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Experimental Study of the Kinematics of a Double-Row Planetary Mechanism Using Two Elliptical External Gears

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Abstract

Introduction. Mechanisms with non-circular gears are of wide interest to researchers and inventors due to their compactness and the implementation of a wide range of transfer functions. The development of this area is stimulated by the advancements and reduction in cost of mechanical processing and additive manufacturing technologies, as well as the use of applied mathematical modeling packages for the analysis and synthesis of non-circular gears. Traditionally, non-circular gears are used to transmit rotational motion between parallel axes with a variable ratio of angular velocities. However, their use in planetary gear schemes provides implementing various types of output link motion. The analysis of the papers on the research area shows that gears with movable rotation axes have not been sufficiently studied from the point of view of kinematics and dynamics. Most research papers reveal the theory of such mechanisms without verifying the results obtained in practice. This work is aimed at the experimental verification of the kinematics of a planetary mechanism with two external engagements, which contains elliptical gears.

Materials and Methods. The kinematic model of the mechanism under study is built on the basis of the velocity diagram of its links, which made it possible to obtain expressions for finding an analogue of the angular velocity and the position function of the output shaft. The experimental study of kinematics was performed on a laboratory stand containing a model of a planetary mechanism with a set of replaceable gear wheels, absolute encoders on the input and output shafts of the mechanism, a controller, and a PC for recording and processing the signal. The analysis of the obtained results was performed on a computer using statistical analysis methods.

Results. As a result of kinematic analysis, position functions were constructed for three alternative planetary mechanisms, which had different geometric parameters of the gears and made it possible to implement various types of motion of the output shaft: swinging motion, discontinuous motion, and unilateral uneven rotation.

Discussion and Conclusion. The analysis of the experimental results showed the adequacy of the constructed mathematical model of kinematics to real mechanisms. The confidence interval of measuring errors at a reliability level of 95% was 0.16±0.08° for the first version of the mechanism, 0.57±0.22° — for the second version, and 0.08±0.26° — for the third. The proposed planetary mechanism with elliptical gears for implementing various types of motion can be used in drives of process equipment in numerous industries: chemical and food (mixers), oil refining (pumping units for crude production), mechanical engineering (compressors, pumps, automated machines), and others. The conducted kinematic studies of the planetary mechanism and their experimental analysis are needed for further dynamic and force investigations, as well as for the design of drives based on the proposed transmission.

Keywords: planetary mechanism, elliptical gears, kinematic analysis, position function, statistical analysis, uncertainty of measurement, confidence interval

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Оригинальное эмпирическое исследование

Экспериментальное исследование кинематики двухрядной планетарной передачи эллиптическими зубчатыми колесами с двумя внешними зацеплениями

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Аннотация

Введение. Механизмы с некруглыми зубчатыми колесами вызывают широкий интерес исследователей и изобретателей вследствие их компактности и реализации широкого спектра передаточных функций. Развитие данной области стимулируется развитием и удешевлением технологий механической обработки и аддитивного производства, а также применением прикладных пакетов математического моделирования для анализа и синтеза некруглых зубчатых колес. Некруглые зубчатые колеса традиционно служат для передачи вращательного движения между параллельными осями с переменным отношением угловых скоростей. Однако их применение в схемах планетарных передач позволяет реализовать различные виды движения выходного звена. Анализ работ по тематике исследования показывает, что передачи с подвижными осями вращения недостаточно исследованы с точки зрения кинематики и динамики. Большинство научных работ раскрывают теорию таких механизмов, не проводя верификации полученных результатов на практике. Целью настоящей работы является экспериментальная верификация кинематики планетарного механизма с двумя внешними зацеплениями, имеющего в своем составе эллиптические зубчатые колеса.

Материалы и методы. Кинематическая модель исследуемого механизма построена на базе плана скоростей его звеньев, который позволил получить выражения для нахождения аналога угловой скорости и функции положения выходного вала. Экспериментальное исследование кинематики выполнено на лабораторном стенде, содержащем макет планетарного механизма с набором сменных зубчатых колес, абсолютные энкодеры на входном и выходном валах механизма, контроллер и ПК для регистрации и обработки сигнала. Анализ полученных результатов проведен на ЭВМ с использованием методов статистического анализа.

Результаты исследования. В результате кинематического анализа построены функции положения для трех вариантов планетарного механизма, имеющих различные геометрические параметры зубчатых колес и позволяющих реализовать различные виды движения выходного вала: возвратно-вращательное движение, движение с остановками и одностороннее неравномерное вращение.

Обсуждение и заключение. Анализ результатов эксперимента показал адекватность построенной математической модели кинематики реальным механизмам. Доверительный интервал ошибок измерения при уровне достоверности 95 % составил для первого варианта механизма $0{,}16 \pm 0{,}08$, для второго варианта — $0{,}57 \pm 0{,}22$ и для третьего — $0{,}08 \pm 0{,}26$. Предложенный планетарный механизм с эллиптическими зубчатыми колесами для реализации различных видов движения может быть применен в приводах технологического оборудования многих отраслей промышленности: химической и пищевой (перемешивающие устройства), нефтеперерабатывающей (станки-качалки для добычи нефти), машиностроительной (компрессоры, насосы, станки-автоматы) и других. Проведенные кинематические исследования планетарного механизма и их экспериментальный анализ необходимы при дальнейшем динамическом и силовом исследованиях, а также при проектировании приводов на базе предложенной передачи.

Ключевые слова: планетарный механизм, эллиптические зубчатые колеса, кинематический анализ, функция положения, статистический анализ, неопределенность измерения, доверительный интервал

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Introduction. Mechanisms with non-circular gears have been known for a long time, but their experimental research and practical implementation have long been difficult due to the focused specialization of such mechanisms, the complexity and high cost of their manufacture. Nowadays, there is an increase in the interest of researchers in this topic. At the same time, most scientific papers on designing non-circular gearing are based on the analysis of geometry [1] and kinematics [2] to obtain the required transfer function for practical application [3]. In [4] and [5], mechanisms for synthesizing angular velocity functions were developed and studied. Researchers proposed robotics with non-circular gears, for example, a jumping robot [6], a hexapod robot [7], an exoskeleton mechanism for knee joint rehabilitation [8]. More efficient planting machines for the agricultural industry were studied from the point of view of kinematics [9] and dynamics [10]. Other numerous devices were developed and designed.

A review of sources has shown that the most common gears are those with an elliptical centrode [11, 12]. Machines [10] and devices [4] with elliptical gears are created, the geometry [13] is studied, and some design [14] and manufacturing issues are resolved [15].

Elliptical gears are mainly used to transmit rotary movement between parallel fixed axes [16]. However, such use of non-circular gears allows for only one-way nonuniform rotary movement, which limits the scope of their application. Planetary gears have broader capabilities for implementing complex types of movement of working bodies. The creation of drives based on them will provide the development and implementation of more efficient and compact machines for various technological purposes.

Based on the results of the analysis of scientific literature in the field of transmissions with non-circular gears, a planetary mechanism with elliptical wheels is proposed as an object of study. The objective of the work is an experimental kinematic analysis of a planetary transmission. It is justified by the need to verify theoretical provisions for their correct use at the following stages of design — in dynamic, force analysis and strength calculations.

Materials and Methods. The kinematic model of the mechanism is constructed on the basis of the velocity diagram of its links (Fig. 1).

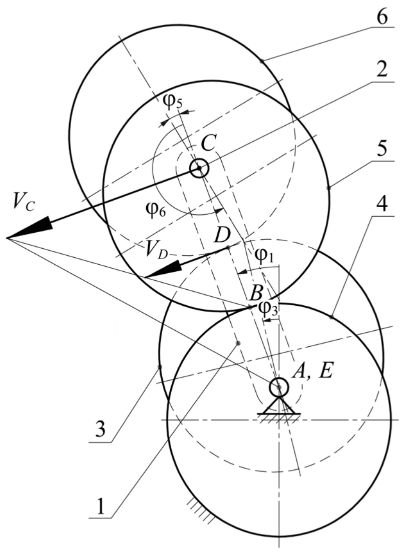


Fig. 1. Velocity diagram for the mechanism under consideration:
1 — carrier; 2 — satellite shaft; 3 — elliptical gear on the output shaft;
4 — sun elliptical wheel; 5, 6 — elliptical wheels of the satellite

The analog of the angular velocity of the output shaft is determined as follows [17]:

$$\varphi_3' = \frac{\omega_3}{\omega_1} = \frac{V_D \cdot AC}{V_C \cdot DE} = \frac{BD \cdot AC}{BC \cdot DE}. \tag{1}$$

Segments *BD, BC* and *DE* are determined through the polar equation of the ellipse [17]:

$$\rho(\varphi) = \frac{a\left(1-e^2\right)}{1-e \cdot cos\,\varphi}, \tag{2}$$

where φ — rotation angle, e and a — eccentricity and semi-major axis of an ellipse.

We determine the engagement radii of gears 5 and 6:

$$BC = \rho_5 = \frac{a\left(1 - e_1^2\right)}{1 - e_1 \cdot cos\,\varphi_5}, \tag{3}$$

$$CD = \rho_6 = \frac{a\left(1 - e_2^2\right)}{1 - e_2 \cdot cos\,\varphi_6}, \tag{4}$$

where e_1 and e_2 — eccentricities for each pair of elliptical gears, whose rotation angles φ_5 and φ_6 are obtained as follows:

$$\varphi_5 = \int \frac{1 - e_1^2}{1 + e_1^2 + 2e_1 \cdot cos\,\varphi_1} d\varphi_1. \tag{5}$$

$$\varphi_6 = \pi + \varphi_5. \tag{6}$$

Based on the velocity plan and taking into account equations (3), (4), we define the required segments as:

$$BD = \rho_6 - \rho_5, \tag{7}$$

$$AC = EC = 2a, \tag{8}$$

$$DE = 2a - \rho_6. \tag{9}$$

Based on (2)–(9), we obtain an expression for determining the analogue of the angular velocity of the output shaft:

$$\varphi_3' = \frac{\left(\rho_6 - \rho_5\right) \cdot 2a}{\rho_5\left(2a - \rho_6\right)}. \tag{10}$$

The rotation angle is determined by integrating equation (10) over joint coordinate φ_1:

$$\varphi_3 = \int \varphi_3' d\varphi_1. \tag{11}$$

Variations of the kinematic scheme of the mechanism include replacing elliptical gears with cylindrical ones in one of the pairs, for the study of which it is required to replace the radius functions with fixed values in the resulting mathematical model.

We consider mechanisms with a pair of elliptical gears 3–6 (e=0.28), and cylindrical wheels 4 and 5 of the following sizes:

– option 1: R_5=25 mm, R_4=25 mm;

– option 2: R_5= 18 mm, R_4= 32 mm;

– option 3: R_5= 16 mm, R_4= 34 mm.

Figure 2 shows the graphs of the position functions obtained using (11).

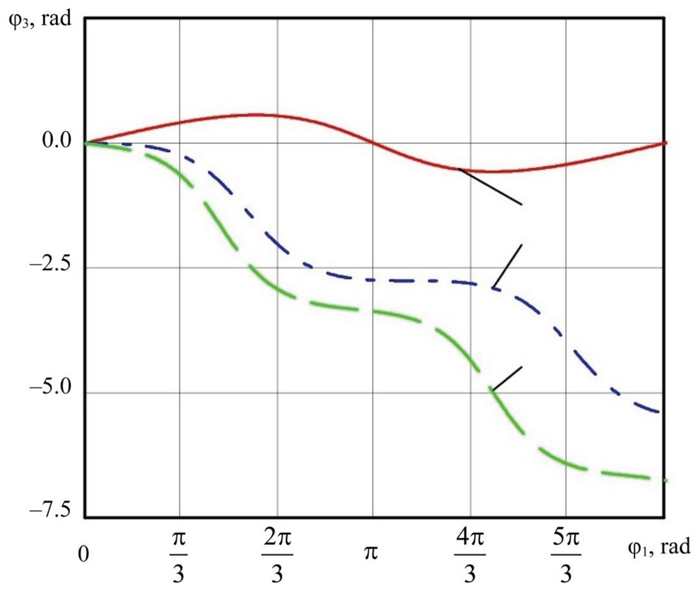


Fig. 2. Dependence graphs for the studied mechanism configurations

The analysis of the graphs shows (in Fig. 2, numbers indicate studied options) that changing the sizes of the gear wheels allows obtaining different types of output shaft movement: reciprocating-rotational (option 1), discontinuous (option 2) and one-way nonuniform rotational movement (option 3).

Research Results. The kinematic analysis confirmed the implementation of various transfer functions, and changing the sizes of gear wheels allows for the kinematic synthesis of new mechanisms.

The object of the experimental study is a prototype, whose details are shown in Figure 3.

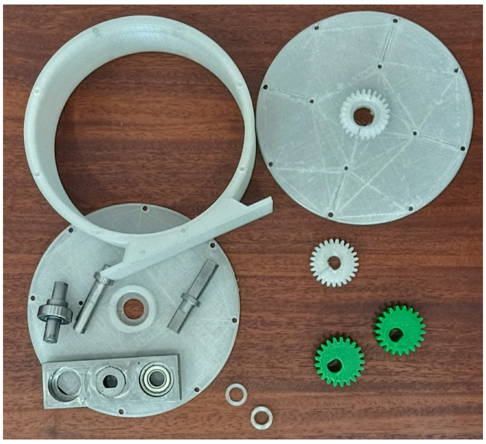


Fig. 3. Details of the prototype under study

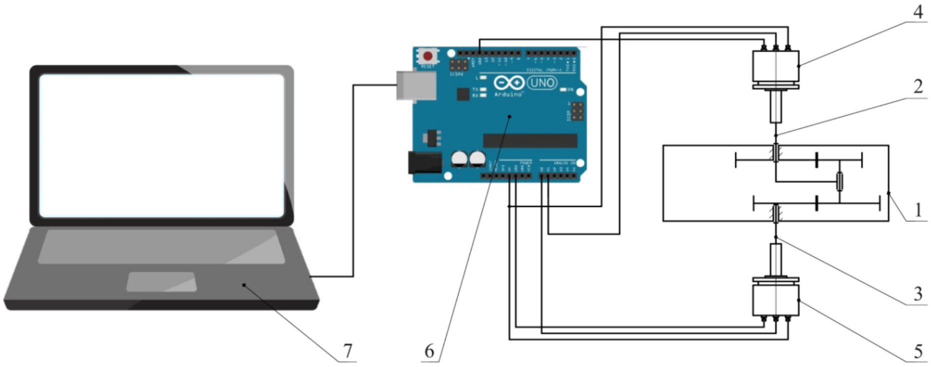
Rotation angles are measured by absolute encoders (Table 1).

Table 1

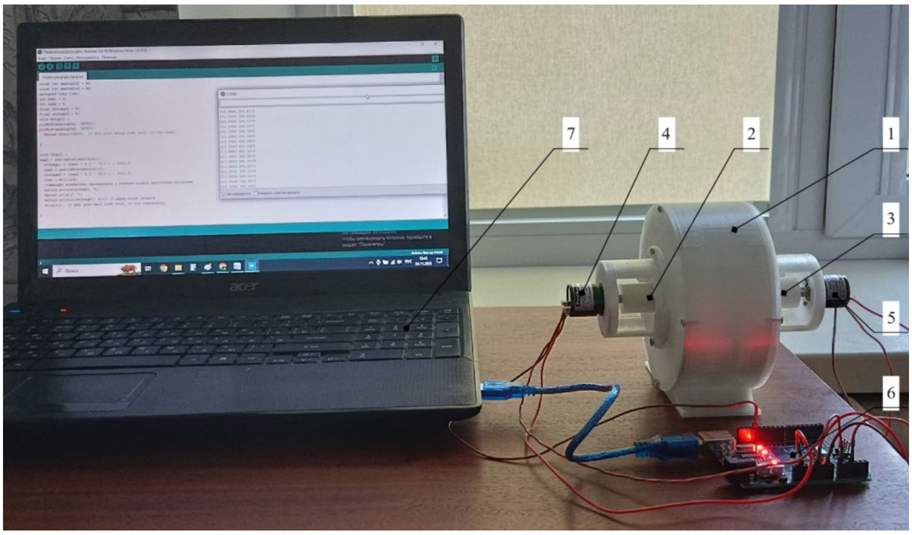
Encoder Characteristics

Diameter	Output signal	Resolution	Linearity	Reading speed
22 mm	0–5 V	360°/4096≈0.088°	0.3%	0.6 ms

The signal is registered by the Arduino controller and processed on the PC (Fig. 4).



a)



b)

Fig. 4. Schematic diagram (*a*) and external view (*b*) of the experimental stand: 1 — housing; 2 — input shaft; 3 — output shaft; 4, 5 — encoders; 6 — Arduino controller; 7 — personal computer

The processing and analysis of the measurement results were performed in the MathCAD application package. The schemes of the studied options and the experiment results are shown in Figure 5.

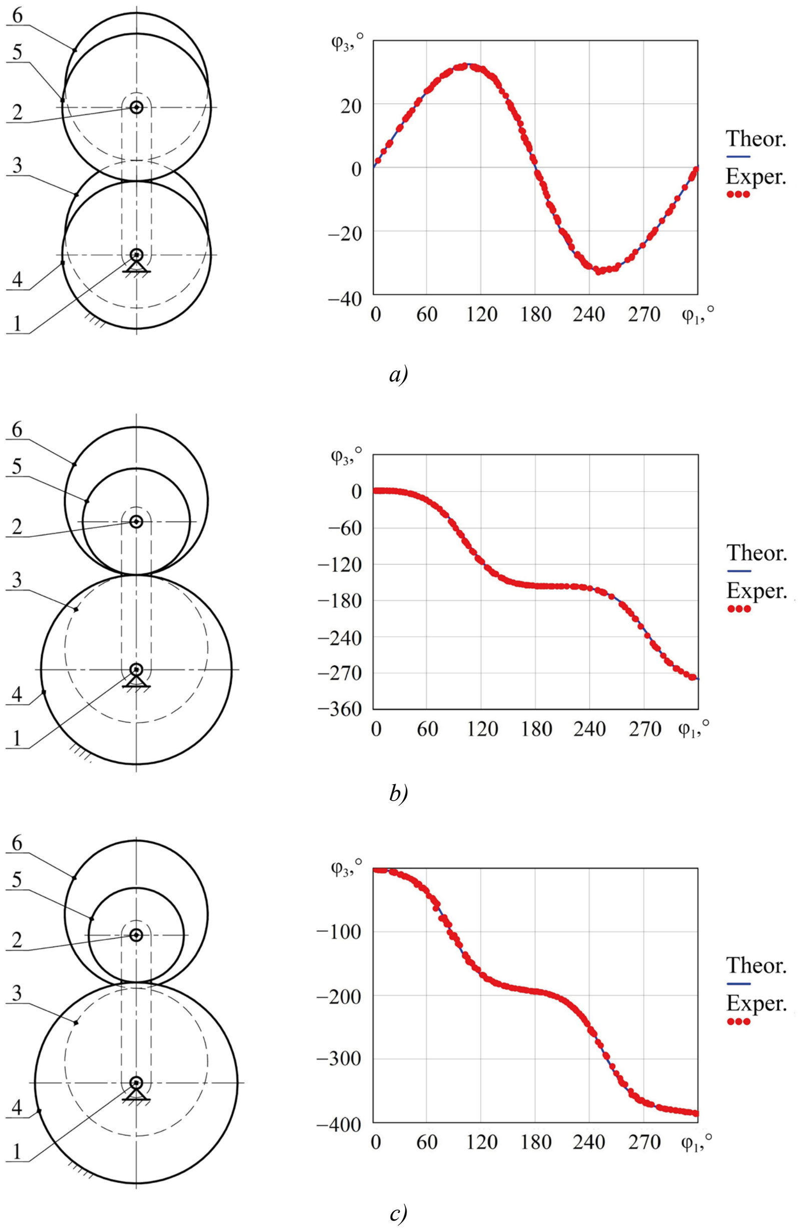


Fig. 5. Research results: *a* — option 1; *b* — option 2; *c* — option 3

As can be seen from the graphs, the measured values of the output shaft rotation angles are adequate to the constructed model. Let us evaluate the results of the experiment using statistical analysis tools.

We determine the average value of measurement errors [18]:

$$\overline{q} = \frac{1}{n}\sum_{k=1}^{n} q_k, \tag{12}$$

where n — number of independent observations q_k.

Let us calculate the sample variance:

$$s^2(q_k) = \frac{1}{n-1}\sum_{k=1}^{n}(q_k - \overline{q})^2. \tag{13}$$

An estimate of the dispersion of the mean value is obtained as follows:

$$s^2(q) = \frac{s^2(q_k)}{n}. \tag{14}$$

The standard uncertainty of Type A measurements is determined [19]:

$$u(q) = \sqrt{s^2(q)}. \tag{15}$$

Taking the measurement errors as values q_k and processing them according to (12)–(15), we calculate the measurement uncertainty. The results of the statistical analysis are given in Table 2.

Table 2

Statistical Analysis of Measurement Errors

Mechanism number	Number of measured values	Mean error value	Uncertainty of measurement
1	195	–0.16°	0.04°
2	168	0.57°	0.11°
3	192	0.08°	0.13°

The confidence interval is defined as $\overline{q} \pm 2u$ for a level of certainty of 95% and is –0.16 ± 0.08° (option 1), 0.57 ± 0.22° (option 2), 0.08 ± 0.26° (option 3). Thus, the constructed mathematical model of kinematics is adequate to the physical prototypes.

Discussion and Conclusion. A planetary gear with elliptical wheels is presented, providing the implementation of nonuniform, discontinuous and reciprocating-rotational movement of the output shaft. The type of movement is determined by the parameters of the gear wheels.

A kinematic model of the transmission was constructed, and the law of movement of the output shaft was obtained. The analysis of mechanisms with different parameters of gear wheels showed the feasibility of a wide range of transmission functions and types of movement of the output shaft.

The correspondence of the results of the conducted kinematic analysis to real mechanisms is confirmed by an experimental study of the output shaft position functions for three options of the physical prototype. The performed assessment of the adequacy of the mathematical model provides its using in dynamic and force analysis of machines based on the proposed transmissions. The studied options of the mechanism can be used in drives of compact and easy-to-balance technological machines.

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Claimed Contributorship:

AA Prikhodko: academic advising, basic concept, research objectives and tasks formulation, revision of the text, correction of the conclusions.

NN Belina: computational analysis, text preparation, formulation of conclusions.

AV Novitskiy: development and making of an experimental stand, design of the calculation part of the paper.

MM Shchetinin: processing of the experimental data, preparation of the experimental part of the paper.

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All authors have read and approved the final manuscript.

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