

# MECHANICS МЕХАНИКА



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## Neural Network Technology for Monitoring the Damaged State of an Extended Structure Based on Analysis of Non-stationary Waves

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### Abstract

**Introduction.** Destruction of the surface layer of concrete structures caused by an aggressive environment is one of the most common types of damage that require remote and automatic monitoring. Acoustic non-destructive testing methods can address this problem. However, the existing approaches, including the contour line intersection method we previously developed, demonstrate instability in the low-porosity region (less than 15%) because of the weak dependence of signal amplitude on time. So, we identified a problem that existing methods cannot reliably identify damage parameters across the whole range of values. The objective of this work is to develop a neural network technology for the robust identification of the depth and porosity of the damaged layer of an extended structure based on the analysis of non-stationary waves, which determines the uniqueness and scientific novelty of the study.

**Materials and Methods.** The study is based on numerical simulation. The mechanical model is a two-layer elastic anisotropic strip of thickness  $H = 1$  cm with an upper damaged layer modeled as a porous composite whose effective properties calculated in the ACELAN-COMPOS package. The damage parameters are layer thickness  $h$  (0.05–0.3 cm) and porosity  $p$  (10–80%). A piezoelectric actuator and sensor made of PZT-4 ceramics with vertical polarization are placed on the surface of the strip. Non-stationary waves are excited by a potential difference on the actuator and recorded by the sensor. To obtain training data, a series of direct problems for 48 parameter combinations were solved using the finite element method in the ACELAN package with quadratic triangular elements (mesh: 5,616 elements; 12,155 nodes). Based on cubic interpolation, training (10,000 points) and test (40,000 points) datasets were generated. A feedforward neural network, a multilayer perceptron with four hidden layers (128-64-32-16 neurons), ReLU activation function, dropout layers (20%), and Adam optimizer was constructed and a weighted loss function was used to reduce positive asymmetry in predictions.

**Results.** On the independent test set, the trained network achieves a mean absolute error of 2.433% for porosity and 0.121 mm for depth ( $R^2 = 0.977$  and 0.910, respectively). In the low-porosity region ( $p < 15\%$ ), the error is 0.405% for porosity and 0.173 mm for depth with a bias of only +0.188%, confirming the reliability of the approach in the region. All 40,000 test points yield unique two-dimensional signatures, empirically confirming the uniqueness of the inverse problem solution.

**Discussion.** The obtained data confirm that the proposed neural network technology overcomes the fundamental limitation of the contour line intersection method, providing stable predictions across the entire porosity range. The use of a weighted loss function effectively reduces systematic overestimation of porosity predictions. A limitation of the approach is the assumption of constant porosity along the depth and length of the layer. Accounting for inhomogeneities would require solving an inverse problem of significantly higher complexity. In future work, we plan to move beyond just two extracted features and instead use the full time-domain signal, possibly with convolutional architectures.

**Conclusion.** The neural network technology developed in this research provides an effective method of identification of damage parameters across the entire range of values, overcoming the limitations of existing methods. The obtained results confirm the applicability of the proposed approach for acoustic monitoring systems of extended concrete structures.

**Keywords:** inverse problem, acoustic monitoring, piezoelectric sensor, neural networks

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Оригинальное теоретическое исследование

## Нейросетевая технология мониторинга поврежденного состояния протяженной конструкции на основе анализа нестационарных волн

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### Аннотация

**Введение.** Разрушение поверхностного слоя бетонных конструкций под воздействием агрессивной среды относится к числу наиболее распространенных видов повреждений, требующих дистанционного автоматизированного мониторинга. Для решения данной задачи могут применяться акустические методы неразрушающего контроля. Вместе с тем существующие подходы, в том числе ранее разработанный нами метод пересечения контурных линий, характеризуются недостаточной устойчивостью в области низкой пористости — менее 15 % — вследствие слабой зависимости амплитуды сигнала от времени. В связи с этим была выявлена проблема, заключающаяся в том, что применяемые методы не обеспечивают надежного определения параметров повреждения во всем диапазоне их значений. Цель настоящей работы — разработка нейросетевой технологии устойчивой идентификации глубины и пористости поврежденного слоя протяженной конструкции на основе анализа нестационарных волн; это определяет уникальность и научную новизну исследования.

**Материалы и методы.** Исследование выполнено на основе численного моделирования. В качестве механической модели рассматривается двухслойная упругая анизотропная полоса толщиной  $H = 1$  см с верхним поврежденным слоем, моделируемым пористым композитом, эффективные свойства которого рассчитаны в пакете ACELAN-COMPOS. Параметрами повреждения являются толщина слоя  $h$  (0,05–0,3 см) и пористость  $p$  (10–80 %). На поверхности полосы расположены пьезоэлектрические актуатор и сенсор из керамики PZT-4 с вертикальной поляризацией. Нестационарные волны возбуждаются разностью потенциалов на актуаторе и регистрируются сенсором. Для получения обучающих данных методом конечных элементов в пакете ACELAN с использованием квадратичных треугольных элементов (сетка: 5616 элементов, 12 155 узлов) решена серия прямых задач для 48 сочетаний параметров. На основе кубической интерполяции сгенерированы обучающий (10 000 точек) и тестовый (40 000 точек) наборы данных. Построена нейронная сеть прямого распространения — многослойный персептрон с четырьмя скрытыми слоями (128–64–32–16 нейронов), функцией активации ReLU, слоями дропаута (20 %) и оптимизатором Adam; для уменьшения положительной асимметрии прогнозов использована взвешенная функция потерь.

**Результаты исследования.** На независимом тестовом наборе обученная сеть достигает средней абсолютной ошибки 2,433 % для пористости и 0,121 мм для глубины ( $R^2 = 0,977$  и 0,910 соответственно). В области низкой пористости ( $p < 15$  %) ошибка составляет 0,405 % для пористости и 0,173 мм для глубины при смещении всего +0,188 %, что подтверждает надежность подхода в данной области. Все 40 000 тестовых точек дают уникальные двумерные сигнатуры, что эмпирически подтверждает единственность решения обратной задачи.

**Обсуждение.** Полученные данные подтверждают, что предложенная нейросетевая технология позволяет преодолеть фундаментальное ограничение метода пересечения контурных линий, обеспечивая устойчивое прогнозирование во всем диапазоне значений пористости. Использование взвешенной функции потерь эффективно снижает систематическое завышение прогнозируемых значений данного параметра. Ограничением предложенного подхода остается допущение о постоянстве пористости по глубине и длине поврежденного слоя. Учет пространственных неоднородностей потребует решения обратной задачи существенно более высокой сложности. В дальнейших исследованиях планируется выйти за рамки анализа только двух выделенных признаков и перейти к использованию полного временного сигнала, в том числе с применением сверточных архитектур.

**Заключение.** Разработанная в данном исследовании нейросетевая технология обеспечивает эффективный метод идентификации параметров повреждения во всем диапазоне значений, преодолевая ограничения существующих методов. Полученные результаты подтверждают применимость предложенного подхода для систем акустического мониторинга протяженных бетонных конструкций.

**Ключевые слова:** обратная задача, акустический мониторинг, пьезоэлектрический датчик, нейронные сети

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**Introduction.** The integration of piezoelectric actuators and sensors for acoustically monitoring building structures, Structural Health Monitoring (SHM), has proven effective for detecting and characterizing damage in concrete structures. Destruction of the surface layer caused by an aggressive environment is one of the most common types of damage requiring remote and automatic monitoring. Acoustic non-destructive testing methods can address this problem. However, existing approaches have noticeable limitations, especially in the low-porosity region, where methods become unstable. This review summarizes recent literature on the application of neural networks for damage identification based on acoustic waves, with emphasis on methods directly applicable to the extension of this study.

Review [1] covers data-intelligence-driven approaches for concrete durability and performance prediction. It clearly highlighted the trend that the field is moving from physics-based models to hybrid data-driven ones. The authors [2] looked at AI in bridge SHM and found that machine learning is playing a significant role in interpreting sensor data from different NDT methods. Paper [3] focused on acoustics as an SHM tool a direction that closely matches the approach we take in this work. Work [4] provides a broad overview of deep learning for SHM, covering CNNs, autoencoders, and RNNs for both vibration and wave-based methods. And finally, [5] reviewed AI-based SHM techniques, observing the shift from traditional machine learning to deep learning in damage detection and classification tasks.

An important development in computational mechanics is the integration of physical laws with neural network architectures. Work [6], for example, introduces physics-informed neural networks or PINNs. The approach works by incorporating the governing partial differential equations into the loss function. This allows the network to handle both forward and inverse problems involving nonlinear PDEs. The authors [7] took this a step further and applied PINNs to ultrasonic defect detection in concrete slabs. They found that embedding wave propagation equations into the network architecture improves both reliability and interpretability. Their method worked well even with limited training data which is especially valuable when experimental data are hard to obtain. This aligns with our inverse problem since the governing equations for wave propagation in a two-layer anisotropic strip could serve as a physical foundation for a PINN-based approach.

Acoustic emission (AE) and ultrasonic testing generate rich temporal signals containing critical information about the location, severity, and type of damage. Paper [8] applied deep residual learning for acoustic emission source localization in a steel-concrete composite slab, showing that convolutional neural networks can accurately determine damage source locations from AE signals. The authors [9] developed an AI-assisted ultrasonic wave analysis system for automated classification of steel corrosion-induced concrete damage, demonstrating that neural networks can distinguish damage mechanisms based on wave characteristics. In [10], structural damage detection and classification using machine learning algorithms, providing early evidence of the effectiveness of supervised approaches for wave-based SHM, are investigated. The authors [11] conducted an experimental study on monitoring damage progression in basalt-FRP reinforced concrete slabs using acoustic emission and machine learning, combining unsupervised clustering with supervised k-nearest neighbor classification, achieving 99.2% accuracy in classifying damage mechanisms. They also used SHAP analysis to quantify the contribution of each acoustic emission feature to crack width prediction — a method directly applicable for identifying porosity and depth from sensor signals. The authors [12] developed a deep learning-based framework for remaining useful life prediction of concrete structures using acoustic emission data, combining a stacked autoencoder deep neural network for state indicator construction and LSTM-RNN for prediction, with a false signal removal process using one-class support vector machines for filtering relevant AE events.

Piezoelectric sensors are central to the experimental setup used in this study. Paper [13] used deep learning in combination with finite element model updating techniques to develop an approach to structural damage detection, demonstrating how neural networks can learn the mapping between sensor measurements and damage parameters. In [14], the authors conducted a comparative study of unsupervised machine and deep learning methods for structural damage detection, and their work offers insights into approaches which do not require labeled damage data. The authors [15] presented a digital twin platform integrating multiscale multiphysics modeling with machine learning for damage detection in piezoelectric composite structures using electrical signal measurements, applying an artificial neural network for constitutive coefficient inference and a convolutional LSTM network for damage prediction at specific locations from electrical signals at limited sensor locations.

In inverse problems where forward simulations need to be run repeatedly, neural networks can serve as surrogate models that approximate the mapping from damage parameters to wave signals. Applying deep learning, the authors [16] could predict concrete crack propagation paths and showed that neural networks can capture complex damage evolution. In study [17], the authors demonstrated deep learning-based prediction method for identifying time-dependent chloride penetration in concrete and highlighted the broader applicability of these methods for long-term durability assessment.

Beyond deep learning, traditional machine learning methods remain relevant for certain SHM applications. In [18], response surface methodology in combination with artificial neural networks to optimize the sensing capabilities of shape memory alloys for crack detection in concrete is applied. Paper [19] investigated magnetic flux leakage signal response for determining the fatigue life of reinforced concrete beams. Although the measurement modality differs, their use of neural networks for extracting damage-sensitive features from complex signals is applicable to piezoelectric sensor data.

Several additional studies provide valuable methodological insights. Study [20] presented a systematic review of integrating artificial intelligence techniques with infrared thermography for defect detection in concrete structures. The authors [21] developed an image-based method for predicting concrete mechanical properties through CNN segmentation and ANN modeling. In [22], the authors applied deep learning and texture analysis for ASR crack identification in bridges. In [23], a machine learning model is developed for controlling thermal cracking, which represents a serious problem for massive monolithic structures during the manufacturing stage.

Thus, the literature analysis reveals several promising directions for the present work. Physics-informed neural networks, which embed wave equations into the training process [6, 7], offer enhanced reliability and interpretability for inverse problems. Direct processing of acoustic emission and ultrasonic signals using deep residual networks [4, 8, 9] eliminates the need for manual feature extraction. Recent advances have demonstrated the possibility of combining acoustic emission with machine learning for damage classification [11], deep learning for remaining useful life prediction [12], and digital twin platforms integrating neural networks with piezoelectric sensor data [15]. Deep learning for damage localization [8, 13] is directly analogous to the damage parameter identification task addressed here. Hybrid methods combining experimental design with neural network prediction [18, 19] offer methodological templates. Surrogate modeling for accelerating forward simulations [16, 17] enables rapid parameter space exploration. Finite element-based deep learning frameworks [13, 15] combine simulation data with neural networks for damage identification. Foundational reviews [1, 2, 4, 5] provide broad context for AI applications in SHM and acoustics.

Each of these approaches can be adapted to the specific problem addressed in this work, a two-layer anisotropic strip with a damaged surface layer, excited by a piezoelectric actuator, with signals recorded by a piezoelectric sensor. Elastic waves in the strip are excited by the piezoelectric actuator, and signal registration after propagation along the strip is performed by the piezoelectric sensor.

Earlier, we developed an approach for identifying damage parameters (depth and porosity) by intersecting level lines of surfaces of the maximum amplitude and its arrival time. But this method becomes unstable in the low-porosity region (less than 15%). The reason is that signal amplitude barely depends on time in that regime. So, we saw a gap: there is no reliable way to identify damage parameters across the full range of values, especially in the low-porosity region.

The objective of this work is to develop a neural network technology that can reliably identify the depth and porosity of the damaged layer of an extended structure based on the analysis of non-stationary waves, an approach that overcomes the limitations of existing methods. To make this happen, we focused on four main tasks. First, we built a finite element model of a two-layer strip with a piezoelectric actuator and sensor to generate training data. Second, we created training and test datasets by solving a series of direct problems. Third, we developed and trained a neural network to map measured signal characteristics to damage parameters. And finally, we evaluated the accuracy and robustness of our approach paying special attention to the low-porosity region where previous methods had struggled.

**Materials and Methods. Model of the Structure.** An elastic anisotropic two-layer strip (Fig. 1) of thickness  $H$  is considered in the Cartesian coordinate system  $x_1, x_2, x_3$ . The upper damaged layer of thickness  $h$  is modeled as a porous composite with a percentage of porosity  $p$ . A piezoelectric actuator and sensor are located on the surface of the layer (marked in Fig. 1 by numbers 1 and 2, respectively). The upper and lower planes of the strip are free from mechanical stresses. Waves in the strip are excited by the difference of electric potentials on the electrodes of the actuator, while the signal (electric potential) that has passed through the structure is received by the sensor, the lower electrode of which is grounded and the upper one is free. The choice of piezoelectric elements as the source and receiver is not accidental: first, they can be implanted into the structure during its manufacturing stage; second, if the structure is subjected to dynamic loads during operation, the piezoelectric elements can simultaneously serve as piezoelectric generators (PEG) for energy harvesting devices [26]. Thus, the monitoring devices corresponding to the proposed scheme are autonomous and do not require an additional power source.

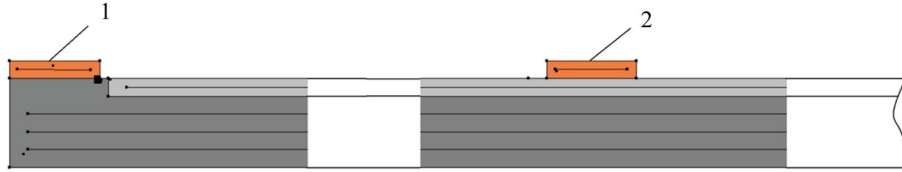


Fig. 1. Schematic of the structure with actuator and sensor

The damaged layer, which is modeled as a porous material, is replaced by a homogeneous one with effective elastic properties. These properties and density were calculated using the ACELAN-COMPOS package [27] and are presented in Table 1. Zero porosity corresponds to the properties of the undamaged strip. The actuator and sensor are made of PZT-4 piezoceramics with vertical polarization (manufactured by Morgan Advanced Materials, UK, or equivalent commercially available material). The calculations were performed under normal conditions (temperature 20°C, atmospheric pressure).

Table 1

Effective Properties [28]

| Porosity (%)                          | 0     | 10    | 20   | 30   | 40   | 50   | 60   | 70   | 80   |
|---------------------------------------|-------|-------|------|------|------|------|------|------|------|
| $\rho, 10^3, \text{kg/m}^3$           | 7.50  | 6.75  | 6.00 | 5.25 | 4.50 | 3.75 | 3.0  | 2.25 | 1.50 |
| $c_{11}^{Eff}, 10^{10}, \text{N/m}^2$ | 13.90 | 11.56 | 9.25 | 6.85 | 5.05 | 3.34 | 2.07 | 1.26 | 0.68 |
| $c_{12}^{Eff}, 10^{10}, \text{N/m}^2$ | 7.78  | 6.15  | 4.66 | 3.14 | 2.10 | 1.16 | 0.62 | 0.28 | 0.13 |
| $c_{13}^{Eff}, 10^{10}, \text{N/m}^2$ | 7.43  | 5.82  | 4.25 | 2.82 | 1.87 | 1.06 | 0.52 | 0.24 | 0.10 |
| $c_{33}^{Eff}, 10^{10}, \text{N/m}^2$ | 11.50 | 9.53  | 7.23 | 5.42 | 3.91 | 2.72 | 1.63 | 0.91 | 0.47 |
| $c_{44}^{Eff}, 10^{10}, \text{N/m}^2$ | 2.56  | 2.23  | 1.83 | 1.44 | 1.10 | 0.74 | 0.44 | 0.23 | 0.10 |

**Mathematical Formulation of the Problem.** This work considers a model for a system of elastic and electroelastic bodies in a plane formulation [29]. The index  $j$  corresponds to the numbering of the bodies. The system of differential equations describing the non-stationary mechanical and electric fields has the form:

$$\begin{aligned} \rho_j \ddot{\mathbf{u}} + \alpha_j \rho_j \dot{\mathbf{u}} - \nabla \cdot \boldsymbol{\sigma} &= \mathbf{f}_j; \nabla \cdot \mathbf{D} = 0; \\ \boldsymbol{\sigma} &= \mathbf{c}_j^E \cdot (\boldsymbol{\varepsilon} + \beta_{dj} \dot{\boldsymbol{\varepsilon}}) - \mathbf{e}_j^T \cdot \mathbf{E}; \mathbf{E} = -\nabla \varphi; \\ \mathbf{D} + \varsigma_d \dot{\mathbf{D}} &= \mathbf{e}_j \cdot (\boldsymbol{\varepsilon} + \varsigma_d \dot{\boldsymbol{\varepsilon}}) + \mathfrak{D}_j^S \cdot \mathbf{E}; \boldsymbol{\varepsilon} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T) / 2, \end{aligned} \quad (1)$$

where  $\boldsymbol{\sigma}$  — stress tensor,  $\rho_j$  — density of the body,  $\boldsymbol{\varepsilon}$  — strain tensor,  $\mathbf{u}$  — displacement vector,  $\mathbf{D}$  — electric induction vector,  $\mathbf{E}$  — electric field strength vector,  $\mathbf{f}_j$  — body force vector,  $\varphi$  — electric potential,  $\alpha, \beta, \varsigma, \beta_{dj}, \varsigma_d$  — damping coefficients,  $\mathbf{c}_j^E, \mathbf{e}_j^T, \mathfrak{D}_j^S$  — tensors of elastic constants, piezomoduli, and dielectric permittivities, respectively, the index  $j$  corresponds to the body number in the model.

When modeling elastic bodies (strip and damaged layer), system (1) takes the form:

$$\rho_{pk} \ddot{\mathbf{u}} + \alpha_{dj} \rho_j \dot{\mathbf{u}} - \nabla \cdot \boldsymbol{\sigma} = \mathbf{f}_j, \quad \boldsymbol{\sigma} = \mathbf{c}_j^E \cdot (\boldsymbol{\varepsilon} + \beta_{dj} \dot{\boldsymbol{\varepsilon}}), \quad \boldsymbol{\varepsilon} = (\nabla \mathbf{u} + \nabla \mathbf{u}^T) / 2. \quad (2)$$

To systems (1), (2), mechanical and electrical boundary conditions are added, which correspond to zero mechanical stresses on the external boundaries of the bodies, constant electric potential on the electrodes, zero normal component of the electric induction vector on non-electroded boundaries, and zero initial conditions. To determine the electric potential on the free electrode  $S_E$  of the sensor, an additional condition of the form is used:

$$\int_{S_E} \dot{D}_n ds = 0. \quad (3)$$

Elastic waves are excited by the variable electric potential  $\varphi = \varphi(t)$  (Fig. 2a) on the upper electrode of the actuator (number 1 in Fig. 1).

**Finite Element Model.** The dataset for training the artificial neural network (ANN) is constructed based on solving direct problems for the strip with known parameters of the damaged state. The solution of these non-stationary problems is carried out using the finite element method (FEM) in the ACELAN package [29] with quadratic triangular elements. The finite element mesh contains 5,616 elements and 12,155 nodes. Figures 1–3 were obtained in the ACELAN package. Figures 4–6 were obtained using the Matplotlib library of the Python programming language based on the calculation results.

Figure 2a shows the time dependence of the potential difference on the actuator (three half-waves of a sinusoid with an amplitude of 100 V), which excites an elastic wave in the strip, and Figure 2b shows the distribution of the vertical displacement of this wave.

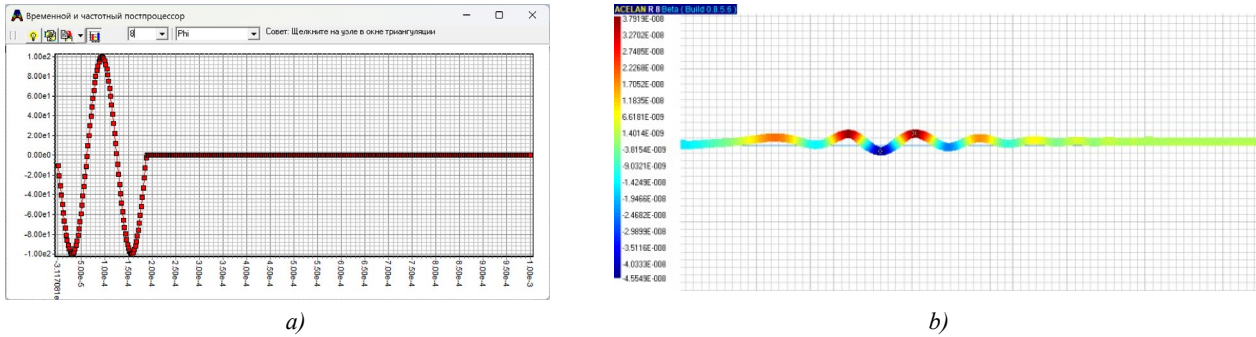


Fig. 2. Time dependence: *a* – potential difference on the actuator; *b* – distribution of vertical displacement

Comparison of the electric potentials on the sensor at 10% and 70% porosity (Fig. 3), calculated using (3), shows significant differences in both amplitude and signal arrival time.

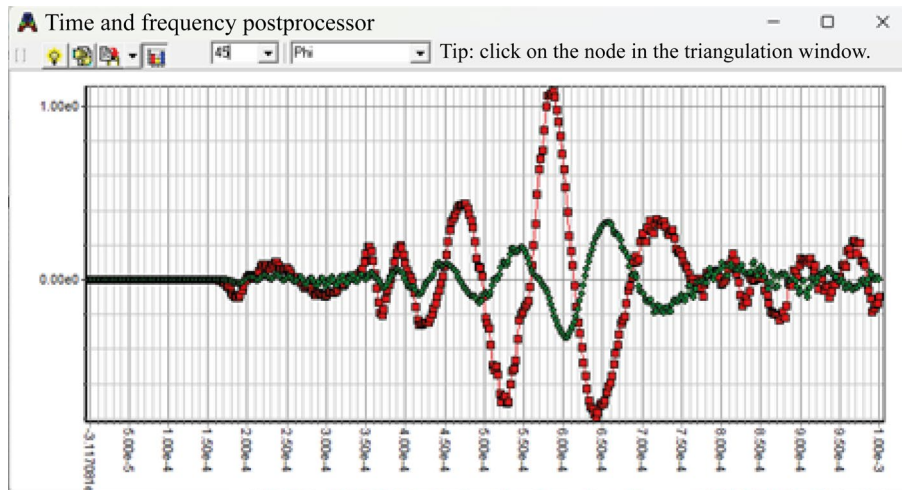


Fig. 3. Time dependence of the electric potential on the sensor at 10% porosity (red curve) and 70% porosity (green curve)

This significant dependence of the measured characteristics on the damage parameters ensures the possibility of identifying porosity and depth of the damaged layer.

**Data Generation for Neural Network Training.** The initial data (the maximum amplitude of the electric potential on the sensor, Fig. 1, number 2, and its arrival time) consist of 48 points obtained from finite element calculations: 6 depth values and 8 porosity values.

Table 2

T — Time ( $\times 10^{-4}$  s) of the Maximum Electric Potential  
Amplitude versus Damage Parameters

| $h$ (mm) $p$ (%) | 10    | 20    | 30    | 40    | 50   | 60    | 70    | 80    |
|------------------|-------|-------|-------|-------|------|-------|-------|-------|
| 0.5              | 5.872 | 5.865 | 5.870 | 5.890 | 5.92 | 5.940 | 5.980 | 6.000 |
| 1                | 5.805 | 5.880 | 5.940 | 6.000 | 6.06 | 6.130 | 6.180 | 6.204 |
| 1.5              | 5.804 | 5.900 | 5.985 | 6.100 | 6.19 | 6.280 | 6.340 | 6.390 |
| 2                | 5.800 | 5.920 | 6.030 | 6.140 | 6.30 | 6.448 | 6.570 | 6.680 |
| 2.5              | 5.804 | 5.940 | 6.050 | 6.200 | 6.37 | 6.590 | 6.780 | 6.890 |
| 3                | 5.804 | 5.940 | 6.075 | 6.285 | 6.52 | 6.770 | 6.930 | 7.160 |

Table 3

V — Amplitude of the Electric Potential (V) versus Damage Parameters

| $h$ (mm) / $p$ (%) | 10    | 20    | 30    | 40    | 50    | 60    | 70    | 80    |
|--------------------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.5                | 1.097 | 1.065 | 1.040 | 0.965 | 0.845 | 0.700 | 0.500 | 0.296 |
| 1                  | 1.092 | 1.070 | 1.010 | 0.890 | 0.770 | 0.590 | 0.405 | 0.232 |
| 1.5                | 1.070 | 1.054 | 0.978 | 0.870 | 0.760 | 0.560 | 0.370 | 0.202 |
| 2                  | 1.058 | 1.044 | 0.96  | 0.860 | 0.740 | 0.532 | 0.340 | 0.190 |
| 2.5                | 1.058 | 1.030 | 0.935 | 0.858 | 0.720 | 0.515 | 0.321 | 0.157 |
| 3                  | 1.060 | 1.030 | 0.935 | 0.843 | 0.720 | 0.467 | 0.320 | 0.155 |

For training and validation of the ANN based on Tables 2 and 3, two datasets were created. The first dataset used two-dimensional spline interpolation. The training dataset, in which cubic interpolation was used, allowed generating a dense training grid of size  $100 \times 100 = 10,000$  points in the parameter domain ( $p$ ,  $h$ ). A second independent test dataset of size  $200 \times 200 = 40,000$  points was generated using the same interpolation method.

**Neural Network Architecture.** For the training dataset, a multilayer feedforward neural network was implemented. The architecture of the feedforward ANN is presented in Table 4.

Table 4

Architecture of the ANN

| Layer             | Neurons | Activation |
|-------------------|---------|------------|
| Input             | 2       | —          |
| Hidden 1          | 128     | ReLU       |
| Dropout           | 20%     | —          |
| Hidden 2          | 64      | ReLU       |
| Dropout           | 20%     | —          |
| Hidden 3          | 32      | ReLU       |
| Hidden 4          | 16      | ReLU       |
| Output (Porosity) | 1       | Linear     |
| Output (Depth)    | 1       | Linear     |

The network has two output branches (porosity and depth) sharing the same hidden layers. Dropout layers (20%) were added after the first two hidden layers to prevent overfitting. The Adam optimizer was used with an initial learning rate of 0.001.

**Weighted Loss Function.** To reduce the positive asymmetry observed in the initial porosity predictions, a weighted mean squared error loss function was used for the porosity output:

$$\mathcal{L}_{\text{porosity}} = \frac{1}{N} \sum_{i=0}^N w_i \left( p_i^{\text{true}} - p_i^{\text{pred}} \right)^2, w_i = \begin{cases} 2, & \text{if } p_i^{\text{true}} - p_i^{\text{pred}} < 0 \\ 1, & \text{otherwise} \end{cases} \quad (4)$$

This loss function penalizes overestimation of predictions twice as much as underestimation, effectively reducing the positive bias in predictions.

**Research Results.** This section presents the results of a numerical experiment on the training and validation of an artificial neural network (ANN) for identifying damage parameters (porosity  $p$  and depth  $h$  of the damaged layer). The results are organized in the following sequence: analysis of the network training process, assessment of overall performance, analysis of accuracy in the low-porosity region, and verification of the uniqueness of the inverse problem solution.

**Analysis of the Neural Network Training Process.** The first stage of the numerical experiment involved investigating the training of the ANN as a function of the number of epochs. The predictions were made using the first training dataset and the second ANN architecture (Table 4). Training was conducted for up to 500 epochs with early stopping and learning rate reduction on a plateau. During the training process, the loss function on the training dataset decreased from 0.120 to 0.003; early stopping was reached at approximately the 280th epoch (Fig. 4a). The loss function on the validation dataset reached its minimum by the early stopping point, after which slight overfitting began, which served as a signal for its activation. The final loss function value on the validation dataset was 0.0045, confirming the good generalization capability of the trained network. Figure 4 shows the training and validation loss curves. The calculations were performed using the Matplotlib library of the Python programming language.

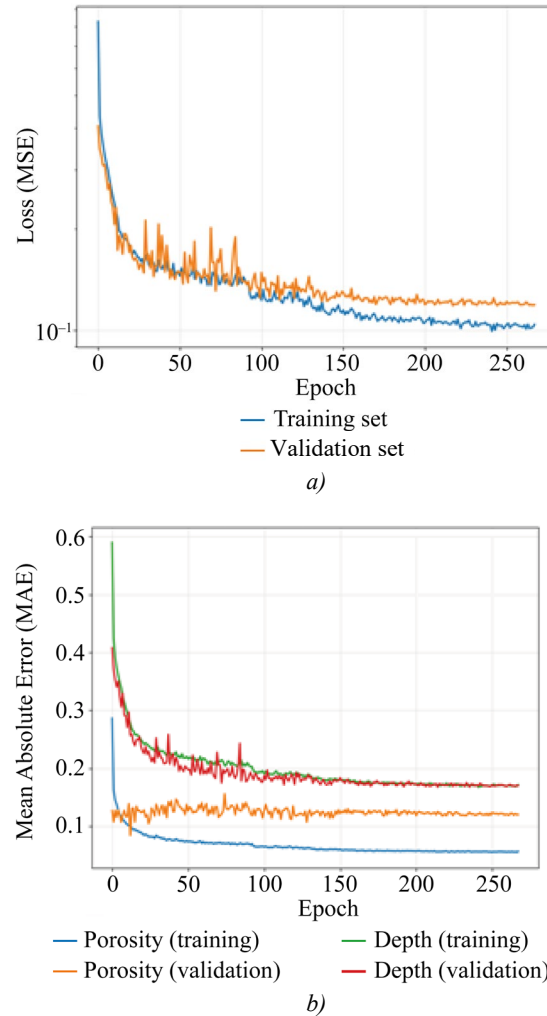


Fig. 4. Training and validation loss curves: *a* — Mean Squared Error (MSE);  
*b* — Mean Absolute Error (MAE) for porosity and depth

**Uniqueness of the Inverse Problem Solution.** The injectivity of the mapping from the measured quantities ( $V$ ,  $T$ ) to the damage parameters ( $p$ ,  $h$ ) was verified on an independent test dataset of 40,000 points. All 40,000 test points yielded unique two-dimensional signatures. These results empirically demonstrate that the inverse problem has a unique solution for all practically significant damage states.

**Overall Performance of the Neural Network.** The trained neural network was evaluated on an independent test dataset of 40,000 points. Table 5 presents the overall performance metrics.

Table 5

Overall Neural Network Performance

| Metric                                 | Porosity | Depth    |
|----------------------------------------|----------|----------|
| Mean Absolute Error (MAE)              | 2.433%   | 0.121 mm |
| Coefficient of Determination ( $R^2$ ) | 0.977    | 0.910    |

Figure 5 presents scatter plots of predicted values versus true values. The excellent alignment with the diagonal line demonstrates the accuracy of the neural network predictions. The calculation was performed using the Matplotlib library of the Python programming language.

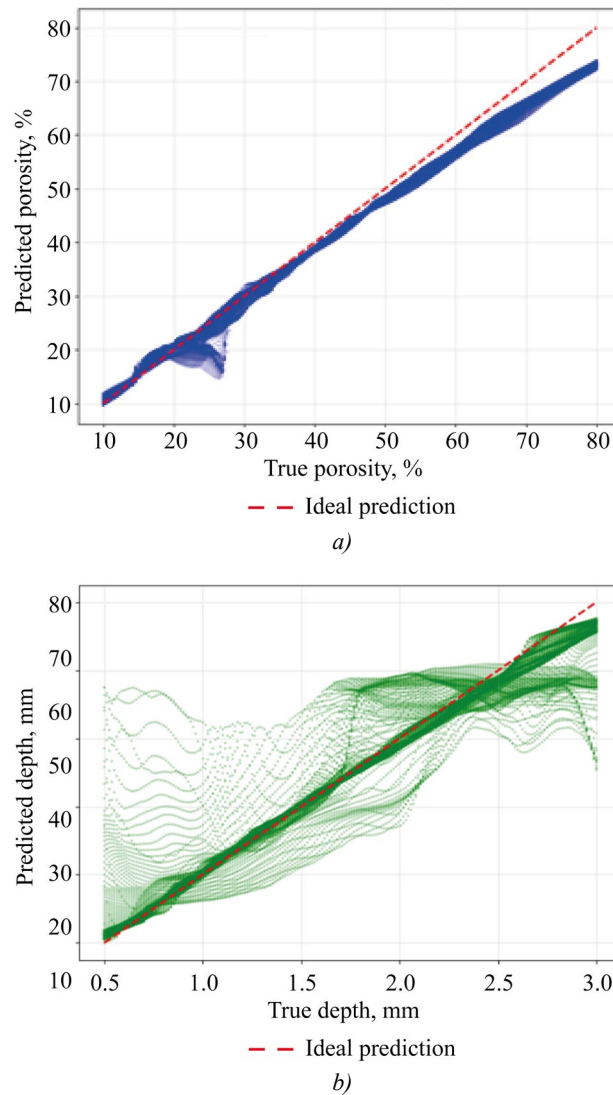


Fig. 5. Neural network predictions compared to true values:  
 a — porosity: MAE = 2.433%,  $R^2 = 0.977$ , bias = -2.191%;  
 b — depth: MAE = 0.121 mm,  $R^2 = 0.910$ , bias = -0.014 mm

**Performance in the Low-Porosity Region.** A key contribution of this work is the improved performance in the low-porosity region ( $p < 15\%$ ), where the method of intersecting contour lines of the surfaces defined in Tables 2 and 3 was unstable. Table 6 presents the performance of the neural network for test points with porosity below 15%, comprising 3000 points.

Table 6

Performance in the Low-Porosity Region

| Metric                                 | Porosity | Depth     |
|----------------------------------------|----------|-----------|
| Mean Absolute Error (MAE)              | 0.405%   | 0.173 mm  |
| Coefficient of Determination ( $R^2$ ) | 0.865    | 0.886     |
| Bias (mean error)                      | +0.188%  | +0.012 mm |

The neural network achieves excellent accuracy in the low-porosity region, with a mean absolute error of only 0.405% for porosity and a bias of only +0.188%. Figure 6 presents the error distribution for test points with porosity less than 15%, confirming the nearly symmetric error distribution. The calculation was performed using the Matplotlib library of the Python programming language.

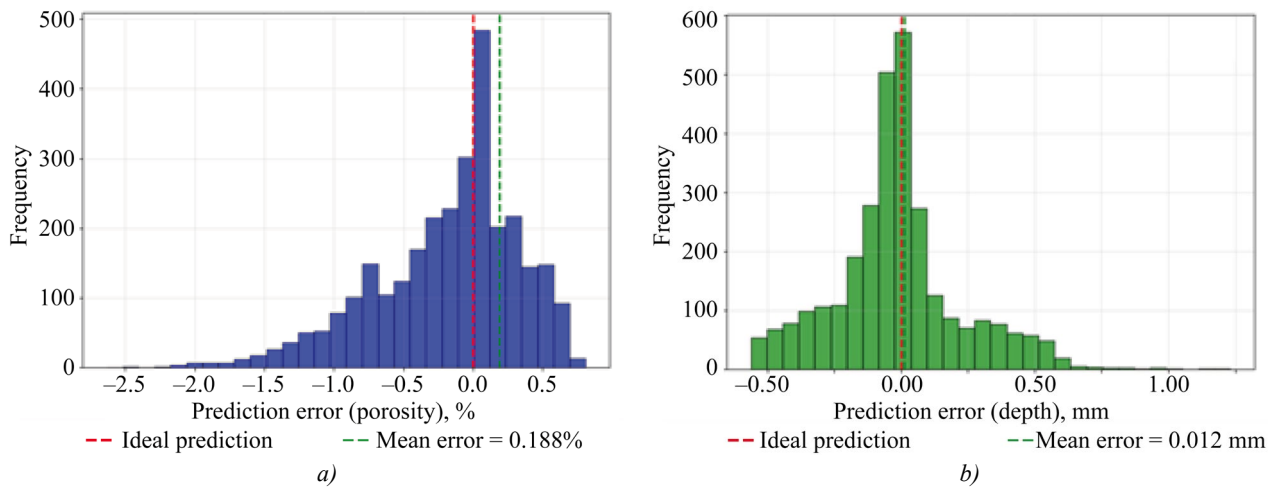


Fig. 6. Error distribution for test points with porosity less than 15%:

*a* — porosity: MAE = 0.405%, bias = 0.188%;

*b* — depth: 0.173 mm, bias = 0.012 mm

**Discussion.** The key result of this work is the development of a neural network technology for acoustic monitoring of the damaged state of extended building structures that allows automatic identification of damage parameters (porosity  $p$  and depth  $h$  of the damaged layer) based on the analysis of non-stationary waves.

The proposed approach successfully overcomes a fundamental limitation of the contour line intersection method developed earlier based on the dependence of the maximum amplitude of the electric potential at the sensor and its arrival time on the damage parameters, which became unstable in the low-porosity region ( $p < 15\%$ ) because of the weak dependence of the signal amplitude on time. In contrast, the trained neural network (a multilayer perceptron with the architecture 128-64-32-16) demonstrates stable and accurate predictions across the entire range of parameter values, including the problematic low-porosity region. This is confirmed by the numerical results. In the region  $p < 15\%$ , the network achieves MAE = 0.405% for porosity and MAE = 0.173 mm for depth, which is comparable to the accuracy in the high-porosity region.

The use of a weighted loss function that penalizes overestimation twice as much as underestimation plays an effective role in reducing the positive skewness of porosity predictions. In the low-porosity region, the bias (mean error) was only +0.188%, indicating practically unbiased estimates which is particularly important for practical applications, where overestimation of the degree of damage can lead to false alarms and unnecessary repair work.

In the output of the python program, all 40,000 test points yield unique two-dimensional signatures, empirically confirming the uniqueness of the inverse problem solution and the reliability of the proposed approach and its suitability for practical use. This property of the mapping  $(V, T) \rightarrow (p, h)$  guarantees that each measured signal corresponds to a unique set of damage parameters, which eliminates ambiguity in the interpretation of monitoring results.

Comparison of the obtained results with the current literature shows that the proposed approach is aligned with the actively developing methods of applying neural networks for Structural Health Monitoring tasks. The studies [4, 8, 9, 11] have demonstrated the effectiveness of deep learning for processing acoustic emission and ultrasound signals. In contrast to these works, where neural networks are used for classification of damage types or localization of sources, the present study addresses a more complex task — quantitative identification of two damage parameters (porosity and depth) from two measured signal characteristics demonstrating that even with limited input information (only two features), a neural network is capable of providing high prediction accuracy.

A key advantage of the proposed approach lies in its energy autonomy. By using piezoelectric elements that function as both actuator and sensor, the system can be embedded into a structure during manufacturing. Under dynamic loading, these same elements act as piezoelectric generators, harvesting energy and making the monitoring setup fully self-sustaining, no external power source is required. This feature is particularly attractive for such locations as bridge components and underground facilities that are hard to reach and changing batteries or maintaining a grid connection is often impractical.

At the same time, this approach has a number of limitations that should be considered in its practical application. First, it assumes constant porosity throughout the depth and length of the damaged layer. In real structures, damage often has a complex spatial structure with a non-uniform porosity distribution. Accounting for such inhomogeneities would lead to an inverse problem of significantly higher complexity, requiring an increase in the number of input features and, possibly, the use of more complex neural network architectures. Second, the study was conducted based on numerical simulation. Experimental validation on real concrete specimens is a necessary next step. Third, the parameter range is limited to ( $10\% \leq p \leq 80\%$ ,  $0.05 \text{ mm} \leq h \leq 0.3 \text{ mm}$ ). To extend this range additional network training may be required.

Future research directions include several avenues. First, the use of the entire amplitude–time characteristic of the electric potential at the sensor (rather than just two extracted features) as input information, and the application of convolutional neural networks (CNNs) for automatic feature extraction from time-domain signals. Second, experimental validation of the developed approach on laboratory concrete specimens with controlled artificial damage. Third, adaptation of the method to elements of complex geometry (cross-sections typical of structural components).

In conclusion, it should be emphasized that the developed neural network technology represents a convenient non-destructive testing tool that can be readily implemented in a physical device. It provides stable and accurate predictions of damage parameters across the entire range of their values, confirming its practical value for acoustic monitoring systems of extended concrete structures. The proposed approach can be integrated into automated structural health monitoring systems that can enable timely damage detection to prevent catastrophic failure consequences.

**Conclusion.** In this work, the problem of automatic determination of the damaged state of extended building structures has been successfully solved. The parameters considered for this purpose were the thickness of the damaged layer and the degree of damage, the porosity of this material. To this end, a physical model of a monitoring device was developed, consisting of piezoelectric actuator and sensor elements. Additional information was selected for solving the inverse geometric and coefficient problem of identifying damage parameters in a finite-dimensional domain.

A mathematical and computational model (within the finite element method) of such a device was developed. Calculations based on these models demonstrated a significant dependence of the maximum amplitude of the electric potential at the sensor and its arrival time on the damage parameters. Using this model within the finite element package ACELAN, the dependencies of the additional information on the damage parameters were obtained. Based on these dependencies, two datasets were generated using cubic interpolation for training the neural network: a training set (10,000 points) and a test set (40,000 points).

The training and validation of the neural network show that it can be effectively applied to the monitoring problem. Training ran for up to 500 epochs with early stopping and learning rate reduction on a plateau; the early stopping point was reached at around the 280th epoch. So, this work offers a neural network-based solution to the inverse problem of identifying damage parameters in extended concrete structures using acoustic monitoring.

### Main Conclusions

1. In this work, we trained a feedforward neural network consisting of four hidden layers (with 128, 64, 32, and 16 neurons, respectively) to predict the porosity  $p$  and the damaged-layer depth  $h$  from the normalized wave propagation time and the normalized electric potential. The network employs ReLU activations and includes dropout layers with a rate of 20%.
2. Evaluated on 40,000 independent test samples, the network gave us a mean absolute error of 2.433% for porosity and 0.121 mm for depth, with corresponding coefficients of determination of  $R^2 = 0.977$  and  $R^2 = 0.910$ .
3. In the low-porosity region ( $p < 15\%$ ), where the contour line intersection method from our previous work was unstable, the neural network achieves a mean absolute error of 0.405% for porosity and 0.173 mm for depth, with a bias of only +0.188%.
4. A weighted loss strategy that penalizes overprediction twice as harshly as underprediction, helped reduce the positive skewness of the porosity predictions.
5. Our neural network solves a key problem that the contour-line method couldn't handle, it gives stable and accurate predictions across the whole porosity range.
6. All 40,000 test points yielded unique two-dimensional signatures. This empirically confirmed the uniqueness of the inverse problem, demonstrating the injectivity of the mapping.

Thus, the result of this work proposes a device scheme and software in the form of a trained ANN that automatically monitor damaged state. The practical importance of this study is the development of a convenient non-destructive testing tool that can be readily implemented in a physical device. The use of piezoelectric elements as both actuator and sensor ensures the energy autonomy of the monitoring system, which is particularly important for hard-to-reach areas of structures.

The proposed method was demonstrated on a simple geometry (a strip) of an extended structural element. Future research directions include the development of monitoring models for elements of complex geometry (cross-sections typical of structural components). Future work may focus on experimental validation using laboratory concrete specimens with controlled artificial damage, as well as the inclusion of additional sensor data (the entire amplitude–time characteristic of the electric potential) and the application of convolutional neural networks to improve prediction accuracy.

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