses, which provides a single common basis for energy analysis in shear and bending modes, is presented in [3]. In [4], the effect of the inhomogeneity of the properties of the plate under shear and torsional vibrations of its central part was studied. In experimental work [5], a multilayer cylindrical piezoelectric shear actuator (MCPSA) operating in shear mode d_{15} , was presented for precision actuation under high

mechanical load. The actuator was made of piezoelectric ceramic rings $\text{Pb}(\text{Zr,Ti})\text{O}_3$ (PZT-51), which were concentrically assembled together in an electrically parallel connection with alternately positive and negative polarization in the axial direction. In [6], metamaterial of identical elementary cells was created, an artificial prototype of a device with characteristic patterned electrodes and piezoceramic subunits arranged in a row was designed and manufactured, which, as proven, ideally generated the synthetic shear deformation of the face. At the same excitation voltage, there was an increase in the displacement of the shear type by more than an order of magnitude, compared to the previous volumetric elements in mode d_{15} . In the static formulation in [7], the field of electromechanical coupling in the shear-bending mode for an annular piezoelectric plate was theoretically established. In accordance with the classical theory of elastic plates of small bending and piezoelectric defining equations, an analytical solution of the bending deformation of the piezoactuator under the action of an electric field and a concentrated or evenly distributed mechanical load was achieved. The mechanism of generating bending deformation was explained by axisymmetric shear deformation, which additionally caused bending deformation of one piezoelectric plate in the form of a ring. This mechanism differs significantly from the mechanism of piezoelectric bimorphic or unimorphic drives, which were previously reported. The design of the annular piezoactuator has been optimized. In [8], a one-dimensional model was used to construct a sensor response function based on shear resonators (quartz cuts) of a bulk acoustic wave, which are promising for in-line measurements of fluid viscosity, e.g., in industrial processes. In [9], using the finite element method, a piezoelectric flying height control converter was investigated using a shear model deformation. In [10], the theory of a functionally graded plate with four-unknown shear deformations was used to express the displacement component. The distribution of the electric potential was a linear function of thickness. The plate was under mechanical load and electrical voltage. The basic equations and boundary conditions were derived using the virtual work principle. The analysis of stresses and deformations from the design parameters was performed. The electromechanical analysis of the stability loss of a piezoelectric nanoplate under shear using a modified theory of paired stresses with different boundary conditions was studied in [11]. To take into account the electrical effects, an external electrical voltage was applied to the piezoelectric nanoplate. A simplified theory of the first-order shear deformation was used. The basic differential equations were obtained using the Hamilton principle and nonlinear Von Karman deformations. Finally, the results showed that the effect of external electrical voltage on the critical shear load arising on the piezoelectric nanoplate was insignificant. In [12], using a combination of two classical approaches to modeling the nonlinear behavior of piezoelectric materials, a piezoelectric shear drive for an atomic-force microscope was investigated. Specifically, the novelty of the proposed method was in the fact that it combined two sources of nonlinearity of the field-dependent model from Müller and Zhang with the frequency-dependent model from Damjanović. The numerical results obtained using the finite element method (FEM) were compared to the experiment.

Vibrations in which there is shear in addition to bending are less studied in the scientific literature, i.e., piezomodule d_{15} , is the “working” one, whose value decreases with increasing porosity, but to a lesser extent than elastic modules. The latter circumstance makes it possible to build an efficient energy conversion device. Therefore, the development of an applied theory of PEG calculation using porous piezoceramics based on simplified models without “heavy” finite element packages seems to be a highly relevant task. The objective of this work was to construct an applied calculation method for steady-state transverse vibrations in the low-frequency region of a porous piezoceramic plate characterized by both shear and bending.

Materials and Methods. The PEG under study was a piezoceramic plate (length l , thickness h), polarized in thickness, cantilevered on the right side, the left side was attached to an inertial mass that performed vertical vibrations and was fixed in the horizontal direction. The electrodes were located on the sides of the plate; therefore, with a potential difference on them and no mechanical load, the principal deformation in the low-frequency region was shear. Vibrations whose frequency was less than the frequency of the first resonance were considered.

Mathematical formulation of the problem

The mathematical formulation of the problem is described by a system of differential equations [13] and the corresponding boundary conditions.

$$\begin{aligned} \rho_{pk} \ddot{\mathbf{u}} + \alpha_{dj} \rho_j \dot{\mathbf{u}} - \nabla \cdot \boldsymbol{\sigma} &= \mathbf{f}_j & \nabla \cdot \mathbf{D} &= 0 \\ \boldsymbol{\sigma} &= \mathbf{c}_j^E \cdot (\boldsymbol{\varepsilon} + \beta_{dj} \dot{\boldsymbol{\varepsilon}}) - \mathbf{e}_j^T \cdot \mathbf{E} & \mathbf{E} &= -\nabla \varphi \\ \mathbf{D} + \zeta_d \dot{\mathbf{D}} &= \mathbf{e}_j \cdot (\boldsymbol{\varepsilon} + \zeta_d \dot{\boldsymbol{\varepsilon}}) + \mathfrak{I}_j^S \cdot \mathbf{E} & \boldsymbol{\varepsilon} &= (\nabla \mathbf{u} + \nabla \mathbf{u}^T) / 2 \end{aligned} \quad (1)$$

When considering porous ceramics of connectivity 3–0, equations (1) use effective physical constants determined through the ACELAN-COMPOS package [14]. These effective properties based on representative volumes (Fig. 1) were obtained in [15]. They are presented in Table 1.

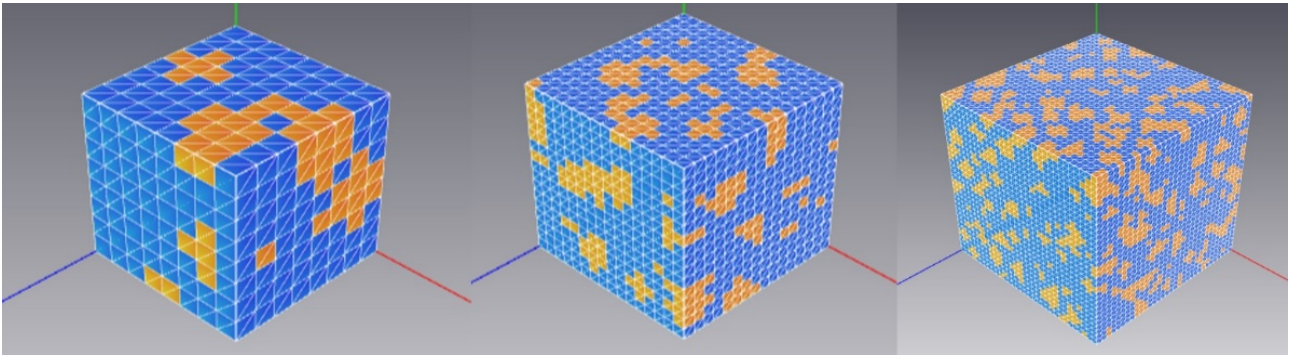


Fig. 1. Representative volumes in ACELAN-COMPOS package [14]

Table 1

Effective properties of porous ceramics

% of porosity	0	10	20	30	40	50	60	70	80
ρ , kg/m ³	7,500	6,750	6,000	5,250	4,500	3,750	3,000	2,250	1,500
$c_{11}^{E\text{eff}}$, 10 ¹⁰ , N/m ²	13.90	11.56	9.25	6.85	5.05	3.34	2.07	1.26	0.68
$c_{12}^{E\text{eff}}$, 10 ¹⁰ , N/m ²	7.78	6.15	4.66	3.14	2.10	1.16	0.62	0.28	0.13
$c_{13}^{E\text{eff}}$, 10 ¹⁰ , N/m ²	7.43	5.82	4.25	2.82	1.70	1.06	0.52	0.24	0.10
$c_{33}^{E\text{eff}}$, 10 ¹⁰ , N/m ²	11.50	9.53	7.23	5.42	3.91	2.72	1.63	0.91	0.47
$c_{44}^{E\text{eff}}$, 10 ¹⁰ , N/m ²	2.56	2.23	1.83	1.44	1.10	0.74	0.44	0.23	0.10
e_{33}^{eff} , C/m ²	15.10	13.38	11.37	9.59	7.68	5.93	3.93	2.30	1.25
e_{31}^{eff} , C/m ²	−5.20	−4.23	−3.14	−2.07	−1.32	−0.75	−0.43	−0.21	−0.10
e_{31}^{eff} , C/m ²	12.70	10.96	8.96	6.91	5.00	3.30	1.95	1.00	0.44
$\kappa_{11}^{S\text{eff}} / \varepsilon_0$	730	663	582	509	439	349	263	191	122
$\kappa_{33}^{S\text{eff}} / \varepsilon_0$	635	567	492	413	345	270	199	130	75

According to Table 1, the dependences of effective elastic modules and piezomodules on the percentage of porosity [15] are plotted. They are shown in Figure 2.

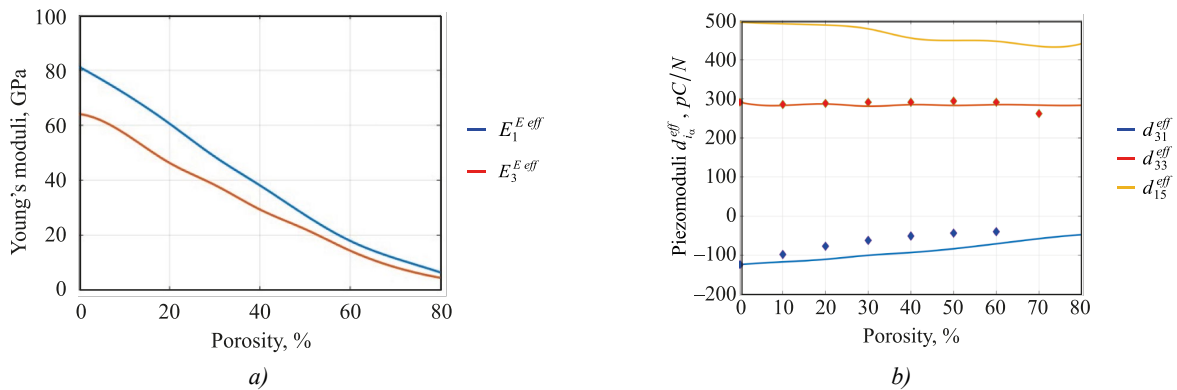


Fig. 2. Dependences of values on percentage of porosity [15]: a — elastic models; b — piezomodules [15]

In accordance with the results shown in Figure 2, elastic modulus E_1 decreases significantly faster than piezomodule d_{15} .

Building an applied theory

The developed calculation method consists of solving two problems: in the first, bending under the effect of a mechanical load with zero potential difference is considered; in the second, the shear caused by a potential difference in the absence of a mechanical load is modeled. In both cases, the absence of charges on the front faces of the plate is taken into account. Under the action of a mechanical load and an electrical potential difference, the results of the two tasks add up due to the linearity of the task.

The action of the mechanical load, the potentials on the electrodes are zero. Hypotheses of the Kirchhoff-Love type are accepted regarding the equality of normal stresses to zero, and for displacements:

$$u_3 = UZ_2(x), u_1 = -\left(\frac{d}{dx}UZ_2(x)\right)z. \quad (2)$$

The distribution of electric potential over the thickness is assumed to be quadratic and symmetrical:

$$\phi = \Phi_1(x)z\left(2, \frac{z}{h} - 1\right)h^{-1} + \Phi_1(x)z\left(1 + 2\frac{z}{h}\right)h^{-1} + \Phi_2(x)\left(1 - 4\frac{z^2}{h^2}\right). \quad (3)$$

To take into account the boundary conditions at the ends of the plate ($x = 0, l$), an expression for the transverse force is obtained:

$$\begin{aligned} Q_1 = & \left(-\frac{1}{12} \left[-\frac{c_{13}e_{33} \left(\frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)} - \frac{8}{h^2} \right)}{c_{33}} + e_{31} \left(\frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{8}{h^2} \right) \right] h^3 - \right. \\ & \left. -\frac{1}{12} e_{15} \left(\frac{1}{2} \frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{4}{h^2} \right) h^3 - e_{15}h \right) \left(\frac{d}{dx} \Phi_2(x) \right) + \\ & + \left(-\frac{1}{12} \left[-c_{11} - \frac{c_{13} \left(\frac{e_{33}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)}{(c_{33}g_{33} + e_{33}^2)h^2} - c_{13} \right)}{c_{33}} + \frac{e_{31}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)}{(c_{33}g_{33} + e_{33}^2)h^2} \right] h^3 - \right. \\ & \left. -\frac{1}{24} \frac{e_{15}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)h}{c_{33}g_{33} + e_{33}^2} \right) \left(\frac{d^3}{dx^3} UZ_2(x) \right). \end{aligned} \quad (4)$$

Taking into account the equality of the normal component of the electric induction vector on the front faces ($z = \pm h$) to zero, the equations for the unknown deflection $UZ_2(x)$ and distribution of the electric potential $\Phi_2(x)$ have the form:

$$\begin{aligned} & \left(-\frac{1}{2} \left[-\frac{c_{13}e_{33} \left(\frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)} - \frac{8}{h^2} \right)}{c_{33}} + e_{31} \left(\frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{8}{h^2} \right) \right] h^3 - \right. \\ & \left. -\frac{1}{2} e_{15} \left(\frac{1}{2} \frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{4}{h^2} \right) h^3 - e_{15}h \right) \left(\frac{d^2}{dx^2} \Phi_2(x) \right) + \\ & + \left(-\frac{1}{2} \left[-c_{11} - \frac{c_{13} \left(\frac{e_{33}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)}{(c_{33}g_{33} + e_{33}^2)h^2} - c_{13} \right)}{c_{33}} + \frac{e_{31}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)}{(c_{33}g_{33} + e_{33}^2)h^2} \right] h^3 - \right. \\ & \left. -\frac{1}{24} \frac{e_{15}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)h}{c_{33}g_{33} + e_{33}^2} \right) \left(\frac{d^4}{dx^4} UZ_2(x) \right) - W^2 \rho h UZ_2(x) - p(x) = 0, \end{aligned} \quad (5)$$

$$\begin{aligned}
 & \left(-\frac{e_{33}^2 \left(\frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{8}{h^2} \right)}{c_{33}} - g_{33} \left(\frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{8}{h^2} \right) \right) \Phi_2(x) + \\
 & + \left(-g_{11} - \frac{1}{2}g_{11} \left(\frac{1}{2} \frac{8c_{33}g_{33} + 8e_{33}^2}{(c_{33}g_{33} + e_{33}^2)h^2} - \frac{4}{h^2} \right) h^2 \right) \left(\frac{d^2}{dx^2} \Phi_2(x) \right) + \\
 & + \left(-\frac{g_{33}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)}{(c_{33}g_{33} + e_{33}^2)h^2} - e_{31} - \frac{e_{33} \left(\frac{(c_{13}e_{33}h^2 - c_{33}e_{31}h^2)}{(c_{33}g_{33} + e_{33}^2)h^2} - c_{13} \right)}{c_{33}} \right) \left(\frac{d^2}{dx^2} UZ_2(x) \right) - \\
 & - \frac{1}{24} \frac{g_{11}(c_{13}e_{33}h^2 - c_{33}e_{31}h^2) \left(\frac{d^4}{dx^4} UZ_2(x) \right)}{c_{33}g_{33} + e_{33}^2} = 0.
 \end{aligned} \tag{6}$$

The potential difference is set, the mechanical load is zero. Independence of lateral displacement from thickness, equality of the longitudinal displacement to zero, and the quadratic distribution of the electric potential over the thickness are assumed (3):

$$u_3 = UZ_2(x), u_1 = 0. \tag{7}$$

Expression for the transverse force:

$$Q_1 = -c_{44} \left(\frac{d}{dx} UZ_2(x) \right) h - e_{15} \left(\frac{d}{dx} \Phi_2(x) \right) h. \tag{8}$$

Taking into account the equality of the normal component of the electric induction vector on the front faces ($z = \pm h$) to zero, the equations for the unknown deflection $UZ_2(x)$ and distribution of the electric potential $\Phi_2(x)$ have the form:

$$-c_{44} \left(\frac{d^2}{dx^2} UZ_2(x) \right) h - e_{15} \left(\frac{d^2}{dx^2} \Phi_2(x) \right) h - p(x) - W^2 \rho h UZ_2(x) = 0, \tag{9}$$

$$e_{15} \frac{d^2}{dx^2} UZ_2(x) - g_{11} \frac{d^2}{dx^2} \Phi_2(x) = 0. \tag{10}$$

Research Results. The results of calculations based on the proposed applied theories were compared to the calculations of piezoelectric element vibrations ($l = 0.1$ m, $h = 0.01$ m) at a frequency equal to 100 s^{-1} by the finite element method in ACELAN [16].

In the first problem, defined by equations (5) and (6), when specifying a uniformly distributed load $p(x) = 1000 \text{ Pa}\cdot\text{m}$ and with boundary conditions:

$$\frac{d}{dx} UZ_2(0) = 0, Q_1|_{x=0} = 0, \Phi_2(0) = \Phi_2(l) = 0, UZ_2(l) = 0, \frac{d}{dx} UZ_2(l) = 0, \tag{11}$$

the following results were obtained, shown in Figures 3, 4.

The calculations showed that the error in determining the vertical displacement was 5.8%, and for the horizontal displacement, it was 1.2%.

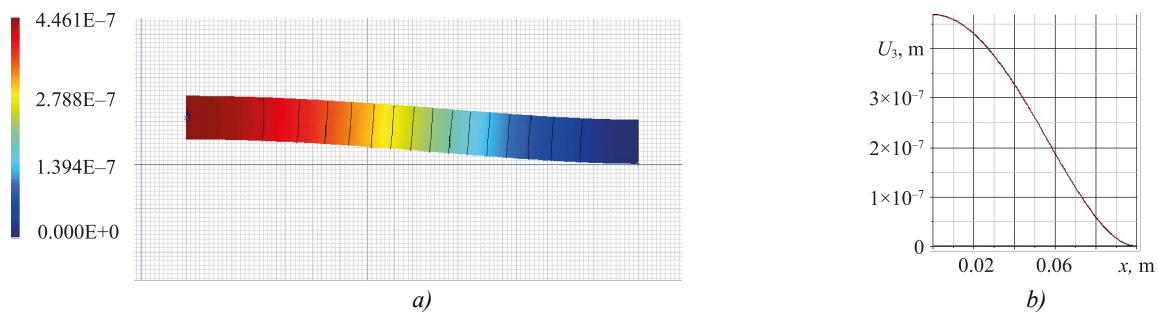


Fig. 3. Vertical displacement in ACELAN in the first problem: a — distribution; b — graph on the upper boundary

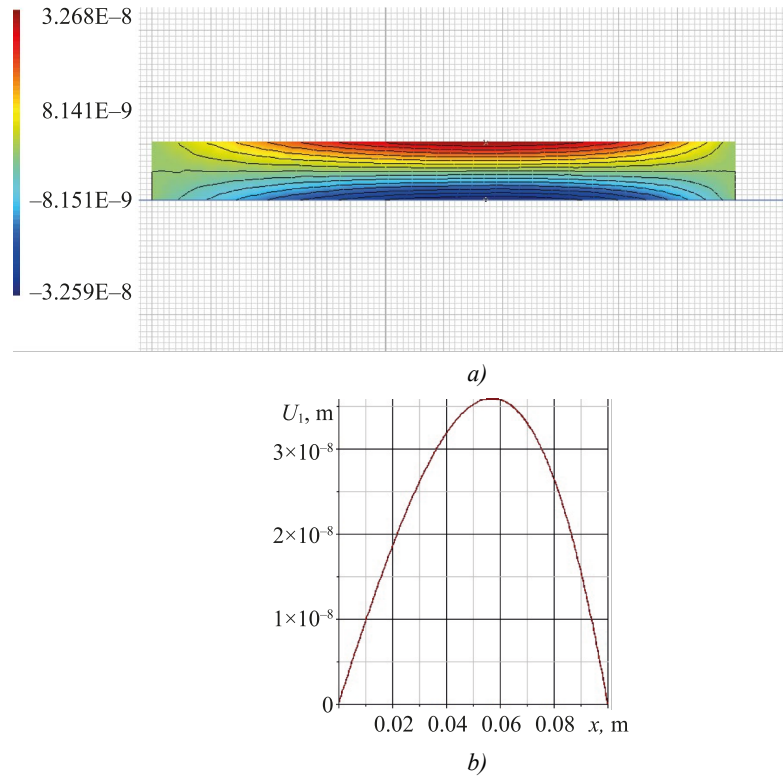


Fig. 4. Horizontal displacement in ACELAN in the first problem: *a* — distribution; *b* — graph on the upper boundary

In the second problem, defined by equations (8) and (9), when setting zero load $p(x) \equiv 0$, potential difference $V_0 = 100$ W and with boundary conditions:

$$Q_1|_{x=0} = 0, \Phi_2(0) = 0, \Phi_2(l) = V_0, UZ_2(l) = 0, \quad (12)$$

the following results were obtained, shown in Figures 5, 6.

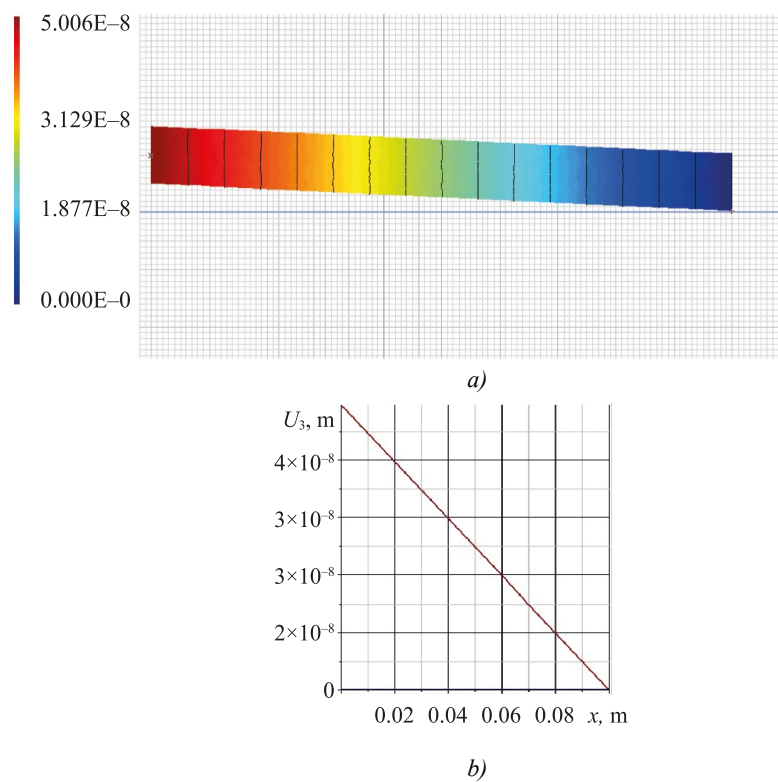


Fig. 5. Vertical displacement in ACELAN in the second problem: *a* — distribution; *b* — graph on the upper boundary

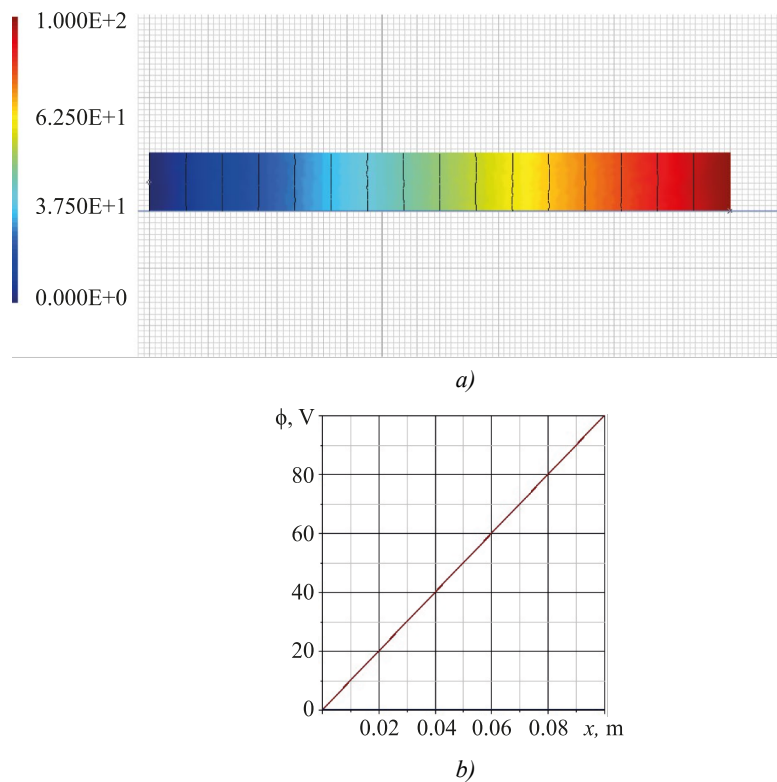


Fig. 6. Electric potential in ACELAN in the second problem: *a* — distribution; *b* — dependence on the longitudinal coordinate in the middle of the thickness

The calculations showed that the error in determining the vertical displacement was 0.8%, and for the electric potential, it was less than 1%. It should be noted that the values of the horizontal displacement calculated in ACELAN turned out to be three orders of magnitude less than the maximum vertical displacement, which indicated the adequacy of the hypothesis (7).

When setting mechanical loads and potential differences, the error of the proposed method turned out to be about 6 % for the displacement and electrical potential components.

Discussion and Conclusion. As noted in the cited literature, the simultaneous use of bending and shear of a piezoelectric element can significantly increase its efficiency. In addition, the use of porous ceramics, due to different dependences of elastic modules and piezomodules on the percentage of porosity, also improves the output characteristics of PEG.

In this paper, due to the linear formulation in the theory of electroelasticity, it was possible to develop an applied theory for calculating bending-shear vibrations of a piezoelectric element in the low-frequency region, which consisted of solving two problems: in the first, mechanical loads operated at zero potentials, and in the second, on the contrary, there were zero mechanical loads, and a potential difference was set. Based on various hypotheses about the distribution of mechanical and electric fields, two boundary value problems for systems of ordinary differential equations were obtained, which were solved analytically. The results of the calculation of displacements and electric potential were compared using the proposed method and the FEM implemented in the ACELAN package. These calculations validated the applicability of the proposed method, the error for which in calculating the above characteristics was 6%. This accuracy is sufficient for engineering calculations; therefore, the proposed method can be applied in the design of piezoelectric devices, including the collection and storage of energy. Further development of this applied theory will be aimed at covering a wider frequency range, including the first flexural shear resonance.

References

1. Liang Zhou, Jing Sun, Xuejun Zheng, Shuifeng Deng, Jihe Zhao, Jihe Zhao, et al. A Model for the Energy Harvesting Performance of Shear Mode Piezoelectric Cantilever. *Sensors and Actuators A: Physical*. 2012;179:185–192. <http://doi.org/10.1016/j.sna.2012.02.041>
2. Dung-An Wang, Nine-Zeng Liu. A Shear Mode Piezoelectric Energy Harvester Based on a Pressurized Water Flow. *Sensors and Actuators A: Physical*. 2011;167(2):449–458. <https://doi.org/10.1016/j.sna.2011.03.003>
3. Shreya Banerjee, Sitikantha Roy. A Timoshenko like Model for Piezoelectric Energy Harvester with Shear Mode. *Composite Structures*. 2018;204:677–688. <https://doi.org/10.1016/j.compstruct.2018.07.117>

4. Yanping Kong, Jinxi Liu. Vibration Confinement of Thickness-Shear and Thickness-Twist Modes in a Functionally Graded Piezoelectric Plate. *Acta Mechanica Sinica*. 2011;24(4):299–307. [https://doi.org/10.1016/S0894-9166\(11\)60031-1](https://doi.org/10.1016/S0894-9166(11)60031-1)
5. Xiangyu Gao, Xudong Xin, Jingen Wu, Zhaoqiang Chu, Shuxiang Dong. A Multilayered-Cylindrical Piezoelectric Shear Actuator Operating in Shear (d_{15}) Mode. *Applied Physics Letters*. 2018;112:152902. <http://doi.org/10.1063/1.5022726>
6. Jikun Yang, Qiang Huan, Yang Yu, Jingen Wu, Zhaoqiang Chu, Mohammadjavad Pourhosseiniasl, et al. Tailoring Artificial Mode to Enable Cofired Integration of Shear-type Piezoelectric Devices. *Advanced Science*. 2020;7(17):2001368. <https://doi.org/10.1002/adv.202001368>
7. Zejun Yu, Shuxiang Dong, Daining Fang. Theoretical Analysis on Shear-Bending Deflection of a Ring-Shape Piezoelectric Plate. *AIP Advances*. 2016;6(2):025124. <https://doi.org/10.1063/1.4943219>
8. Ewald Benesa, Helmut Nowotny, Stefan Braun, Stefan Radel, Martin Greöschl. Analytical Sensor Response Function of Viscosity Sensors Based on Layered Piezoelectric Thickness Shear Resonators. *Physics Procedia*. 2015;70:236–240. <http://doi.org/10.1016/j.phpro.2015.08.144>
9. Hui Lia, Shengnan Shenb, Kensuke Amemiya, Bo Liu, Hejun Du. Simulation of Piezoelectric Flying Height Control Slider Using Shear-Mode Deformation. *Physics Procedia*. 2011;16:101–110. <https://doi.org/10.1016/j.phpro.2011.06.115>
10. Zenkour AM, Alghanmi RA. Stress Analysis of a Functionally Graded Plate Integrated with Piezoelectric Faces via a Four-Unknown Shear Deformation Theory. *Results in Physics*. 2019;12:268–277. <https://doi.org/10.1016/j.rinp.2018.11.045>
11. Mohammad Malikan. Electro-Mechanical Shear Buckling of Piezoelectric Nanoplate Using Modified Couple Stress Theory Based on Simplified First Order Shear Deformation Theory. *Applied Mathematical Modelling*. 2017;48:196–207. <https://doi.org/10.1016/j.apm.2017.03.065>
12. Bo Xue, Emmanuel Brousseau, Chris Bowen. Modelling of a Shear-type Piezoelectric Actuator for AFM-based Vibration-assisted Nanomachining. *International Journal of Mechanical Sciences*. 2023;243:108048. <https://doi.org/10.1016/j.ijmecsci.2022.108048>
13. Belokon' AV, Ereemeev VA, Nasedkin AV, Solov'ev AN. Partitioned Schemes of the Finite-Element Method for Dynamic Problems of Acoustoelectroelasticity. *Journal of Applied Mathematics and Mechanics*. 2000;64(3):381–393.
14. Kudimova AB, Nadolin DK, Nasedkin AV, Nasedkina AA, Oganessian PA, Soloviev AN. Finite Element Homogenization of Piezocomposites with Isolated Inclusions Using Improved 3-0 Algorithm for Generating Representative Volumes in ACELAN-COMPOS Package. *Materials Physics and Mechanics*. 2020;44(3):392–403. https://doi.org/10.18720/MPM.4432020_10
15. Nasedkin AV, Oganessian PA, Soloviev AN. Analysis of Rosen Type Energy Harvesting Devices from Porous Piezoceramics with Great Longitudinal Piezomodulus. *Zeitschrift für Angewandte Mathematik und Mechanik*. 2021;101(3):e202000129. <https://doi.org/10.1002/zamm.202000129>
16. Nasedkin AV, Skaliukh AS, Soloviev AN. ACELAN Package and Finite Element Modeling of Hydroacoustic Piezoelectric Transducers. *Bulletin of Higher Education Institutes. North-Caucasian Region. Natural Sciences*. 2001;S1:122–125. (In Russ.).

About the Authors:

Arkadiy N. Soloviev, Dr.Sci. (Phys.-Math.), Professor of the Mathematics and Physics Department, Crimean Engineering and Pedagogical University named after Fevzi Yakubov (8, Uchebnyi Lane, Simferopol), Professor of the Theoretical and Applied Mechanics Department, Don State Technical University (1, Gagarin sq., Rostov-on-Don, 344003, Russian Federation), Chief Researcher, Southern Federal University (105/42, Bolshaya Sadovaya Str., Rostov-on-Don, 344006, Russian Federation), SPIN-code: [8087-8998](https://orcid.org/0807-8998), [ORCID](https://orcid.org/0807-8998), [ResearcherID](https://orcid.org/0807-8998), [ScopusID](https://orcid.org/0807-8998), solovievarc@gmail.com

Valery A. Chebanenko, Cand.Sci. (Phys.-Math.), Senior Researcher, Laboratory of Transport, Composite Materials and Structures, Southern Research Center, Russian Academy of Sciences (41, Chekhova Av., Rostov-on-Don, 344006, RF), SPIN-code: [6487-3095](https://orcid.org/6487-3095), [ORCID](https://orcid.org/6487-3095), [ResearcherID](https://orcid.org/6487-3095), [ScopusID](https://orcid.org/6487-3095), valera.chebanenko@yandex.ru

Pavel A. Oganessian, Cand.Sci. (Phys.-Math.), Associate Professor of the Mathematical Modeling Department, Institute of Mathematics, Mechanics and Computer Science, Southern Federal University (8a, Milchakova Str., Rostov-on-Don, 344058, Russian Federation), SPIN-code: [3870-0861](https://orcid.org/3870-0861), [ORCID](https://orcid.org/3870-0861), [ResearcherID](https://orcid.org/3870-0861), [ScopusID](https://orcid.org/3870-0861), oganesyan@hey.com

Elizaveta I. Fomenko, graduate student of the Mathematical Modeling Department, Institute of Mathematics, Mechanics and Computer Science, Southern Federal University (8a, Milchakova Str., Rostov-on-Don, 344058, Russian Federation), SPIN-code: [6176-3556](#), [ORCID](#), [ScopusID](#), elfomenko@sfedu.ru

Об авторах:

Аркадий Николаевич Соловьев, доктор физико-математических наук, профессор кафедры математики и физики Крымского инженерно-педагогического университета им. Февзи Якубова (г. Симферополь, пер. Учебный, д. 8), профессор кафедры теоретической и прикладной механики Донского государственного технического университета (344003, Российская Федерация, г. Ростов-на-Дону, пл. Гагарина, 1), главный научный сотрудник Южного федерального университета (344006, Российская Федерация, г. Ростов-на-Дону, ул. Большая Садовая, 105/42). SPIN-код: [8087-8998](#), [ORCID](#), [ResearcherID](#), [ScopusID](#), solovievare@gmail.com

Валерий Александрович Чебаненко, кандидат физико-математических наук, старший научный сотрудник лаборатории транспорта, композиционных материалов и конструкций федерального исследовательского центра «Южный научный центр Российской академии наук» (344006, Российская Федерация, г. Ростов-на-Дону, пр. Чехова, 41), SPIN-код: [6487-3095](#), [ORCID](#), [ResearcherID](#), [ScopusID](#), valera.chebanenko@yandex.ru

Павел Артурович Оганесян, кандидат физико-математических наук, доцент кафедры математического моделирования института математики, механики и компьютерных наук им. И.И. Воровича Южного Федерального университета (344058, Российская Федерация, г. Ростов-на-Дону, ул. Милячакова, 8 а), SPIN-код: [3870-0861](#), [ORCID](#), [ResearcherID](#), [ScopusID](#), oganesyan@hey.com

Елизавета Ивановна Фоменко, магистрант кафедры математического моделирования института математики, механики и компьютерных наук им. И.И. Воровича Южного Федерального университета (344058, Российская Федерация, Ростов-на-Дону, ул. Милячакова, 8 а), SPIN-код: [6176-3556](#), [ORCID](#), [ScopusID](#), elfomenko@sfedu.ru

Claimed Contributorship:

AN Soloviev: task setting, literature review, research objectives and tasks.

VA Chebanenko: derivation of equations of the applied theory and performing the calculations using them, formulation of conclusions.

PA Oganessian: building a finite element model, formulation of conclusions.

EI Fomenko: computational analysis in the ACELAN package, revision of the text.

Заявленный вклад авторов:

А.Н. Соловьев: постановка задачи, обзор литературы, цели и задачи исследования.

В.А. Чебаненко: вывод уравнений прикладной теории и проведение расчетов с их использованием, формирование выводов.

П.А. Оганесян: построение конечно элементной модели, формирование выводов.

Е.И. Фоменко: проведение расчетов в пакете ACELAN, доработка текста.

Conflict of Interest Statement: the authors declare no conflict of interest.

Конфликт интересов: авторы заявляют об отсутствии конфликта интересов.

All authors have read and approved the final manuscript.

Все авторы прочитали и одобрили окончательный вариант рукописи.

Received / Поступила в редакцию / 15.03.2024

Revised / Поступила после рецензирования 27.03.2024

Accepted / Принята к публикации 01.04.2024