

MACHINE BUILDING AND MACHINE SCIENCE МАШИНОСТРОЕНИЕ И МАШИНОВЕДЕНИЕ



UDC 621.914.3

Original Empirical Research

<https://doi.org/10.23947/2687-1653-2026-26-2-2662>

Experimental and Curvature-Based Analysis for Accuracy Optimization in 3-Axis CNC Milling of Concave Parabolic Freeform Surfaces

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Abstract

Introduction. Concave parabolic freeform surfaces are integral to high-performance optical reflectors and precision mechanical components, where stringent geometric accuracy is paramount to functional efficiency. Despite their widespread application, achieving high surface fidelity in 3-axis CNC milling remains a significant technical challenge. This difficulty arises from the intricate, non-linear interactions between cutting tool geometry, machining parameters, and the varying local curvature of the surface. While conventional research often focuses on global parameter optimization, there exists a notable scientific gap in understanding how localized curvature gradients dictate error distribution during the material removal process. This study addresses this gap by establishing a curvature-aware analytical framework aimed at identifying the fundamental drivers of machining inaccuracies in parabolic geometries.

Materials and Methods. The research methodology integrates theoretical modeling with rigorous experimental validation. Initially, a mathematical model based on principal deviation theory was formulated to characterize the geometric deviations inherent in the 3-axis milling process. This analytical foundation allowed for the mapping of theoretical errors against surface differential geometry. Subsequently, an experimental investigation was executed using a Taguchi L₉ orthogonal array design to systematically evaluate the influence of three primary machining factors: tool diameter, feed rate, and step-over. Concave parabolic samples were machined and measured using high-precision metrology equipment. The resulting data were processed via Analysis of Variance (ANOVA) and Signal-to-Noise (S/N) ratio analysis to quantify the individual and interactive statistical contributions of each parameter to the total machining error.

Results. The findings demonstrate that tool diameter and step-over are the predominant factors, accounting for the majority of the variance in machining accuracy, whereas the influence of feed rate is found to be statistically marginal within the tested range. Crucially, the results have revealed that machining errors are not uniformly distributed but are highly correlated with the local principal curvatures of the parabolic profile. MATLAB simulations further corroborated these findings, showing that the maximum deviation occurs in regions of high curvature where the tool-surface engagement geometry is most constrained. These specific results provide a quantitative map of how tool geometry interacts with parabolic concavity to produce predictable error patterns.

Discussion. This research provides a novel interpretation of machining errors through the lens of surface differential geometry, successfully bridging the gap between theoretical surface modeling and practical CNC manufacturing. The findings demonstrate that “one-size-fits-all” toolpath strategy is inherently insufficient for concave freeform geometries due to localized geometric complexities. Instead, this study validates that the implementation of an adaptive step-over strategy, dynamically adjusted based on local curvature values, significantly enhances surface forming accuracy. The core scientific novelty lies in the proposed integration of curvature-based compensation directly into the toolpath planning phase, allowing for proactive error mitigation rather than reactive adjustments.

Conclusion. In conclusion, the proposed approach offers a robust and scalable solution for improving the precision of complex freeform components in real-world industrial environments. By adopting this methodology, high-precision manufacturing processes can potentially reduce post-processing requirements and overall production costs. While this study focuses on static and geometric accuracy, future research will extend this model to incorporate the assessment of dynamic processing errors. This expansion will provide a more comprehensive framework for accuracy optimization in multi-axis CNC machining of complex surface.

Keywords: freeform surface with concave parabolic structure, tool diameter, feed rate, step-over, surface curvature, geometric deviation, Taguchi method

Acknowledgments. The authors would like to appreciate the Technical Practice Center for providing the test samples and equipment used in the experimental process. In addition, we would like to thank the Editorial Board of the Journal and the anonymous reviewers for their insightful professional evaluations and valuable recommendations, which significantly contributed to the improvement and enhancement of the quality of this article.

For Citation. Ngoc Tuyen Bui, Nguyen Tho Van. Experimental and Curvature-Based Analysis for Accuracy Optimization in 3-Axis CNC Milling of Concave Parabolic Freeform Surfaces. *Advanced Engineering Research (Rostov-on-Don)*. 2026;26(2):2662. <https://doi.org/10.23947/2687-1653-2026-26-2-2662>

Оригинальное эмпирическое исследование

Экспериментальный анализ и подход на основе кривизны для оптимизации точности при 3-осевом фрезеровании с ЧПУ вогнутых параболических поверхностей произвольной формы

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Аннотация

Введение. Вогнутые параболические поверхности свободной формы играют ключевую роль в высокоэффективных оптических отражателях и прецизионных механических узлах, где жесткие требования к геометрической точности определяют функциональную эффективность изделий. Несмотря на широкое распространение таких поверхностей, обеспечение высокого качества их обработки на 3-осевых фрезерных станках с ЧПУ остается существенной технической проблемой. Данная сложность обусловлена комплексными нелинейными взаимодействиями между геометрией режущего инструмента, параметрами обработки и изменяющейся локальной кривизной поверхности. В то время как традиционные исследования обычно сосредоточены на глобальной оптимизации параметров, существует заметный научный пробел в понимании того, как локальные градиенты кривизны определяют распределение погрешностей в процессе съема материала. Настоящее исследование устраняет этот пробел, предлагая аналитическую основу, учитывающую кривизну и направленную на выявление фундаментальных факторов погрешностей обработки параболических геометрий.

Материалы и методы. Методология исследования сочетает теоретическое моделирование со строгой экспериментальной проверкой. На первом этапе на основе теории главных отклонений была разработана математическая модель, характеризующая геометрические отклонения, свойственные процессу 3-осевой фрезеровки. Данная аналитическая база обеспечила возможность отображения теоретических погрешностей относительно дифференциальной геометрии поверхности. Затем было проведено экспериментальное исследование с применением ортогонального плана Taguchi L₉ для систематической оценки влияния трех основных технологических факторов: диаметра инструмента, подачи и поперечного смещения. Были изготовлены образцы вогнутых параболических поверхностей, которые затем подверглись измерениям на высокоточном метрологическом оборудовании. Обработка полученных данных выполнялась методами дисперсионного анализа (ANOVA) и анализа отношения сигнал/шум (S/N ratio), что позволило количественно оценить индивидуальный и интерактивный статистический вклад каждого из рассматриваемых параметров в суммарную погрешность обработки.

Результаты исследований. Полученные результаты показали, что диаметр инструмента и поперечный шаг являются доминирующими факторами, на долю которых приходится основная часть дисперсии точности обработки, в то время как влияние скорости подачи в пределах исследуемого диапазона оказалось статистически незначительным. Важно отметить, что погрешности обработки распределены не равномерно, а существенно коррелируют с локальной главной кривизной параболического профиля. Моделирование в MATLAB дополнительно подтвердило эти выводы, показав, что максимальное отклонение происходит в областях с высокой кривизной, где геометрия взаимодействия инструмента с поверхностью наиболее ограничена. Данные результаты обеспечивают количественное отображение того, как взаимодействие геометрии инструмента с параболической вогнутой поверхностью приводит к образованию предсказуемых закономерностей распределения погрешностей.

Обсуждение. Настоящее исследование предлагает новую интерпретацию погрешностей механической обработки сквозь призму дифференциальной геометрии поверхности, успешно преодолевая разрыв между теоретическим моделированием поверхности и практическим фрезерованием на станках с ЧПУ. Результаты показывают, что универсальная стратегия построения траектории инструмента («один размер для всех») является принципиально недостаточной для вогнутых свободных поверхностей из-за присущих им локальных геометрических сложностей. Данная работа подтверждает, что применение адаптивной стратегии изменения поперечного шага, динамически подстраиваемого в зависимости от локальных значений кривизны, существенно повышает точность формообразования поверхности. Ключевая научная новизна состоит в предложенной интеграции компенсации, основанной на кривизне, непосредственно на этапе планирования траектории движения инструмента, что позволяет упреждающе минимизировать погрешности вместо того, чтобы исправлять их постфактум.

Заключение. В заключение следует отметить, что предлагаемый подход представляет собой надежное и масштабируемое решение для повышения точности изготовления сложных компонентов произвольной формы в реальных промышленных условиях. Внедрение данной методологии в высокоточные производственные процессы позволит потенциально сократить объем последующей обработки и снизить общие производственные издержки. Несмотря на то, что настоящее исследование сосредоточено на статической и геометрической точности, в будущем предполагается расширить данную модель за счет включения оценки динамических погрешностей обработки. Это расширение создаст более комплексную основу для оптимизации точности при многокоординатной фрезерной обработке сложных поверхностей на станках с ЧПУ.

Ключевые слова: поверхность произвольной формы с вогнутой параболической структурой, диаметр инструмента, скорость подачи, поперечный шаг, кривизна поверхности, геометрическое отклонение, метод Тагучи

Благодарности. Авторы выражают искреннюю признательность Техническому практическому центру за предоставление образцов для испытаний и оборудования, использованного в экспериментальной работе. Авторы также благодарят редакционную коллегию журнала и анонимных рецензентов за их глубокие профессиональные заключения и ценные замечания, которые внесли существенный вклад в доработку и повышение качества настоящей статьи.

Для цитирования. Нгок Туен Буй, Нгуен Тхо Ван. Экспериментальный анализ и подход на основе кривизны для оптимизации точности при 3-осевом фрезеровании с ЧПУ вогнутых параболических поверхностей произвольной формы. *Advanced Engineering Research (Rostov-on-Don)*. 2026;26(2):2662. <https://doi.org/10.23947/2687-1653-2026-26-2-2662>

Introduction. Freeform surfaces are increasingly employed in advanced engineering applications such as precision molds, optical components, and aerodynamic structures due to their superior functional and aesthetic performance. Among various local geometrical features, concave parabolic structures are widely utilized in components such as parabolic reflectors and antenna devices, where geometric accuracy directly affects functional efficiency. In particular, the machining of such surfaces using 3-axis CNC milling remains a challenging task due to the complex interaction between tool geometry, surface curvature, and toolpath strategy.

In the context of ball-end milling, surface generation errors are strongly influenced by geometric factors such as tool radius, feed rate, and step-over, as well as by the local curvature of the machined surface. Previous studies have extensively investigated the influence of machining parameters on freeform surface quality. Recently, significant contributions from the Russian metalworking school have addressed both the experimental characterisation and the predictive modelling of geometric accuracy in the milling of complex-shape parts. In [1], the authors developed an experimental methodology and a mathematical model for controlling the geometric accuracy indicators of die impressions in ball-end milling, with the resulting model employed to adjust the toolpath. In related study [2], empirical models were established linking spherical end-mill life to geometric accuracy and surface roughness, using these models to define both tool-replacement schedules and criteria for surface analysis. Furthermore, paper [3] explored the influence of tool orientation angles on surface roughness when processing complex-profile surfaces, highlighting the critical role of tool-surface contact geometry in determining final quality.

Specifically, the effects of toolpath strategies were evaluated in [4], while the impact of feed rate was analyzed in [5]. The role of step-over in determining geometric deviation was established in [6]. Other studies applied experimental design approaches, such as the Taguchi method combined with analysis of variance (ANOVA), to determine optimal machining conditions for minimizing surface errors [7]. These approaches have demonstrated that toolpath strategy is often a primary

factor affecting machining accuracy [8]. In addition, the significant contribution of step-over to surface forming quality has been confirmed through statistical analysis [9]. Furthermore, some researchers focus on comparing toolpath strategies in various CAM systems [10] or proposing solutions to determine scallop heights from CNC part programs [11]. Strategies to reduce the variation of the effective tool diameter have also been proposed to ensure uniform machining accuracy [12].

However, most existing studies focus on general freeform surfaces or specific geometries such as impellers [13]. Other research has targeted specific profiles like saddle surfaces [14] or the optimization of geometric characteristics for specialized cycloidal profiles [15]. Meanwhile, the machining characteristics of concave parabolic structures have received limited attention. More importantly, the relationship between machining parameters and surface accuracy is often analyzed from a purely experimental perspective, without explicitly considering the role of surface curvature in governing geometric deviations. As a result, the underlying mechanisms that explain why certain parameters dominate the machining accuracy remain insufficiently clarified.

To address these limitations, this study focuses on the machining of freeform surfaces with concave parabolic structures, characterized by a first principal curvature of zero and a second principal curvature with a variable negative value. Based on the theory of principal deviations and surface geometry [16], an experimental model is developed to investigate the influence of key machining parameters, including tool diameter, feed rate, and step-over, on surface forming accuracy in 3-axis CNC milling. A systematic experimental design based on the Taguchi orthogonal array is employed, and the results are analyzed using ANOVA to quantify the contribution of each parameter [17].

The objective of this study is to evaluate the influence of surface curvature, tool radius, step-over, and feedrate on the geometric error of a concave parabolic surface structure during 3-axis CNC milling. Theoretical error models and MATLAB simulations aim to assess the distribution of feedrate error, step-over error, and total error on the surface for parallel toolpaths in two principal curvature directions. From this, the appropriate toolpath direction is determined, and an adaptive toolpath solution based on the developed theoretical model is proposed. Experimental models are used to evaluate the extent to which tool radius, step-over, and feedrate affect the shaping accuracy. In addition to conventional experimental analysis, this work emphasizes the geometric interpretation of machining errors by linking them to curvature-related factors, thereby providing deeper insight into the interaction between tool geometry and surface characteristics. The findings of this study contribute to a better understanding of parameter sensitivity in machining concave parabolic surfaces and provide a basis for improving toolpath planning and machining efficiency in practical applications.

Materials and Methods. The investigated surface is generated by sweeping a parabolic curve along a straight line, resulting in a parabolic cylindrical surface. The parametric representation of the surface is defined as:

$$\mathbf{P}(u, v) = (x(u), y(v), z(u)) = (60u - 30, 70v, 240u^2 - 240u + 60). \quad (1)$$

$$0 \leq u, v \leq 1.$$

This construction implies that the surface exhibits curvature only along the parametric direction u , while remaining geometrically linear along the direction v .

A sample experimental model with the concave parabolic freeform surface $\mathbf{P}(u, v)$ was constructed using NX12 as shown in Figure 1.

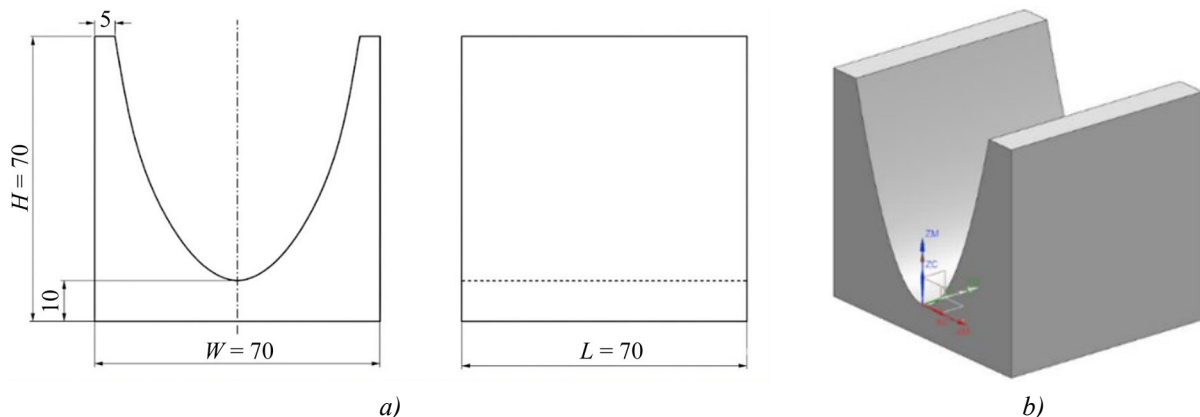


Fig. 1. Drawing of the experimental sample *a* — 2D drawing; *b* — 3D model

The first-order partial derivatives are expressed as:

$$\mathbf{P}_u = (60, |0, |480u - 240), \mathbf{P}_v = (0, |70, |0). \quad (2)$$

The second-order derivatives are:

$$\mathbf{P}_{uu} = (0, |0, |480), \mathbf{P}_{vv} = (0, |0, |0).$$

Along the v -direction (generator direction), the first principal curvature is:

$$k_1 = 0.$$

Along the u -direction (profile curve direction), the second principal curvature is:

$$k_2 = \frac{z''(u)}{\left[1 + (z'(u))^2\right]^{3/2}} = \frac{480}{\left[1 + (480u - 240)^2\right]^{3/2}}. \quad (3)$$

The sign of the curvature depends on the surface orientation; for concave configurations, it is considered negative. Importantly, k_2 varies continuously along the parameter u , reflecting the non-uniform curvature distribution of the parabolic profile.

The Gaussian curvature is given by:

$$K = k_1 \cdot k_2 = 0. \quad (3)$$

This confirms that the surface is a developable surface, characterized by zero Gaussian curvature.

The mean curvature is:

$$H = \frac{1}{2}(k_1 + k_2) = \frac{k_2}{2}, \quad (4)$$

which varies along the surface due to the dependence of k_2 on u .

Surface curvature characteristics significantly influence the geometric accuracy in CNC milling, leading to critical implications for overall machining precision.

Since $k_1 = 0$, there is no curvature-induced deviation along the toolpath direction when it is aligned with the generator direction.

The varying curvature $k_2(u)$ leads to non-uniform tool-surface contact conditions, which significantly influence geometric deviations.

Regions with higher curvature (smaller radius) are more sensitive to tool diameter and step-over, resulting in larger scallop height and form error.

Therefore, the geometric deviation in ball-end milling can be interpreted as a curvature-dependent phenomenon, where the second principal curvature plays a dominant role.

The optimization of machining accuracy is achieved through a comprehensive analysis of toolpath strategies and curvature-based modeling. Two representative toolpath strategies are investigated for machining the concave parabolic cylindrical surface. In Case A, the feed direction is aligned with the zero-curvature direction ($k_1 = 0$), while the step-over is applied along the parabolic profile. In Case B, the feed direction follows the parabolic profile, and the step-over is aligned with the zero-curvature direction.

In ball-end milling of freeform surfaces, the geometric deviation is strongly influenced by the local curvature of the machined surface. For the considered parabolic cylindrical surface, the curvature is anisotropic, with one principal curvature equal to zero and the other varying along the surface.

Let the second principal curvature be:

$$k(u) = k_2(u), R(u) = \frac{1}{|k(u)|}, \quad (5)$$

where $R(u)$ is the local radius of curvature.

The total geometric deviation in ball-end milling can be approximated as the sum of two components:

$$h_{\Sigma}(u) = h_f + h_s,$$

where h_f, h_s are feed-direction deviation and step-over (scallop) deviation in turn.

The geometric deviations can be approximated using a second-order Taylor expansion of the surface profile [18]. This approach leads to the classical relation $h \approx s^2/(8R)$, which is widely documented in manufacturing theory [19].

The effective curvature radius $R_{eff}(u)$ is defined by combining the curvature of the machined surface and the ball-end tool. Based on curvature superposition, the effective radius can be expressed as:

$$\frac{1}{R_{eff}(u)} = \frac{1}{R(u)} + \frac{1}{R_T}, \quad (6)$$

which leads to:

$$R_{eff}(u) = \frac{R(u)R_T}{R(u) + R_T}. \quad (7)$$

where $R(u)$ and R_T are curvature radius of the machined surface and the ball-end tool, in turn.

Due to the anisotropic curvature characteristics of the surface, the geometric deviation exhibits distinct behaviors in the feed rate and step-over directions.

For Case A, the feed-direction error h_f^A remains constant along the toolpath because the curvature in this direction is zero, leading to an effective radius equal to the tool radius. Feed-direction deviation

$$h_f^A \approx \frac{f_z^2}{8R_{eff,f}^A}, \quad (8)$$

where the effective radius is:

$$R_{eff,f}^A = R_T.$$

The dominant geometric error in finishing is the step-over deviation (scallop height) $h_s^A(u)$ caused by the step-over S .

$$h_s^A(u) = \frac{S^2}{8R_{eff,s}^A(u)}, \quad (9)$$

where the effective radius is:

$$R_{eff,s}^A(u) = \frac{R(u)R_T}{R(u) + R_T}.$$

Total curvature dependent deviation h_A in Case A is:

$$h_A = \frac{f_z^2}{8R_T} + \frac{S^2}{8R_{eff,s}^A(u)}. \quad (10)$$

For Case B, the situation is reversed. The step-over error remains constant, while the feed-direction error varies significantly with the local curvature. As the curvature reaches its maximum near the vertex of the parabola, the effective radius decreases, resulting in a sharp increase in feed-direction error. This leads to a non-uniform error distribution, with pronounced peaks in regions of high curvature.

The feed-direction deviation $h_f^B(u)$ depends on the feed per tooth f_z , tool radius R_T , and curvature radius $R(u)$:

$$h_f^B(u) \approx \frac{f_z^2}{8R_{eff,f}^B(u)}, \quad (11)$$

where the effective radius is:

$$R_{eff,f}^B(u) = \frac{R(u)R_T}{R(u) + R_T}.$$

The step-over deviation h_s^B , caused by the step-over S in this case can be approximated as:

$$h_s^B \approx \frac{S^2}{8R_{eff,s}^B}, \quad (12)$$

where the effective radius is:

$$R_{eff,s}^B = R_T.$$

Total curvature-dependent deviation in case B is

$$h_B = \frac{f_z^2}{8R_{eff,f}^B(u)} + \frac{S^2}{8R_T}. \quad (13)$$

Regarding the curvature-based optimization model for machining accuracy, the total error $h_{\Sigma}(u)$ must satisfy the following constraint to meet the precision requirements:

$$h_{\Sigma}(u) \leq [h], \quad (14)$$

where $[h]$ is maximum allowable geometric deviation.

Substituting deviation h_A into this inequality, we get:

$$\frac{f_z^2}{8R_T} + \frac{S(u)^2}{8R_{eff,s}^A(u)} \leq [h]. \quad (15)$$

Solving for $S(u)$, the curvature-based adaptive step-over is obtained as:

$$S(u) = \sqrt{8R_{eff,s}^A(u) \left([h] \frac{f_z^2}{8R_T} \right)}. \quad (16)$$

For the experimental study, the experimental setup was established to validate the proposed model.

Machine tool: CNC milling machine DX-6080 from WANTAI (Taiwan) with some basic technical characteristics as follows: machine table size of 600 x 800 (mm); main spindle motor power of 5.5 kw, spindle speed of 600–18000 rev/min; max feed rate of 6000 mm/min; repeatable positioning accuracy of 0.005 mm.

Cutting tool: from the formula for determining curvature k_2 above, the smallest radius of curvature of the surface is determined as $R_{pmin} = 7.5$ mm. Therefore, to avoid undercutting, the ball end mills used in this study are chosen as $\Phi 6$, $\Phi 10$, $\Phi 14$ from G.T. cutting tools (Taiwan) with number of teeth $Z = 2$.

Workpiece: $L \times W \times H = 70 \times 70 \times 70$ (mm), aluminum alloy A6061

CAM software: NX12

Equipment for measurement: 3D Scanner SIMSCAN from SCANTECH (HANGZHOU) CO., LTD, with accuracy up to 0.020 mm, scanning rate up to 2020000 measurements/sec, resolution up to 0.025 mm, scanning area up to 410 mm $\times$ 400 mm.

Experiment plan: the workpieces are rough machined and semi-finished to form a cylindrical surface with a finish allowance of 0.1 mm.

The experimental finishing process was performed with zigzag parallel cutting strategy (Case A) according to the experimental plan designed based on the Taguchi $OA_9(3^3)$ orthogonal array with 3 factors: Step-over (S), Feed rate (F) and Tool diameter (D). The factors and experimental levels are presented in Table 1.

Table 1.

Factors and Levels

Factors	Levels		
	1	2	3
Step-over S (mm)	0.1	0.4	0.7
Feed rate F (mm/min)/ f_z (mm/tooth)	100/(0.025)	500/(0.125)	900/(0.225)
Tool diameter D (mm)	6	10	14

The remaining technological conditions are unchanged for all 9 samples: velocity of spindle $n = 2000$ rpm, depth of cut of 0.1 mm, flushing with cutting fluid. The output indicators of this experimental planning are the machining accuracy assessed by the geometric error of the surface of the machined sample compared to the surface of the designed sample.

The finished machined samples (Figure 4) were digitized using a SIMSCAN handheld 3D scanner. These 9 point cloud data files were used for sequential registration with the design CAD model surface to determine the geometric errors of the samples using Geomagic Quality 2003. Geomagic Quality performed iterative rigid-body registration between the measured point cloud and the nominal CAD surface by minimizing the least-squares distance between corresponding geometric entities, following the ICP framework. The criterion for evaluating geometric errors was the average deviation determined by the formula: average deviation = (max dev – min dev)/2.

Results and Discussion. Based on the above analysis, a machining error simulation program was written in MATLAB with input parameters tool radius $R_T = 7$ mm, feed rate $f_z = 0.2$ mm/tooth, step over $S = 0.1$ mm. The program results for machining errors for the two cases A and B are shown in Figure 2. Comparison of the total machining error shows that both strategies produce errors that vary along the surface. However, Case A consistently yields lower error values than Case B across the entire parameter domain. More importantly, the error distribution in Case A is smoother and more stable, whereas case B exhibits strong sensitivity to curvature variations. This difference is critical from a manufacturing perspective, as large variations in feed-direction error are directly associated with surface waviness and form deviation.

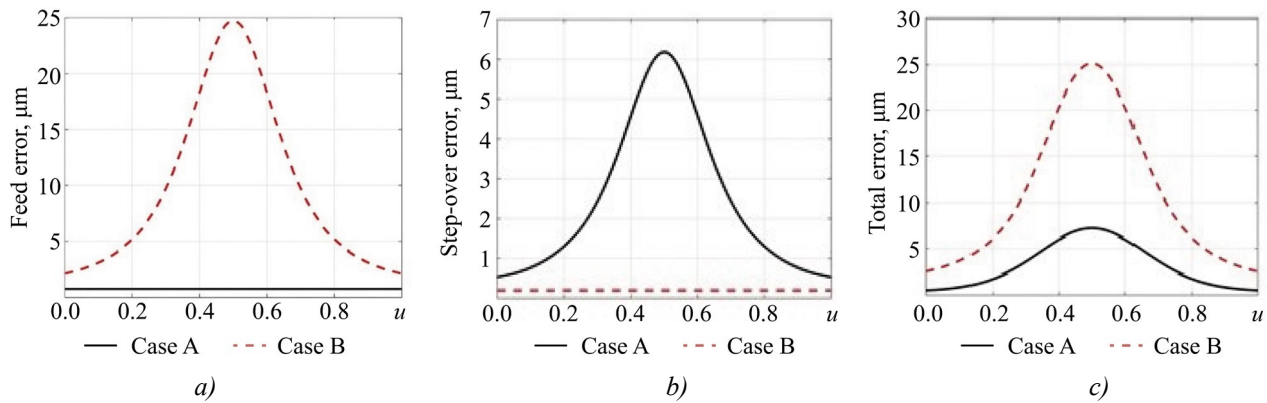


Fig. 2. Errors simulation for two Cases
a — Feed error; b — Step-over error; c —Total error

The results indicate that the feed-direction error plays a dominant role in determining the overall form accuracy. Although the total error magnitude may appear comparable under certain parameter combinations, the redistribution of error components significantly affects the final surface quality. By aligning the feed direction with the minimum-curvature direction, Case A effectively decouples the most critical error component from curvature variations, thereby improving machining stability and accuracy. Therefore, the toolpath strategy in Case A is better than the toolpath strategy in Case B.

From a practical standpoint, this finding supports the strategy commonly adopted in advanced CAM systems such as Siemens NX and PowerMill, where toolpaths are preferentially oriented along directions of minimal curvature. Therefore, for the machining of parabolic cylindrical surfaces, the optimal strategy is to align the feed direction with the zero-curvature direction and to control the step-over according to the local curvature.

Figure 3 shows the result of running the program with input data $R_T = 7$ mm, $f_z = 0.02$ mm/tooth, allowable error $[h] = 0.02$ mm. With the step-over that adapts according to the surface curvature as in the diagram, the geometric surface error will remain unchanged at 0.02 mm. Currently, commercial CAM systems like Siemens NX and PowerMill also have adaptive step-over in an indirect and heuristic way, but not explicitly formulated as here.

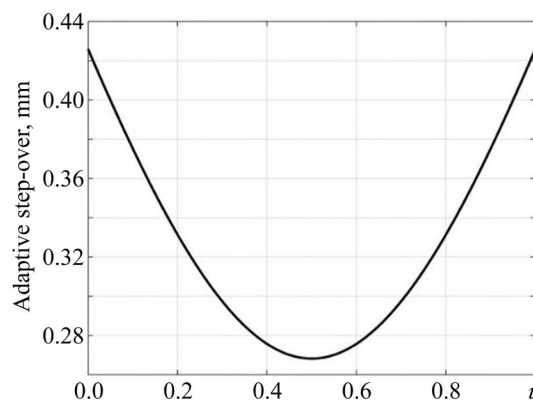


Fig. 3. Adaptive step-over simulation for Case A

Regarding the experimental results and discussion, the machined samples are presented in Figure 4, the measured geometric errors for Sample 2 are illustrated in Figure 5, while the comprehensive error data are summarized in Table 2.



Fig. 4. Nine experimental samples

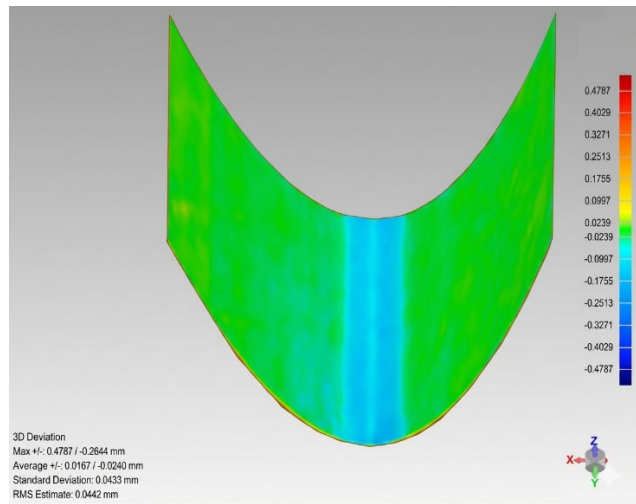


Fig. 5. Geometrical error of sample 2

In the analysis and assessment of machining accuracy, the performance is evaluated based on specific machining error criteria. The smaller the machining error, the higher the machining accuracy. According to the choice of output as small as possible (smaller is better), the S/N ratio for each experiment is determined (Table 2).

Table 2

Experimental Results Obtained from the Orthogonal Array Taguchi $OA_9(3^3)$

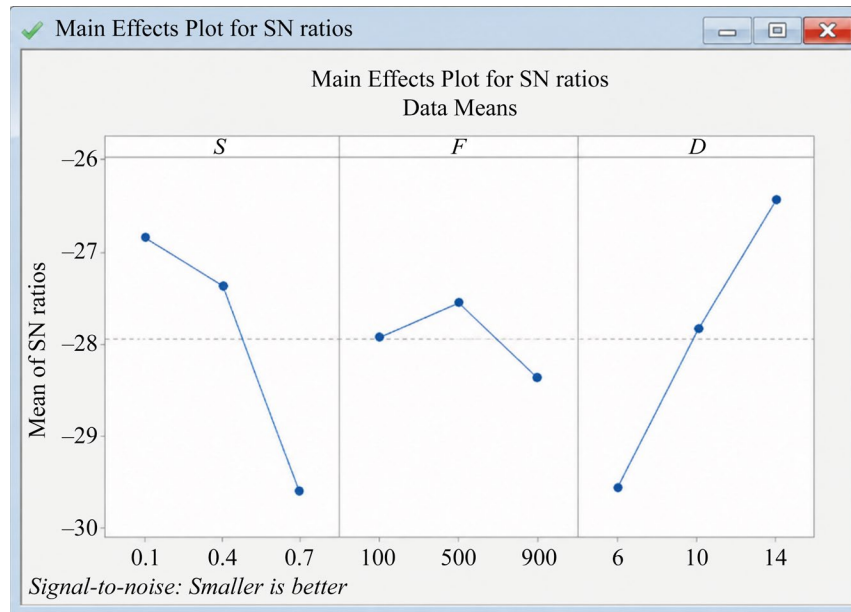
N^0	S (mm)	F (mm/min)	D	Geometric Deviation (μm)			$(S/N)_\Delta$
				Upper	Lower	Average Δ	
1	0.1	100	6	18.9	35.3	27.1	-14.3297
2	0.1	500	10	16.7	24	20.35	-13.0856
3	0.1	900	14	20.4	18	19.2	-12.833
4	0.4	100	10	16.8	28.7	22.75	-13.5698
5	0.4	500	14	16.6	22.1	19.35	-12.8668
6	0.4	900	6	18.2	39.2	28.7	-14.5788
7	0.7	100	14	15.5	34	24.75	-13.9358
8	0.7	500	6	22.6	45.8	34.2	-15.3403
9	0.7	900	10	25.7	38.2	31.95	-15.0447

The average S/N ratio for each factor at levels 1, 2 and 3 can be calculated by averaging the S/N ratios of the corresponding experiments. The influence of each factor on the machined surface geometric error is determined using Minitab as shown in Figure 6. A large S/N ratio indicates that the machined surface has small geometric errors. Analysis of variance (Table 3) shows that the coefficients of determination are quite high (98.92% and 95.68%), confirming that the model fits the data well.

Table 3

Analysis of Variance (ANOVA) for $(S/N)_\Delta$
 Model Summary R-Sq = 98.92%, R-Sq(adj) = 95.68%

Source	DF	Seq SS	Adj SS	Adj MS	F	P
S	2	12.4878	12.4878	6.2439	41.36	0.024
F	2	0.9044	0.9044	0.4522	2.99	0.250
D	2	14.2398	14.2398	7.1199	47.16	0.021
Residual Error	2	0.3020	0.3020	0.1510		
Total	8	27.9339				


Fig. 6. Main effects plot for (S/N) Δ ratios

Through the data in this table, the influence of each factor on machining accuracy can also be determined. Diameter of Tool (D) has the largest influence (51%), followed by the influence of Step-over (44.7%). The influence of Feed rate is the smallest (3%) (Table 4).

Table 4

Influence of Factors on the Machining Accuracy

N ⁰	Factor	Contribution %
1	Step-over (S)	44.7
2	Feed rate (F)	3
3	Diameter of Tool (D)	51
4	Errors	1.3

From here, it can be seen that the conformity of a generating tool and the machined surface has a significant influence on shaping accuracy. With the S/N value as large as possible, we can choose the optimal parameter set $S_1F_2D_3$. The optimal parameters to have the best geometrical accuracy of the machined surface are step-over of 0.1 mm, feed rate of 500 mm/min, and the tool diameter of 14 mm.

The predicted average deviation in this case would be:

$$\Delta_{min} = \bar{\Delta}(S_1) + \bar{\Delta}(F_2) + \bar{\Delta}(D_3) - 2 \frac{\sum \Delta_i}{9} = 22.22 + 24.63 + 21.1 - 2 \cdot 228.35 / 9 \approx 17.2 \text{ (}\mu\text{m)}.$$
 (17)

Conclusion. This study investigated the influence of machining parameters on the forming accuracy of concave parabolic freeform surfaces in 3-axis CNC milling. In contrast to conventional experimental approaches, a curvature-based analytical framework was introduced to interpret machining errors from a geometric perspective.

The results confirm that the considered surface exhibits anisotropic curvature characteristics, where one principal curvature is zero and the other varies continuously along the surface. This geometric property leads to non-uniform tool–surface interaction conditions, which significantly affect machining accuracy. Experimental analysis based on the Taguchi method and ANOVA indicates that step-over and tool diameter are the dominant factors influencing geometric deviation, while feed rate has a comparatively smaller effect.

To provide a theoretical explanation for these observations, a curvature-based optimization model was developed. The model relates machining errors to local curvature through an effective radius formulation, demonstrating that both feed-direction deviation and scallop height are inversely proportional to the curvature radius. Based on this relationship, a curvature-adaptive step-over strategy was proposed to maintain a constant allowable error across the surface.

The comparison between constant and curvature-adaptive step-over strategies shows that the adaptive approach significantly reduces error variation and improves overall surface accuracy. This finding highlights the importance of incorporating geometric characteristics into machining parameter selection, rather than relying solely on empirical optimization methods.

Although the direct implementation of continuous curvature-adaptive step-over is not supported by standard CNC controllers, the proposed model provides a theoretical foundation for constant scallop height toolpath generation available in modern CAM systems. Therefore, the results of this study have practical relevance for improving machining efficiency and accuracy in industrial applications.

Future work will focus on extending the proposed approach to more complex freeform surfaces with non-zero Gaussian curvature, as well as integrating curvature-based optimization into automated toolpath generation and intelligent machining systems.

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Conflict of Interest Statement: the authors declare no conflict of interest.

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Все авторы прочитали и одобрили окончательный вариант рукописи.

Received / Поступила в редакцию 16.03.2026

Reviewed / Поступила после рецензирования 29.04.2026

Accepted / Принята к публикации 20.05.2026