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Original Theoretical Research

Optical Flow Parameter Identification in Vision Systems under Measurement Noise with Unknown Probabilistic Characteristics

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Abstract

Introduction. Methods for processing information contained in variations in the optical flow intensity during object motion are widely used in numerous technical applications, including space research, technical diagnostics, machine (technical) vision, object tracking in digital images, autonomous navigation of unmanned vehicles, etc. Among these methods, monocular techniques for estimating the parameters of the video camera own motion have demonstrated the greatest practical efficiency, both based on a cartographic analysis of the underlying surface and using various algorithms for estimating optical flow (velocity field) parameters. Although existing velocity field estimation algorithms offer advantages such as operability in the absence of terrain maps and computational costs easily implemented in onboard processors, their practical application is significantly complicated by the inevitable noise in optical measurements, which can have a wide variety of physical origins and reduce dramatically the accuracy of optical flow parameter estimation. Therefore, the objective of the research discussed in this paper is to address the problem of simultaneous high-precision estimation of the intensity of optical flow and identification of its parameters under conditions of measurement noise with unknown probabilistic characteristics. A theoretical solution to this problem enables the development of a new general approach to the synthesis of robust algorithms for high-precision optical flow processing in video monitoring systems. This, when applied in practice, will ensure noise immunity and the required accuracy characteristics for machine vision systems, autonomous navigation systems of unmanned vehicles, and other applications.

Materials and Methods. The solution was obtained by monocular methods for determining the camera proper motion and minimizing a regularized quadratic criterion. The starting-point for the solution was the formulation of the problem as one of stochastic estimation and parametric identification of a discrete linear nonstationary system while observing its state vector under interference with an unknown probability distribution. The synthesis of the "estimation-identification" algorithm in this formulation was implemented as a procedure that guaranteed the highest accuracy in the minimax sense. Minimizing the resulting minimax criterion allowed us to construct an algorithm for estimating and parametrically identifying the optical flow as a stable vector recurrent procedure, easily implemented in onboard computers of moving objects.

Results. The problem of simultaneous high-precision estimation of the optical flow intensity and identification of its parameters under measurement noise with unknown probabilistic characteristics is solved. This problem has not been considered in the scientific literature to date. The solution will enable the development of an approach to the synthesis of robust algorithms for high-precision processing of optical flows in video monitoring system. In practical use, it will ensure noise immunity and the required accuracy characteristics of machine vision systems, autonomous navigation systems of unmanned vehicles, etc. The assessment of the practical applicability of the developed algorithm for estimating and identifying optical flow parameters was performed under conditions of non-Gaussian measurement noise through numerical simulation. Despite the specified high level of measurement noise, the errors in estimating the optical flow intensity at all tested coordinate points proved to be both rapidly converging to steady-state values and very small in the steady-state mode (a few percent of the maximum value of the optical flow intensity).

Discussion. The obtained data confirm that the proposed algorithm has such advantages over known optical flow processing algorithms as the ability to estimate the intensity of the optical flow and identify its parameters under noise conditions, whose probability distributions are a priori unknown. It is characterized by high accuracy and robustness, and it does not require high computational costs.
Conclusion. The practical significance of the developed algorithm consists, firstly, in the possibility of high-precision stable processing of the optical flow under the conditions of uncertain probabilistic nature of measurement errors, and secondly, in the computational efficiency of the developed "estimation-identification" procedure. This, in turn, provides its successful practical application in solving optical information processing problems in vision systems, navigation of autonomous robotic systems, space exploration, technical diagnostics, and other fields.

Keywords: optical flow, optical flow parameters, measurement disturbances, uncertain probabilistic characteristics, regularization, quadratic criterion

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Оригинальное теоретическое исследование

Идентификация параметров оптического потока в системах технического зрения в условиях помех измерения с неизвестными вероятностными характеристиками

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Аннотация
Введение. Методы обработки информации, содержащейся в изменении интенсивности оптического потока, регистрируемого в процессе движения объекта, широко востребованы в различных сферах деятельности: в космических исследованиях, технической диагностике, машинном (техническом) зрении, отслеживании объектов на цифровых изображениях, автономной навигации беспилотных объектов и др. В классе данных методов наибольшую эффективность на практике демонстрируют монокулярные методы оценки параметров собственного движения видеокамеры, как построенные на картографическом анализе подстилающей поверхности, так и использующие различные алгоритмы оценки параметров оптического потока (поля скоростей). Несмотря на то, что существующие алгоритмы оценки поля скоростей обладают такими преимуществами, как работоспособность при отсутствии карт местности и вычислительные затраты, легко реализуемые в бортовых процессорах, их применение на практике существенно осложнено неизбежными помехами оптических измерений, имеющих самую разную физическую природу и резко снижающих точность оценки параметров оптического потока. В связи с этим цель представленного исследования — решение задачи одновременной высокоточной оценки интенсивности оптического потока и идентификации его параметров в условиях помех измерения с неизвестными вероятностными характеристиками. Теоретическое решение данной задачи позволит сформировать оригинальный подход к синтезу робастных алгоритмов высокоточной обработки оптических потоков в системах видеомониторинга, что при практическом использовании данных алгоритмов обеспечит помехоустойчивость и требуемые точностные характеристики систем технического зрения, автономных навигационных систем беспилотных объектов и др.
Материалы и методы. Решение основано на монокулярных методах определения собственного движения видеокамеры и минимизации регуляризованного квадратичного критерия. Его отправной точкой стала формулировка поставленной задачи как задачи стохастической оценки и параметрической идентификации дискретной линейной нестационарной системы при наблюдении ее вектора состояния в условиях помех с неизвестным вероятностным распределением. Синтез алгоритма оценки-идентификации в подобной постановке был реализован в виде процедуры, гарантирующей наивысшую точность в минимаксном смысле. Минимизация полученного минимаксного критерия позволила построить алгоритм оценки и параметрической идентификации оптического потока в виде устойчивой векторной рекуррентной процедуры, легко реализуемой в бортовых вычислителях подвижных объектов.

Результаты исследования. Авторами решена задача одновременной высокоточной оценки интенсивности оптического потока и идентификации его параметров в условиях помех измерения с неизвестными вероятностными характеристиками, которая до настоящего времени не рассматривалась в научной литературе. Это решение позволит сформировать подход к синтезу робастных алгоритмов высокоточной обработки оптических потоков в системах видеомониторинга и обеспечить при практическом использовании помехоустойчивость и требуемые точностные характеристики систем технического зрения, автономных навигационных систем беспилотных объектов и др. Оценка возможности практического применения разработанного алгоритма оценки-идентификации параметров оптического потока была проведена в условиях негауссовских помех измерения путем численного моделирования. Несмотря на заданный высокий уровень помех измерения, погрешности оценки интенсивности оптического потока во всех протестированных координатных точках оказались как быстро сходящимися к стационарным значениям, так и весьма малыми в установившемся режиме (единицы процентов от максимального значения интенсивности оптического потока).

Обсуждение. Полученные данные подтверждают, что предложенный алгоритм обладает такими преимуществами перед известными алгоритмами обработки оптического потока, как возможность оценивания интенсивности оптического потока и идентификации его параметров в условиях помех, вероятностные распределения которых априори неизвестны. Ему присущи высокая точность и устойчивость, и он не требует больших вычислительных затрат.

Заключение. Практическая значимость разработанного алгоритма состоит, во-первых, в возможности высокоточной устойчивой обработки оптического потока в условиях неопределенного вероятностного характера помех измерения, во-вторых, в вычислительной эффективности разработанной процедуры оценки-идентификации. Это, в свою очередь, обеспечивает ее успешное использование при решении задач обработки оптической информации в системах машинного зрения, в задачах навигации автономных робототехнических комплексов, в космических исследованиях, технической диагностике и др.

Ключевые слова: оптический поток, параметры оптического потока, помехи измерения, неопределенные вероятностные характеристики, регуляризация, квадратичный критерий

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Introduction. The rapid adoption of unmanned technologies has driven explosive growth in the global unmanned aerial vehicle market. Forecasts indicate that it will increase more than triple or quadruple by 2030. For example, according to GlobalData[1], the market will grow from $32.2 billion in 2024 to $89.8 billion by 2030. A report from Research Nester[2], in turn, provides an even more significant long-term forecast — growth from $42.39 billion in 2025 to $191.89 billion by 2035.

At the same time, the adoption of autonomous systems across various industries is increasing by more than 10–20% annually. Thus, according to Market Report Analytics[3], as of January 2026, the market for autonomous agricultural machinery is growing at a compound annual growth rate (CAGR) of nearly 18.5%, while the segment of autonomous agricultural robots is showing a growth rate of 18–20%. According to forecasts from Fortune Business Insights and GM Insights[4], the market for automated logistics and autonomous mobile robots in 2025–2026 will grow by 12.8–17.9%, and the market for autonomous construction equipment, according to estimates by The Business Research Company[5], will show a steady CAGR growth of nearly 9.1% by 2026.

Such intensive use of unmanned technologies requires high precision in autonomous navigation systems, in which a key role is played by estimating the parameters of the video camera own motion [1–3], associated with the linear and angular velocities of the unmanned object [4, 5]. These parameters are used to determine its position and orientation in space and to calculate the motion trajectory based on analyzing pixel displacement in the video stream. This provides independence from external sensors and reference to visual landmarks on the underlying surface (which is critically

[1] *Globaldata.* URL: https://www.globaldata.com/store/report/drones-theme-analysis (accessed: 30.01.2026).
[2] *Research Nester.* URL: https://www.researchnester.com/reports/unmanned-aerial-vehicle-market/6364 (accessed: 30.01.2026).
[3] *Market Report Analytics.* URL: https://www.datainsightsmarket.com/reports/autonomous-agriculture-robots-283675 (accessed: 30.01.2026).
[4] *Fortune Business Insights and GM Insights.* URL: https://www.thebusinessresearchcompany.com/report/autonomous-construction-equipment-global-market-report (accessed: 30.01.2026).
[5] *Business Research Company.* URL: https://www.fortunebusinessinsights.com/logistics-automation-market-105991 (accessed: 30.01.2026).

important, for example, for the accuracy of hovering and landing), as well as instantaneous response to changes in the vehicle motion. This is most significant for high-speed objects, where the slightest error in estimating self-motion leads to a catastrophe. At the same time, the required error in determining optical flow parameters [6, 7] for modern autonomous objects does not exceed 1–2%. Such accuracy indicators make it possible to maintain the trajectory within acceptable limits over long distances, perform safe obstacle avoidance at high speed, and achieve a stable position at a single point in hover mode without using satellite navigation. This demonstrates the reliability of operation under real, rather than laboratory, conditions.

Achieving this level of accuracy requires advanced algorithms that are robust to various types of noise. In real-world conditions, however, measurements of optical flow intensity are accompanied by noise with unknown probabilistic characteristics, which significantly complicates the identification of these parameters, especially under conditions of artificially induced noise.

Modern approaches to optical flow identification are primarily focused on estimating the velocity vector field under known noise conditions. Thus, stochastic methods, which include the application of Kalman filters and Bayesian estimates [8, 9], require apriori knowledge of the noise characteristics, which reduces their practical applicability. Robust methods [10, 11] account for uncertainty but do not allow for the simultaneous estimation of the intensity and parameters of the velocity field.

Existing monocular methods for estimating the self-motion parameters of a video camera can be conditionally divided into methods of cartographic analysis of the underlying surface [12] and methods of estimating optical flow parameters (velocity field) [8].

Cartographic analysis methods involve comparing the current frame and a prior map stored in memory. This requires preliminary geodetic survey of the terrain and significant computational resources, making them unsuitable for onboard implementation.

Methods for estimating optical flow parameters are based either on the analysis of image features (for example, identifying key points or scene edges [13]) or on the analysis of flow parameters across video frames using mathematical models.

The fundamental mathematical expression for constructing algorithms for the estimation and identification of optical flow and its parameters u, v [2] is the so-called optical flow constraint equation [10]:

$$\frac{\partial I}{\partial t} + u\frac{\partial I}{\partial x} + v\frac{\partial I}{\partial y} = 0, \qquad (1)$$

where $u(x, y, t)$, $v(x, y, t)$ — current components of the brightness pattern velocity at point (x, y); $\frac{\partial I(x, y, t)}{\partial t}$, $\frac{\partial I(x, y, t)}{\partial x}$, $\frac{\partial I(x, y, t)}{\partial y}$ — partial derivatives of the brightness function $I(x, y, t)$ (intensity of the optical flow) of the scanned scene.

The lack of a unique solution to equation (1) with respect to two unknown variables u, v has led to the creation of a whole range of methods and algorithms that provide the required accuracy only in specific practical situations. Among them, the most common are:

– gradient methods, which allow for real-time calculation of the camera motion vector based on the analysis of intensity gradients in each frame;

– correlation methods, which use a direct search for similar fragments among adjacent frames through a correlation function or root-mean-square deviation;

– variational methods, based on various assumptions regarding the spatiotemporal dynamics of optical flow intensity.

The basic approach to constructing these algorithms is, as a rule, the introduction of various restrictions imposed on the dynamics of changes in the intensity of the optical flow of the scanned underlying surface. Thus, for example, the assumption of equal displacement of all points of the scanned surface, combined with the approximation of the intensity in the neighborhood of each pixel by a quadratic form, provides the specified accuracy of optical flow parameter identification for low-speed motion [4]. And the additional introduction of the assumption of constant scene brightness along the object motion trajectory, using the minimization of the objective functional by the least squares method, allows for even greater accuracy at small displacements (the Lucas-Kanade method) [3]. A more general assumption is that of optical flow “smoothness” (the absence of abrupt changes in the velocity field), which provides, with accuracy acceptable for a wide range of practical applications, the calculation of optical flow based on the so-called energy functional proposed by Horn and Schunck [2]:

$$E(u,v) = \int_D \left(\frac{\partial I}{\partial t} + \nabla I \begin{vmatrix} u \\ v \end{vmatrix} \right)^2 + \lambda\left(|\nabla u|^2 + |\nabla v|^2 \right) dxdy,$$

where D — domain of existence of variables x, y; ∇ — Hamilton operator; λ — regularization parameter.

But since this method uses optimization algorithms based either on discrete representations of the integral and partial derivatives, or on the Euler-Lagrange equations with reflecting boundary conditions, the computational costs of such an approach often prove to be unfeasible for onboard computers [2].

After analyzing the existing methods for estimating optical flow parameters, it can be summarized that their key disadvantages are, firstly, the large amount of computational costs, which does not allow estimating optical flow parameters in real time, and, secondly, critical sensitivity to measurement noise of optical flow intensity, which is inevitable in real navigation systems.

The presence of such problems indicates the lack of a methodological approach capable of solving the problem of joint stochastic estimation of intensity and identification of optical flow parameters under conditions of unknown noise, which represents a significant gap in scientific knowledge.

The objective of this work is to solve the problem of simultaneous high-precision estimation of optical flow intensity and identification of its parameters under conditions of measurement noise with unknown probabilistic characteristics. The research tasks include:

1) development of an estimation-identification algorithm capable of operating without prior knowledge of the underlying surface;

2) providing robustness and high accuracy of optical flow parameter estimation under conditions of unknown noise;

3) implementation of a method focused on minimal computational costs, making it suitable for onboard data processing.

Materials and Methods. The solution to the stated problems is considered in the coordinate system of the video camera image (x, y). Since the analyzed image is a matrix of pixels, by discretizing the coordinate plane, we obtain the basic optical flow equation (1) in a discrete form with respect to the scene coordinates:

$$
\begin{gathered}
\frac{\partial I(x_1,y_1,t)}{\partial t}=-u(x_1,y_1,t)\left[\frac{I(x_1,y_1,t)}{\Delta x}-0\right]-v(x_1,y_1,t)\left[\frac{I(x_1,y_1,t)}{\Delta y}-0\right]\\
\frac{\partial I(x_1,y_2,t)}{\partial t}=-u(x_1,y_2,t)\frac{I(x_1,y_2,t)}{\Delta x}-v(x_1,y_2,t)\frac{I(x_1,y_2,t)-I(x_1,y_1,t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_1,y_i,t)}{\partial t}=-u(x_1,y_i,t)\frac{I(x_1,y_i,t)}{\Delta x}-v(x_1,y_i,t)\frac{I(x_1,y_i,t)-I(x_1,y_{i-1},t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_1,y_N,t)}{\partial t}=-u(x_1,y_N,t)\frac{I(x_1,y_N,t)}{\Delta x}-v(x_1,y_N,t)\frac{I(x_1,y_N,t)-I(x_1,y_{N-1},t)}{\Delta y}\\
\frac{\partial I(x_2,y_1,t)}{\partial t}=-u(x_2,y_1,t)\frac{I(x_2,y_1,t)-I(x_1,y_1,t)}{\Delta x}-v(x_2,y_1,t)\frac{I(x_2,y_1,t)}{\Delta y}\\
\frac{\partial I(x_2,y_2,t)}{\partial t}=-u(x_2,y_2,t)\frac{I(x_2,y_2,t)-I(x_1,y_2,t)}{\Delta x}-v(x_2,y_2,t)\frac{I(x_2,y_2,t)-I(x_2,y_1,t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_2,y_i,t)}{\partial t}=-u(x_2,y_i,t)\frac{I(x_2,y_i,t)-I(x_1,y_i,t)}{\Delta x}-v(x_2,y_i,t)\frac{I(x_2,y_i,t)-I(x_2,y_{i-1},t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_2,y_N,t)}{\partial t}=-u(x_2,y_N,t)\frac{I(x_2,y_N,t)-I(x_1,y_N,t)}{\Delta x}-v(x_2,y_N,t)\frac{I(x_2,y_N,t)-I(x_2,y_{N-1},t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_j,y_1,t)}{\partial t}=-u(x_j,y_1,t)\frac{I(x_j,y_1,t)-I(x_{j-1},y_1,t)}{\Delta x}-v(x_j,y_1,t)\frac{I(x_j,y_1,t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_j,y_i,t)}{\partial t}=-u(x_j,y_i,t)\frac{I(x_j,y_i,t)-I(x_{j-1},y_i,t)}{\Delta x}-v(x_j,y_i,t)\frac{I(x_j,y_i,t)-I(x_j,y_{i-1},t)}{\Delta y}\\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\\
\frac{\partial I(x_j,y_N,t)}{\partial t}=-u(x_j,y_N,t)\frac{I(x_j,y_N,t)-I(x_{j-1},y_N,t)}{\Delta x}-v(x_j,y_N,t)\frac{I(x_j,y_N,t)-I(x_j,y_{N-1},t)}{\Delta y}
\end{gathered}, \qquad (2)
$$

where j=1,….,N, (x_i,y_j) — coordinates of ij-th point in the adopted coordinate system; $I(x_i,y_j,t)$ — current intensity of the optical flow at ij-th point; Δx, Δy — current intensity of the optical flow at axes Ox, Oy, respectively (pixel sizes of the scanned scene in the corresponding directions).

To simplify the subsequent notation, the following designations are introduced:

$$u\left(x_i, y_j, t\right) = u_{ij}, v\left(x_i, y_j, t\right) = v_{ij}, I\left(x_i, y_j, t\right) = I_{ij},$$

$$U = \left|u_{11}u_{12}\dots u_{1N}u_{21}u_{22}\dots u_{2N}\dots\dots u_{N1}u_{N2}\dots u_{NN}\right|^{\mathrm{T}},\ V = \left|v_{11}v_{12}\dots v_{1N}v_{21}v_{22}\dots v_{2N}\dots\dots v_{N1}v_{N2}\dots v_{NN}\right|^{\mathrm{T}},$$

$$I = \left|I_{11}I_{12}\dots I_{1N}I_{21}I_{22}\dots I_{2N}\dots\dots I_{N1}I_{N2}\dots I_{NN}\right|^{\mathrm{T}},$$

$$D\left(I\right) = \left|\begin{array}{cccccc}
-\frac{I_{11}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-1} & -\frac{I_{11}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-1} & & \\
0 & -\frac{I_{12}}{\Delta x}\ \underbrace{0\dots\dots0}_{N^2-2} & 0 & -\frac{I_{12}-I_{11}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-2} & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{N-1} & -\frac{I_{1N}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-N} & \underbrace{0\dots\dots0}_{N-1} & -\frac{I_{1N}-I_{1(N-1)}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-N} \\
\underbrace{0\dots\dots0}_{N} & -\frac{I_{21}-I_{11}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-N-1} & \underbrace{0\dots\dots0}_{N} & -\frac{I_{21}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-N-1} \\
\underbrace{0\dots\dots0}_{N+1} & -\frac{I_{22}-I_{12}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-N-2} & \underbrace{0\dots\dots0}_{N+1} & -\frac{I_{22}-I_{21}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-N-2} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{2N-1} & -\frac{I_{2N}-I_{1N}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-2N} & \underbrace{0\dots\dots0}_{2N-1} & -\frac{I_{2N}-I_{2(N-1)}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-2N} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{N(j-1)} & -\frac{I_{j1}-I_{(j-1)1}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-N(j-1)-1} & \underbrace{0\dots\dots0}_{N(j-1)} & -\frac{I_{j1}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-N(j-1)-1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{N(j-1)+(i-1)} & -\frac{I_{ji}-I_{(j-1)i}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-N(j-1)-i} & \underbrace{0\dots\dots0}_{N(j-1)+(i-1)} & -\frac{I_{ji}-I_{j(i-1)}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-N(j-1)-i} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{Nj-1} & -\frac{I_{jN}-I_{(j-1)N}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-Nj} & \underbrace{0\dots\dots0}_{Nj-1} & -\frac{I_{jN}-I_{j(N-1)}}{\Delta y} & \underbrace{0\dots\dots0}_{N^2-Nj} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{N^2-N} & -\frac{I_{N1}-I_{(N-1)1}}{\Delta x} & \underbrace{0\dots\dots0}_{N-1} & \underbrace{0\dots\dots0}_{N^2-N} & -\frac{I_{N1}}{\Delta y} & \underbrace{0\dots\dots0}_{N-1} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{N^2-N-1+i} & -\frac{I_{Ni}-I_{(N-1)i}}{\Delta x} & \underbrace{0\dots\dots0}_{N-i} & \underbrace{0\dots\dots0}_{N^2-N-1+i} & -\frac{I_{Ni}-I_{N(i-1)}}{\Delta y} & \underbrace{0\dots\dots0}_{N-i} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\underbrace{0\dots\dots0}_{N^2-1} & -\frac{I_{NN}-I_{(N-1)N}}{\Delta x} & \underbrace{0\dots\dots0}_{N^2-1} & -\frac{I_{NN}-I_{N(N-1)}}{\Delta y} & &
\end{array}\right|,$$

Equation (2) can be represented in vector form:

$$\frac{\partial I}{\partial t} = D\left(I\right)\begin{vmatrix} U \\ V \end{vmatrix}. \tag{3}$$

In the subsequent synthesis of the algorithm for estimating and identifying the optical flow intensity and its parameters, we use a discrete description of all the above time functions, which is due to the digital nature of information processing in onboard computers. In this case, the discrete form of equation (3) takes the form:

$$I_k = I_{k-1} + \Delta t \cdot D\left(I_{k-1}\right)\begin{vmatrix} U_{k-1} \\ V_{k-1} \end{vmatrix},$$

where I_k — value of the intensity vector of the optical flow at the k-th moment of time; U_k, V_k — values of the vectors of the optical flow parameters at the k-th moment of time; Δt — time sampling step.

To simplify further discussion, we denote vector $\begin{vmatrix} U_k \\ V_k \end{vmatrix}$ as ξ_k, $k = 1\dots N$, $\xi_0 = 0$, and write the resulting discrete equation in the form:

$$I_k = I_{k-1} + \Delta t \cdot D\left(I_{k-1}\right)\xi_{k-1}. \tag{4}$$

At each *ij*- th scanning point, the optical flow intensity is measured with error w_{ij}, whose probabilistic characteristics (particularly, type of distribution density) are generally unknown. Introducing the measurement noise vector analogously W_k, to the above:

$$W_k = \left| w_{11} w_{12} \ldots w_{1N} w_{21} w_{22} \ldots w_{2N} \ldots\ldots w_{N1} w_{N2} \ldots w_{NN} \right|^{\mathrm{T}},$$

measurement vector Z_k of the optical flow intensity I_k, captured from the entire scanned scene, is represented as

$$Z_k = I_k + W_k .$$

Thus, in the used definitions, the problem of estimating optical flow intensity and identifying its parameters represents a problem of stochastic estimation and parametric identification of a discrete linear nonstationary system (4) when observing its state vector under noise conditions with an uncertain probability distribution density.

It is important to note that the application of traditional discrete estimation methods, in particular Kalman filtering, is impossible here due to, firstly, the presence of an unknown parameter vector ξ_k, and, secondly, the uncertainty of the probabilistic distribution of the measurement noise W_k [14].

Assuming that in the problem under consideration, the probabilistic distribution of the measurement noise of the optical flow intensity belongs to the class of distributions with bounded squares due to the limited power of real noise [14], the estimate of the optical flow intensity $\hat{I}_k$ with simultaneous determination of the parameter vector ξ_k, will be sought as an estimate that guarantees the highest estimation accuracy in the minimax sense (i.e., the minimum error in the most unfavorable situation determined by the given class of distribution), selecting the quadratic function as the minimized objective function: $[Z_k - \hat{I}_k]^T [Z_k - \hat{I}_k]$. To provide the robustness of the developed algorithm when forming the criterion for optimal estimation $(\hat{I}_k)$ and identification (ξ_k), we use regularizing additive components: a Tikhonov regularization component based on L_2 norm — $\alpha \cdot \xi_k^T \Delta\xi_k$ and a component that takes into account the natural constraint on the continuity of changes in the parameter vector ξ_k over time: $\beta \cdot \Delta\xi_k^T \Delta\xi_k$, $\Delta\xi_k = \xi_k - \Delta\xi_{k-1}$, where α, β — are regularization parameters that provide adaptation to the specific operating conditions of the dynamic system.

In accordance with the above, we formulate the minimax optimality criterion J_k in the following form:

$$\begin{gathered} J_k = \left[Z_k - \hat{I}_k \right]^{\mathrm{T}} \left[Z_k - \hat{I}_k \right] + \alpha\, \xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right)^{\mathrm{T}} \xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right) + \beta \cdot \Delta\xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right)^{\mathrm{T}} \Delta\xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right), \\ \Delta\xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right) = \xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right) - \xi_{k-2}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right). \end{gathered} \tag{5}$$

For the subsequent definition of the vector function $\xi_{k-1}(\hat{I}_k, \hat{I}_{k-1})$ that determines the current estimate of the intensity of the optical flow $\hat{I}_k$:

$$\hat{I}_k = \hat{I}_{k-1} + \Delta t \cdot D\left(\hat{I}_{k-1}\right) \xi_{k-1}, \tag{6}$$

we differentiate J_k (5) with respect to ξ_{k-1}:

$$\begin{aligned} \frac{dJ_k}{d\xi_{k-1}} &= -\frac{d\left(\left[Z_k - \hat{I}_k \right]^{\mathrm{T}} \left[Z_k - \hat{I}_k \right]\right)}{d\left[Z_k - \hat{I}_k \right]} \frac{d\hat{I}_k}{d\xi_{k-1}} + 2\alpha \xi_{k-1}^{\mathrm{T}}\left(\hat{I}_k, \hat{I}_{k-1}\right) + 2\beta \left[\xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right) - \xi_{k-2}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right) \right]^{\mathrm{T}} = \\ &= -2\left[Z_k - \hat{I}_k \right]^{\mathrm{T}} \Delta t\, D\left(\hat{I}_{k-1}\right) + 2(\alpha + \beta) \xi_{k-1}^{\mathrm{T}}\left(\hat{I}_k, \hat{I}_{k-1}\right) - 2\beta \xi_{k-2}^{\mathrm{T}}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right). \end{aligned}$$

Equating the resulting expression to 0, we arrive at the equality:

$$\left[Z_k - \hat{I}_k \right]^{\mathrm{T}} \Delta t\, D\left(\hat{I}_{k-1}\right) + \beta \xi_{k-2}^{\mathrm{T}}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right) = (\alpha + \beta) \xi_{k-1}^{\mathrm{T}}\left(\hat{I}_k, \hat{I}_{k-1}\right),$$

from which we determine the desired recurrent vector function $\xi_{k-1}(\hat{I}_k, \hat{I}_{k-1})$:

$$\frac{\Delta t}{\alpha + \beta} D^{\mathrm{T}}\left(\hat{I}_{k-1}\right)\left[Z_k - \hat{I}_k \right] + \frac{\beta}{\alpha + \beta} \xi_{k-2}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right) = \xi_{k-1}\left(\hat{I}_k, \hat{I}_{k-1}\right).$$

Substituting the found expression $\xi_{k-1}(\hat{I}_k, \hat{I}_{k-1})$ into estimation equation (6), we have:

$$\hat{I}_k = \hat{I}_{k-1} + \frac{\Delta t^2}{\alpha + \beta} D\left(\hat{I}_{k-1}\right) D^{\mathrm{T}}\left(\hat{I}_{k-1}\right) Z_k - \frac{\Delta t^2}{\alpha + \beta} D\left(\hat{I}_{k-1}\right) D^{\mathrm{T}}\left(\hat{I}_{k-1}\right) \hat{I}_k + \frac{\Delta t\, \beta}{\alpha + \beta} D\left(\hat{I}_{k-1}\right) \xi_{k-2}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right).$$

Since the right side of the resulting equation depends on the current estimate, $\hat{I}_k$, then for the final determination of $\hat{I}_k$ we transform it as follows:

$$\left(E + \frac{\Delta t^2}{\alpha + \beta} D\left(\hat{I}_{k-1}\right)) D^{\mathrm{T}}\left(\hat{I}_{k-1}\right) \right) \hat{I}_k = \hat{I}_{k-1} + \frac{\Delta t^2}{\alpha + \beta} D\left(\hat{I}_{k-1}\right) D^{\mathrm{T}}\left(\hat{I}_{k-1}\right) Z_k + \frac{\Delta t\, \beta}{\alpha + \beta} D\left(\hat{I}_{k-1}\right) \xi_{k-2}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right),$$

where E — identity matrix of dimension N^2, from which we finally obtain a stochastic estimate of the intensity of the optical flow $\hat{I}_k$:

$$\hat{I}_k = \left(E + \frac{\Delta t^2}{\alpha+\beta} D\left(\hat{I}_{k-1}\right) D^{\mathrm{T}}\left(\hat{I}_{k-1}\right) \right)^{-1} \left\{ \hat{I}_{k-1} + \frac{\Delta t^2}{\alpha+\beta} D\left(\hat{I}_{k-1}\right) D^{\mathrm{T}}\left(\hat{I}_{k-1}\right) Z_k + \frac{\Delta t \beta}{\alpha+\beta} D\left(\hat{I}_{k-1}\right) \xi_{k-2}\left(\hat{I}_{k-1}, \hat{I}_{k-2}\right) \right\}, \quad (7)$$

where the identification of the vector of optical flow parameters $\xi_{k-2}(\hat{I}_k, \hat{I}_{k-1})$ is carried out in accordance with the expression:

$$\frac{\Delta t}{\alpha+\beta} D^T\left(\hat{I}_{k-2}\right)\left[Z_{k-1} - \hat{I}_{k-1}\right] + \frac{\beta}{\alpha+\beta} \xi_{k-3} = \xi_{k-2}. \quad (8)$$

The key advantageous features of the estimation-identification algorithm (7), (8) are, firstly, the ability to construct the velocity field not only for any part of the image scene but also for the entire scanned scene, and, secondly, the relatively low computational costs due to the determination of vector recursion instead of implementing multidimensional nonlinear optimization algorithms in known methods.

The efficiency evaluation of the proposed approach was obtained through a series of numerical experiments with various nonlinear functions and different values of the mean and variance of the Laplace distribution, whose totality revealed a consistent trend. As an illustration, we use one of the typical scenarios.

The modeling of the spatiotemporal change in the intensity of the optical flow $I(x,y,k)$ was performed in the coordinate grid $\{x{:}[0;5],\ y{:}[0;5]\}$ with a uniform step equal to 0.5, on a time interval of [0;600] seconds with a step of $\Delta t = 0.1$ in accordance with the expression:

$$I(x,y,k) = 0.33 e^{-(x-2.7)^2-(y-2.7)^2}\left(cos\left(0.9k\Delta t\right)+1\right) \text{ lm}. \quad (9)$$

The formation of the measurement noise vector W_k was performed on the basis of modeling a spatiotemporal random field described as

$$G(x,y,k) = \left(cos\left(1.3x\right)cos\left(0.97y\right)sin\,k\Delta t + 1\right)L_t,$$

where L_t — random sequence with a Laplace distribution with zero mean and variance $D = (8.7 \cdot 10^{-2})^2$ (lm)^2.

The optical flow intensity estimation was performed in real time according to algorithm (7) with parallel implementation of identification procedure (8) for the optical flow parameter vector ξ_k at regularization coefficient values of $\alpha = 0.033$, $\beta = 0.0017$. The accuracy analysis of the estimation-identification algorithm (7), (8) was performed through comparing the values of the optical flow intensity estimation vector $\hat{I}_k$ and the true optical flow intensity values determined by expression (9).

Figures 1 and 2 show the graphs of changes in the optical flow parameters u, v, characteristic of the selected spatiotemporal change in optical flow intensity (9) and presented for the point $\{x=4,\ y=3\}$, while Figures 3 and 4 show the error graphs when estimating the components of the optical flow intensity vector for two arbitrarily selected points of the coordinate grid $\{i,j\}$: {1,4},{5,2}.

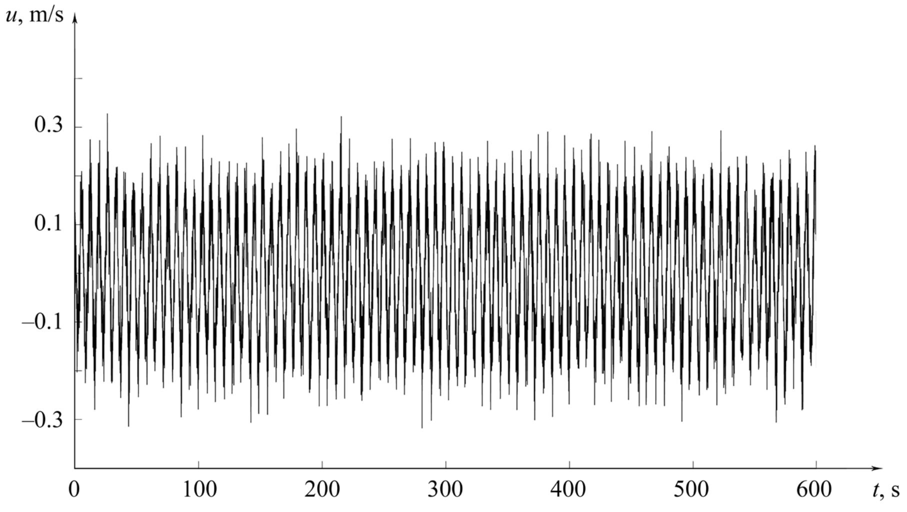

Fig. 1. Graph of change in optical flow parameter u

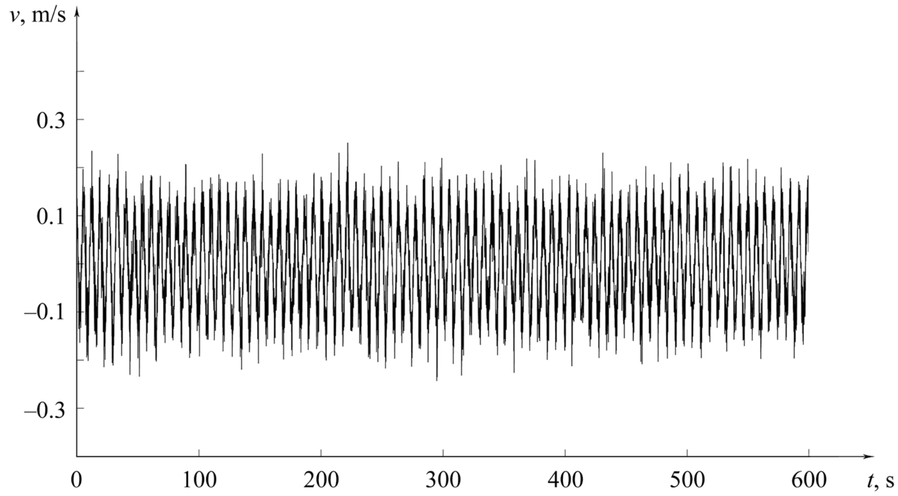

Fig. 2. Graph of change in optical flow parameter *v*

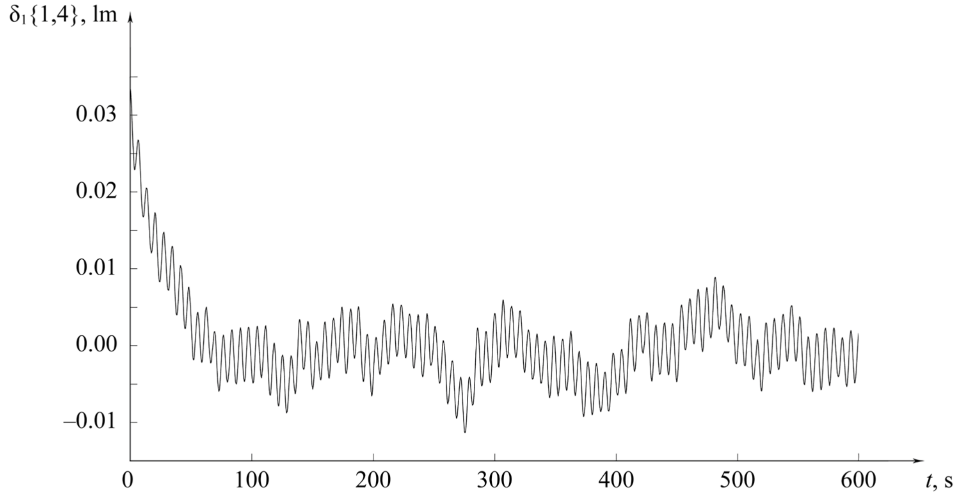

Fig. 3. Error graph for estimating optical flow intensity at coordinate point {1,4}

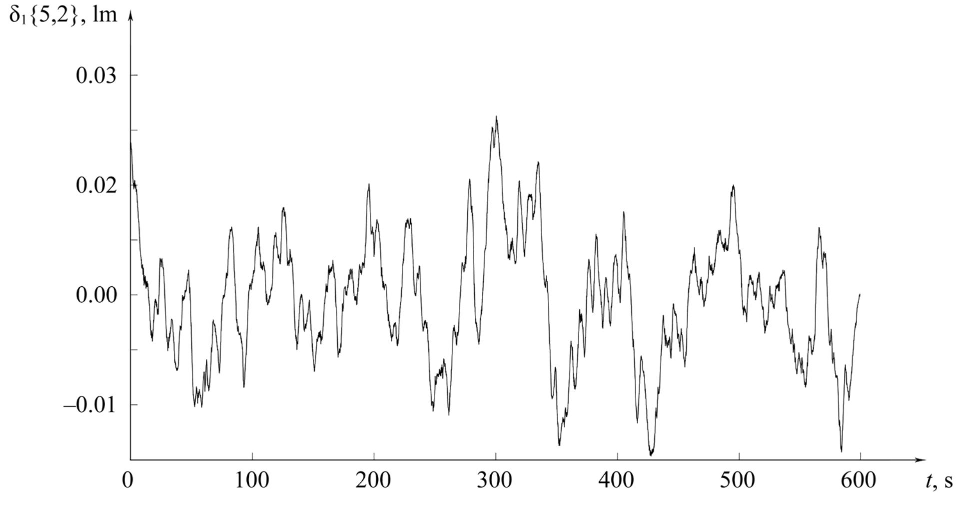

Fig. 4. Error graph for estimating optical flow intensity at coordinate point {5,2}

The presented results allow us to conclude that, despite a fairly high level of measurement noise, the errors in estimating the optical flow intensity at all tested coordinate points turned out to be, firstly, rapidly converging to steady-state values, and, secondly, very small in the steady-state mode: at point {1,4} — no more than 0.7% of the maximum value of the optical flow intensity I_k, and at point {5,2} — no more than 2,1 %.

To analyze the computational efficiency of the developed algorithm, a comparative assessment of computational costs was performed with the Lucas-Kanade method [3], for which the computational costs turned out to be 6.7 times higher.

Research Results. Thus, it can be concluded that the main objective of the work, which consisted in developing a method to solve the problem of simultaneous stochastic estimation of optical flow intensity and identification of its parameters under conditions of measurement noise with unknown probabilistic characteristics — a problem not previously considered in the literature — has been achieved. The basic results of the work are:

– estimation-identification algorithm that implements an integrated approach to estimating optical flow parameters and differs from previously known ones by its recurrent structure with regularization constraints;

– robust estimates of the optical flow parameters $u(x,y,t)$, $v(x,y,t)$ with an error level not exceeding 2.1% under conditions of incomplete apriori information and without accurate knowledge of the noise distribution or the characteristics of the underlying surface;

– efficient algorithmic implementation of the method, providing a reduction in computational resources to 17% (more than six times) of the costs of known gradient methods.

Discussion. The obtained results confirm the feasibility of applying the method of simultaneous optical flow intensity estimation and its parameter identification under conditions of unknown noise. The proposed solution provides the robustness of optical flow parameter estimation even in the absence of accurate information about the probabilistic characteristics of the noise, which is particularly important in real-world observation conditions where measurement noise has unknown characteristics and traditional methods prove inapplicable.

The practical significance of the solution is in the absence of any need for preliminary geodetic survey of the terrain and in ensuring high accuracy and robustness of optical flow parameter estimation under conditions of unknown noise, which is consistent with the theoretical premises outlined in [10, 11]. The computational efficiency of the proposed algorithm meets theoretical expectations, and its recurrent structure suggests minimal computational costs. This makes it suitable for on-board implementation under limited resource conditions, thereby confirming the validity of the proposed approach.

A distinctive feature of the algorithm is the implementation of a minimax optimality criterion, which provides the highest estimation accuracy in the most adverse situation determined by a given class of noise distribution. An important element is the use of Tikhonov regularization and consideration of constraints on the continuity of the parameter vector variation over time, which allows the algorithm to adapt to changing observation conditions. Moreover, the use of these regularizing components and constraints on the continuity of parameter variation requires the selection and preliminary estimation of regularization coefficients, which may somewhat increase the computational costs.

Conclusion. The conducted numerical experiments with various nonlinear functions, different values of the mean and variance of the Laplace distribution, including the special case described in the paper with fixed noise conditions and a limited number of points practically demonstrate the consistency of the proposed concept. Prospects for further research in this area include testing the proposed solutions on real datasets [15], extending the algorithm synthesis framework for practical implementation in specific autonomous navigation systems requiring high-precision and robust estimation of optical flow parameters, including space exploration, machine vision [16], and technical diagnostics.

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