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## Analysis of a Four-Legged Robot Kinematics during Rotational Movements of Its Body

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### Abstract

**Introduction.** Walking robots are widely used in industry due to their unique capabilities for moving on uneven and complex surfaces. To provide high precision in controlling their movement, it is required to develop mathematical models and algorithms for planning the robot movement along various trajectories. A key aspect of the motion control system of walking robots is the planning of their leg movements. Despite significant advances in the field of modeling the kinematics of quadruped robots, existing scientific publications do not provide a complete kinematic model for robots similar to the Mini Cheetah. This research was aimed at the development of a kinematic model of a quadruped robot based on Mini Cheetah, as well as the formulation of recommendations for optimizing its gait to provide rotation around various axes. The creation of such a model will improve the smoothness and accuracy of the robot movements, which, in turn, will increase its efficiency under real production conditions.

**Materials and Methods.** The process of constructing a kinematic model of the robot was based on the use of formulas for the geometry of spatial motion of solids. To test the efficiency of the proposed algorithms for moving the robot legs when performing rotational movements of its body, numerical modeling of the robot kinematics was used. Numerical calculations were performed using the *Wolfram Mathematica* package.

**Results.** The laws of changing the endpoints of the robot legs during its rotation around the vertical axis were proposed. The conducted numerical modeling of the robot kinematics covered the rotation of the body at the course, roll and pitch angles. Based on the simulation results, it was established that the dependences of the rotation angles of the leg links were periodic functions. The considered rotational movements of the robot platform could take place without the occurrence of singular configurations.

**Discussion and Conclusion.** The results of numerical modeling of the robot platform rotation movements confirmed the operability of the proposed leg transfer plan, which allowed for smooth movement of the robot body and avoidance of singular configurations. The resulting kinematic model can be used to control the robot motion at the kinematic level when moving along curvilinear trajectories. As a prospect for further research, it is worth highlighting the development of a mathematical model of the dynamics of a four-legged robot, as well as the creation of laws for controlling its movement at a dynamic level. This will significantly expand the functionality of the robot and increase its efficiency under various operating conditions.

**Keywords:** four-legged robot, inverse kinematics, mobile robot, kinematic model, motion planning

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**Анализ кинематики четвероногого робота при поворотных движениях его корпуса**М.Ж. Фернандо , Г.Р. Сайпулаев  , М.Р. Сайпулаев 

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**Введение.** Шагающие роботы находят широкое применение в промышленности благодаря своим уникальным возможностям передвижения по неровным и сложным поверхностям. Для обеспечения высокой точности управления их движением необходимо разработать математические модели и алгоритмы планирования перемещения робота по различным траекториям. Ключевым аспектом системы управления движением шагающих роботов является планирование перемещений их ног. Несмотря на значительные достижения в области моделирования кинематики четвероногих роботов, в существующих научных публикациях не представлено полноценной кинематической модели для роботов, аналогичных Mini Cheetah. Цель данной работы заключается в разработке кинематической модели четвероногого робота на основе Mini Cheetah, а также в формулировании рекомендаций по оптимизации его походки для обеспечения вращения вокруг различных осей. Создание такой модели позволит улучшить плавность и точность движений робота, что, в свою очередь, повысит его эффективность в реальных производственных условиях.

**Материалы и методы.** Процесс построения кинематической модели робота основан на использовании формул геометрии пространственного движения твердых тел. Для проверки эффективности предложенных алгоритмов перемещения ног робота при осуществлении поворотных движений его корпуса было применено численное моделирование кинематики робота. Численные расчеты выполнены с использованием математического пакета *Wolfram Mathematica*.

**Результаты исследования.** Предложены законы изменения конечных точек ног робота при его вращении вокруг вертикальной оси. Проведенное численное моделирование кинематики робота охватывало повороты корпуса по углам курса, крена и тангажа. По результатам моделирования установлено, что зависимости углов поворота звеньев ног являются периодическими функциями.

Рассмотренные поворотные движения платформы робота могут происходить без возникновения сингулярных конфигураций.

**Обсуждение и заключение.** Результаты численного моделирования поворотных движений платформы робота подтвердили работоспособность предложенного плана переноса ног, который позволяет осуществлять плавное движение корпуса робота и избегать сингулярных конфигураций. Полученная кинематическая модель может быть использована для управления движением робота на кинематическом уровне при перемещениях по криволинейным траекториям. В качестве перспективы для дальнейших исследований стоит выделить разработку математической модели динамики четвероногого робота, а также создание законов управления его движением на динамическом уровне. Это позволит значительно расширить функциональные возможности робота и повысить эффективность его работы в различных условиях эксплуатации.

**Ключевые слова:** четвероногий робот, обратная кинематика, мобильный робот, кинематическая модель, планирование движения

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**Introduction.** Currently, walking mobile robots are widely used to solve problems of observation and exploration of territories [1]. An example of one of the common designs of such robots is the four-legged robot *Mini Cheetah* (Fig. 1), developed at *MIT Biomimetic Robotics Lab* (<https://biomimetics.mit.edu>). These four-legged robots can move on various types of terrain, unlike wheeled or tracked robots.

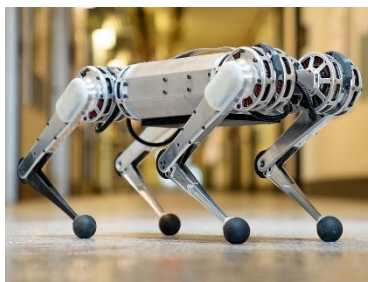


Fig. 1. Four-legged robot *Mini Cheetah* <sup>1</sup>

The design of the walking robots under consideration includes a platform to which four legs are attached. Each of the legs is a three-link manipulator. One of the key aspects of a walking robot is the topology of its leg structure [2, 3]. The topology of the legs refers to the composition and arrangement of the links, as well as the hinges of the limbs of a four-legged robot, which play an important role in providing its movement and stability. The development of an optimal topology for the leg structure of quadruped robots is one of the challenges in engineering. The solution to this problem not only helps to provide stable and efficient gait (locomotion) of the robot on various surfaces, but also makes its mechanical design more reliable and energy efficient.

Another aspect that needs to be considered is the drive system applied in quadruped robots [4]. Different types of drives are widely used in such robots, including pneumatic [5], hydraulic [6, 7], and electric [2, 6], each of which has its own advantages and disadvantages.

Four-legged robots have complex kinematics due to a large number of degrees of freedom. The robot considered in this paper has 18 degrees of freedom. To provide successful locomotion of four-legged robots, it is necessary to solve both direct and inverse problems of kinematics, which allows organizing dynamic control of the position of the legs of these robots. This control should provide the robot balancing, the ability to overcome various surfaces, and adaptation to various types of locomotion, including walking, running and jumping. The direct problem of walking robot kinematics is to determine the position and orientation of the robot platform based on the known rotation angles of the leg links. The inverse problem is to find the rotation angles of the leg links that provide a given position and orientation of the robot platform.

Paper [8] described in detail the application of the Denavit-Hartenberg method for modeling the forward and inverse kinematics of a walking robot. The study on the kinematic model of the robot was conducted with the aim of creating a gait algorithm.

The authors of [9] developed advanced kinematic and dynamic modeling methodologies based on the screw theory for quadruped robots. The proposed methods accounted for different gaits and leg mechanism topologies using simplified models of foot contact (e.g., a ball-and-socket model). Models were described for three separate phases of robot motion: standing, walking, and unsupported (flying) phase. The developed control strategies used the presented models for gait planning and jogging.

In recent years, scientists have been working on control and trajectory planning systems. The motion control method is the basis of a four-legged robot, directly affecting flexibility, stability and the ability to adapt to various support surfaces [3].

The authors of [10] specified the design of a regulator for the force control of a four-legged robot. The main purpose of the regulator was to equalize the forces acting on the robot during symmetrical gait, as well as to reduce disturbances and absorb impacts.

In [11], a new motion control strategy for a quadruped robot that enabled it to navigate efficiently on uneven terrains regardless of visual perception conditions was proposed. Paper [12] discussed the design of a motion controller for a quadruped robot that used torque optimization and performance control to account for the impact of unpredictable external forces such as surface irregularities.

Experimental studies devoted to the application of hierarchical controllers for motion control of quadruped robots were presented in [13], illustrated by the ANYmal robot, and in [14], exemplified by the StarlETH robot.

In [15], the authors discussed the development of a quadruped robot designed for construction and emergency response tasks. Their paper examined the statics, kinematics, and control system for the specific design of this robot.

Finally, in [16], a new central pattern generator model controller and gait switching tactic based on the Wilson-Cowan model were presented. This development aimed to create a smooth robot gait and reduce the adjustment time in the oscillating mechanical system.

Despite the development of the problem of modeling the kinematics of quadruped robots, in existing publications there was no complete kinematic model of the motion of robots whose leg design was similar to the *Mini Cheetah* robot (Fig. 1). Therefore, the objective of this work was to develop a kinematic model of a quadruped robot using the example of the *Mini Cheetah* and to develop proposals for planning the movements of the robot links that could provide the rotation of the robot around various axes.

<sup>1</sup> Massachusetts Institute of Technology. URL: <https://news.mit.edu/2019/mit-mini-cheetah-first-four-legged-robot-to-backflip-0304> (accessed: 07.11.2024).

To achieve the stated goal, we needed to formulate the laws of change of coordinates of the endpoints of the robot legs and angular coordinates of its body. It was also necessary to solve the inverse problem of robot kinematics to check the operability of the proposed motion plan. The results obtained can be used to develop robot control at the kinematic level.

**Materials and Methods. System Definition.** The object of the study is a robotic system consisting of a rigid body (robot platform) and four legs with three degrees of freedom for each of the legs. The links of the legs are connected to the robot platform by several hinges, which are driven by electric drives. Angle  $\beta_i$  is the angle of rotation of link  $O_iA_i$  around the longitudinal axis of the robot body and characterizes the rotation of the plane of the leg (containing points  $A_i, B_i, K_i$ ) relative to the vertical plane of symmetry  $CXZ$ . And angles  $\alpha_i, \beta_i$  characterize the rotation of links  $A_iB_i$  and  $B_iK_i$  in the plane of the leg. The hinges of the leg links at points  $O_i, A_i, B_i$  are controlled, i.e., the rotation of the leg links at angles  $\varphi_i, \alpha_i, \beta_i$  is provided independently by the corresponding drives.

Fixed coordinate system  $XYZ$  and moving coordinates  $Cxyz$  are introduced, where point  $C$  is the geometric center of the robot platform (Fig. 2 a). The orientation of the robot platform is described by course angles  $\psi$  (rotation around axis  $z$ ), roll  $\theta$  (rotation around axis  $x$ ) and pitch  $\gamma$  (rotation around axis  $y$ ), and the position of the robot platform is determined by the coordinates of the geometric center  $x_C, y_C$  and  $z_C$  in the fixed coordinate system.

The distances along the longitudinal and transverse axes of the robot platform from the center of mass  $C$  and the attachment points of the legs to the robot platform  $O_i$  in the moving axes  $x_iy_iz_i$  are specified by  $\rho_{xi}$  and  $\rho_{yi}$ . The lengths of the links are designated as  $l_0 = |O_iA_i|, l_1 = |A_iB_i|, l_2 = |B_iK_i|$  ( $i = \overline{1,4}$ ) (Fig. 2 b).

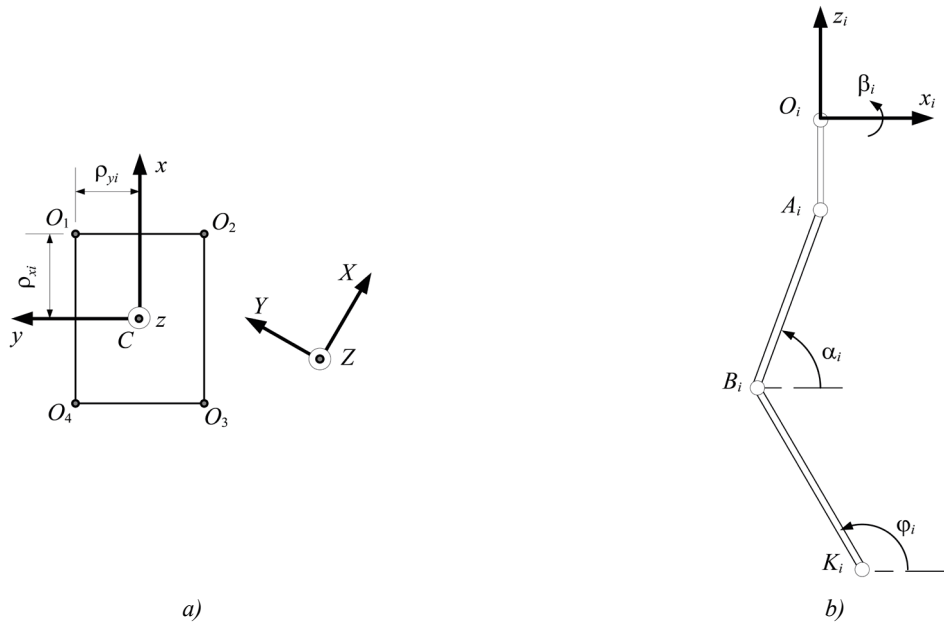


Fig. 2. Kinematic diagram of the robot:  
a — top view of the robot body; b — side view of the robot leg

In projections onto the axes of the fixed coordinate system  $XYZ$ , the position vector of the endpoint of the leg  $K_i$  (or the point in contact with the surface) is determined from the formula:

$$\vec{r}_{Ki} = \vec{r}_C + \Gamma_{\psi\theta\gamma} (\vec{r}_{CO_i} + \vec{r}_{O_iA_i} + \Gamma_{\beta_i} (\vec{r}_{A_iB_i} + \vec{r}_{B_iK_i})), \quad (1)$$

where  $\vec{r}_C = (x_C \ y_C \ z_C)^T$  — position vector of the geometric center of the robot platform  $C$ ;  $\Gamma_{\psi\theta\gamma} = \Gamma_{\psi}\Gamma_{\theta}\Gamma_{\gamma}$  — rotation matrix, which is calculated as the product of the elementary rotation matrices by the course, roll, and pitch angles. The rotation matrices included in equation (1), have the form:

$$\Gamma_{\psi} = \begin{pmatrix} \cos \psi & -\sin \psi & 0 \\ \sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \Gamma_{\theta} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{pmatrix}, \quad (2)$$

$$\Gamma_{\gamma} = \begin{pmatrix} \cos \gamma & 0 & \sin \gamma \\ 0 & 1 & 0 \\ -\sin \gamma & 0 & \cos \gamma \end{pmatrix}, \quad \Gamma_{\beta_i} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta_i & -\sin \beta_i \\ 0 & \sin \beta_i & \cos \beta_i \end{pmatrix}.$$

Other radius vectors included in equation (1) are determined from the expressions:

$$\begin{aligned}\vec{r}_{COi} &= (\rho_{xi} \ \rho_{yi} \ 0)^T, & \vec{r}_{O_iA_i} &= (0 \ -l_0 \sin \beta_i \ -l_0 \cos \beta_i)^T, \\ \vec{r}_{A_iB_i} &= (-l_1 \cos \alpha_i \ 0 \ -l_1 \sin \alpha_i)^T, & \vec{r}_{B_iK_i} &= (-l_2 \cos \varphi_i \ 0 \ -l_2 \sin \varphi_i)^T,\end{aligned}\quad (3)$$

here, the superscript “T” means the vector transpose operation.

After performing intermediate calculations on the right side of equation (1), taking into account expressions (2) and (3), we obtain projections of the position vector  $r_{Ki}$  for each leg.

Differentiating equation (1) with respect to time, we obtain a kinematic model in the form:

$$v_{Ki} = v_C + J_0(q_{\psi\theta\gamma}, q_{\alpha\varphi\beta i}) \dot{q}_{\psi\theta\gamma} + J_i(q_{\psi\theta\gamma}, q_{\alpha\varphi\beta i}) \dot{q}_{\alpha\varphi\beta i}, \quad (4)$$

where  $q_{\psi\theta\gamma} = (\psi \ \theta \ \gamma)^T$  — vector of angular generalized coordinates of the robot platform;  $q_{\alpha\varphi\beta i} = (\beta_i \ \alpha_i \ \varphi_i)^T$  — vector of angular generalized coordinates of the robot leg links;  $v_{Ki} = (\dot{x}_{Ki} \ \dot{y}_{Ki} \ \dot{z}_{Ki})^T$  — vector of linear velocity of the endpoint of the  $i$ -th leg;  $v_C = (\dot{x}_C \ \dot{y}_C \ \dot{z}_C)^T$  — vector of linear velocity of the geometric center of the robot platform;  $J_0(q_{\psi\theta\gamma}, q_{\alpha\varphi\beta i}), J_i(q_{\psi\theta\gamma}, q_{\alpha\varphi\beta i})$  — matrices of coefficients for the corresponding vectors of generalized velocities.

**Research Results.** To analyze the rotational motions of the robot by solving the equations of motion geometry (1) (or integrating the kinematic equations (4)), we set the initial conditions for the rotation angles of the leg links and the laws of change in the velocities of the endpoints of the legs  $v_{Ki}$ , the geometric center of the robot platform  $v_C$  and the vector of angular generalized coordinates  $q_{\psi\theta\gamma}$ . Considering the rotational movements of the robot platform, we assume that the geometric center is stationary during the movement  $v_C = 0$ .

As the initial position of the leg links, we take the following angle values  $\varphi_i, \alpha_i, \beta_i$  ( $i = \overline{1,4}$ ) for each of the legs:

$$\varphi_i(0) = 120^\circ, \alpha_i(0) = 60^\circ, \beta_i(0) = 0^\circ. \quad (i = \overline{1,4}). \quad (5)$$

The given combination of angle values  $\varphi_i, \alpha_i, \beta_i$  ( $i = \overline{1,4}$ ) corresponds to the robot standing on semi-bent legs, which is presented in a simplified visualization (Fig. 3) made in *Wolfram Mathematica*. The following values of the robot geometric parameters were used in the visualization:  $l_0 = 0.15$  m,  $l_1 = 0.45$  m,  $l_2 = 0.45$  m,  $\rho_{x1} = \rho_{x2} = -\rho_{x3} = -\rho_{x4} = 0.5$  m,  $\rho_{y1} = -\rho_{y2} = -\rho_{y3} = \rho_{y4} = 0.25$  m.

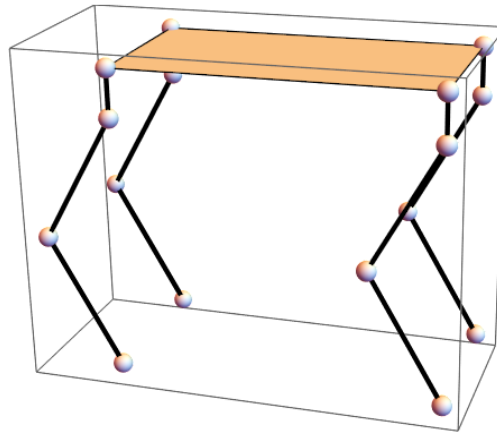


Fig. 3. Simplified visualization of the robot initial position

To perform periodic rotary movements of the robot platform, limited only by the roll angle, we set the laws for changing the orientation angles of the platform in the form:

$$\theta(t) = \theta_0 + A \sin\left(2\pi \frac{t}{T}\right), \quad \gamma(t) = \gamma_0, \quad \psi(t) = \psi_0, \quad (6)$$

where  $\psi_0 = 0^\circ, \theta_0 = 0^\circ, \gamma_0 = 0^\circ$  — initial values of orientation angles;  $A = 0.2$  rad and  $T = 20$  s — constants characterizing the amplitude and period of oscillations by angle. Since it is optional to move the legs when the robot platform is rotated along the roll angle, the coordinates of the contact points of the legs with the support surface remain unchanged ( $v_{Ki} = 0$ ):

$$x_{Ki}(t) = x_{Ki}(0), \quad y_{Ki}(t) = y_{Ki}(0), \quad z_{Ki}(t) = 0. \quad (i = \overline{1,4}), \quad (7)$$

where  $x_{Ki}(0)$  and  $y_{Ki}(0)$  — initial coordinates of the contact points of the robot legs with the support surface.

Solving equations (1) with respect to (2) and (3), when substituting the program motion (6) and (7), we obtain the dependences of the rotation angles  $\varphi_i, \alpha_i, \beta_i$  of the leg links (Fig. 4).

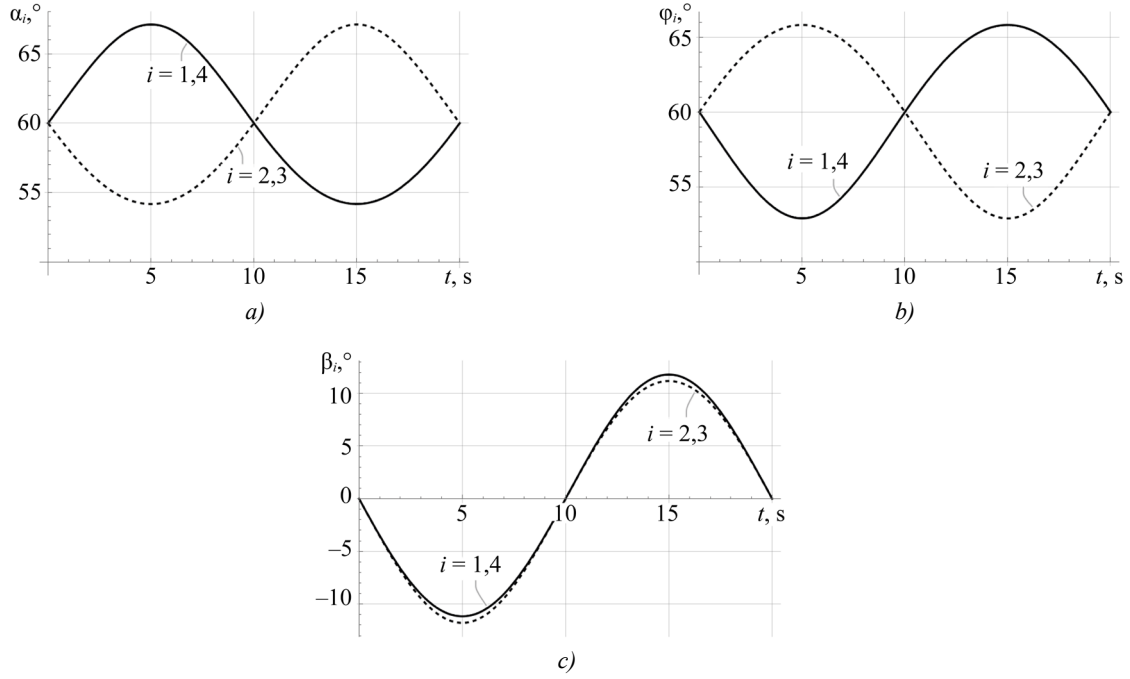


Fig. 4. Roll angle turn simulation results: *a* — angles  $\alpha_i(t)$ ; *b* — angles  $\varphi_i(t)$ ; *c* — angles  $\beta_i(t)$

According to the results of the simulation presented in Figure 4, it can be noted that the rotation angles of the leg links are harmonic functions. As a consequence, it can be mentioned that the rotations of the robot platform around the longitudinal axis of symmetry of the robot (6) can be performed without tearing the contact points of the legs from the surface.

To perform periodic rotary movements of the robot platform, only in pitch angle, we set the laws for changing the orientation angles of the platform in the form:

$$\theta(t) = \theta_0, \quad \gamma(t) = \gamma_0 + A \sin\left(2\pi \frac{t}{T}\right), \quad \psi(t) = \psi_0. \quad (8)$$

When the robot platform rotates along the pitch angle, the transfer of the legs is optional, and the coordinates of the points of contact of the legs with the supporting surface remain unchanged.

Solving equations (1) taking into account (2) and (3), and substituting program movements (7) and (8), we obtain the dependences of the rotation angles  $\varphi_i$ ,  $\alpha_i$ ,  $\beta_i$  of the leg links (Fig. 5).

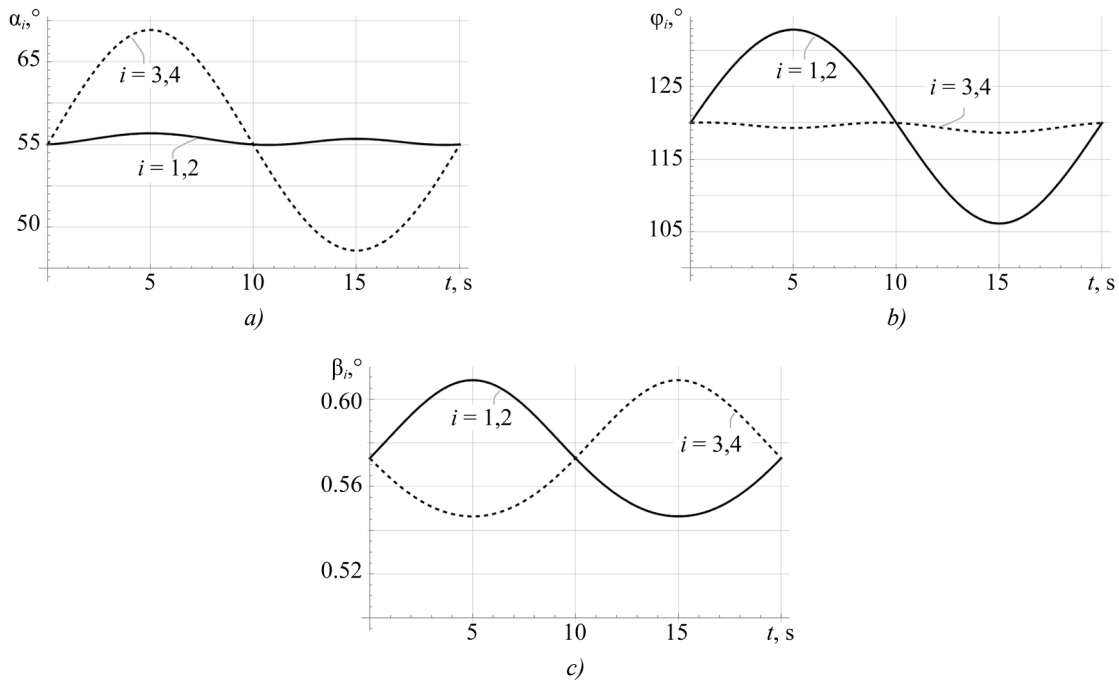


Fig. 5. Pitch angle turn simulation results: *a* — angles  $\alpha_i(t)$ ; *b* — angles  $\varphi_i(t)$ ; *c* — angles  $\beta_i(t)$



According to the simulation results presented in Figure 5, it can be noted that the rotation angles of the leg links are harmonic functions. As a consequence, it can be mentioned that the rotations of the robot platform around the transverse axis of symmetry of the robot (8) can be performed without tearing the contact points of the legs from the surface.

To perform rotation around axis  $Z$ , we consider the laws of changing the orientation angles of the robot platform in the form:

$$\theta(t) = \theta_0, \quad \gamma(t) = \gamma_0, \quad \psi(t) = \psi_0 - \omega_z t, \quad (9)$$

where  $\omega_z = 0.085$  rad/s angular velocity of rotation of the robot platform.

To provide the rotation of the robot around the vertical axis, it is required to plan the movement of the legs. Let us consider the case when the trajectories of the contact points of the legs in projections onto the horizontal plane  $XY$  belong to a circle of radius  $\rho$ . In this case, we will assume that the axis, around which the rotation of the robot platform occurs, passes through the geometric center of the platform. Then, the expressions for the coordinates of the endpoints of the legs can be determined from the relations:

$$\begin{aligned} x_{K1} &= \rho \sin(\omega_z \Delta_{13} + \alpha_1), & y_{K1} &= -\rho \cos(\omega_z \Delta_{13} + \alpha_1), \\ x_{K2} &= \rho \sin(\omega_z \Delta_{24} + \alpha_2), & y_{K2} &= -\rho \cos(\omega_z \Delta_{24} + \alpha_2), \\ x_{K3} &= \rho \sin(\omega_z \Delta_{13} + \alpha_3), & y_{K3} &= -\rho \cos(\omega_z \Delta_{13} + \alpha_3), \\ x_{K4} &= \rho \sin(\omega_z \Delta_{24} + \alpha_4), & y_{K4} &= -\rho \cos(\omega_z \Delta_{24} + \alpha_4), \\ z_{K1} &= z_{K3} = \frac{1}{2} \left[ \operatorname{sgn} \left( \sin \left( \frac{2\pi t}{T_1} \right) \right) + 1 \right] \cdot h_K \sin \left( \frac{2\pi t}{T_1} \right), \\ z_{K2} &= z_{K4} = \frac{1}{2} \left[ \operatorname{sgn} \left( \sin \left( \frac{2\pi t}{T_1} - \pi \right) \right) + 1 \right] \cdot h_K \sin \left( \frac{2\pi t}{T_1} - \pi \right), \end{aligned} \quad (10)$$

where  $\rho = \sqrt{\rho_{Xi}^2 + \rho_{Yi}^2}$ ;  $\alpha_i = \arctan(\rho_{Yi}/\rho_{Xi})$  — angle characterizing the position of the endpoint of the  $i$ -th leg at the initial moment of time;  $h_K = 0.3$  m — maximum height to which the endpoints of the robot legs rise;  $T_1 = 6$  s — time of one step;  $\Delta_{13}, \Delta_{24}$  — auxiliary functions used to provide a piecewise assignment of the change in the coordinates of the endpoints of the legs:

$$\begin{aligned} \Delta_{13} &= \left[ \operatorname{sgn} \left( \sin \left( \frac{2\pi t}{T_1} \right) \right) + 1 \right] \cdot (t \bmod T_1) + \left( t + \frac{T_1}{2} - \left( t - \frac{T_1}{2} \right) \bmod T_1 \right), \\ \Delta_{24} &= \left[ \operatorname{sgn} \left( \sin \left( \frac{2\pi t}{T_1} - \pi \right) \right) + 1 \right] \cdot \left( t + \frac{T_1}{2} \right) \bmod T_1 + (t - (t - T_1) \bmod T_1). \end{aligned} \quad (11)$$

Here, function  $(t \bmod T_1)$  returns the remainder of the integer division  $t$  by  $T_1$ . The piecewise character of the coordinate change assignment is associated with the consideration of gaits (elementary leg movements), in which a sequential transfer of the crossed legs is performed.

Solving equation (1) with respect to (2) and (3), when substituting the program movement (9)–(11), we obtain the dependences of the rotation angles  $\varphi_i, \alpha_i, \beta_i$  of the leg links (Fig. 6).

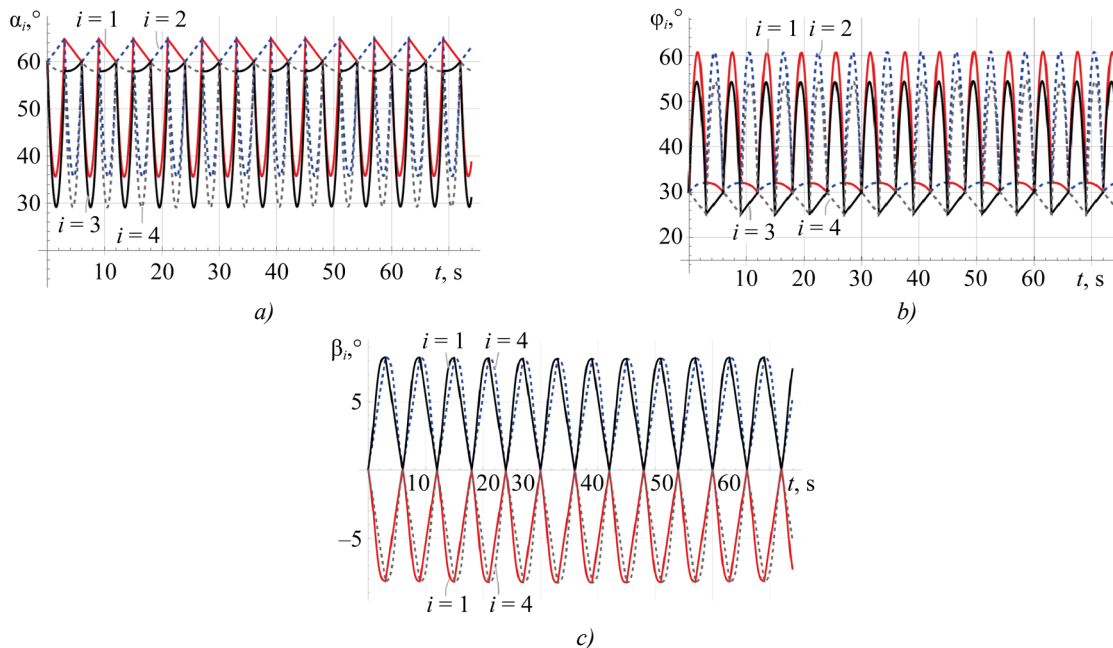


Fig. 6. Course angle turn simulation results:  $a$  — angles  $\alpha_i(t)$ ;  $b$  — angles  $\varphi_i(t)$ ;  $c$  — angles  $\beta_i(t)$

The simulation results show that the dependences of the rotation angles of the leg links are periodic functions and, as a consequence, the considered rotational movements of the robot platform can take place without the occurrence of singular configurations. Thus, the proposed plan for transferring the legs (10), (11) during rotation around vertical axis  $Z$  is suitable for implementing the rotation of the robot at an arbitrary angle  $\psi$ .

**Discussion and Conclusion.** The constructed mathematical model of kinematics allows for the determination of the angles in the joints of the robot leg links, which must be provided to implement the desired movement of the robot platform. The obtained results of numerical modeling of the rotary movements of the robot body confirmed the operability of the developed kinematic model for assessing the implementation of the specified robot movements, taking into account the limitations and dimensions of the structure.

In cases where the robot platform performs a flat motion (at zero pitch and roll angles), the constructed mathematical model of kinematics coincides with the published results of other authors. The results obtained in this paper are a generalization and refinement of the equations that couple the rotation angles of the robot leg links with the position and orientation of the robot platform.

The application of the developed kinematic model (1), (4), as well as the proposed plan for transferring legs (10), (11) during rotation around the vertical axis, will allow constructing control at the kinematic level for the robot movement along curvilinear trajectories. This approach will significantly expand the robot functionality and increase its performance efficiency under various operating conditions.

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