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THREE-DIMENSIONAL CONTACT PROBLEM FOR A TWO-LAYERED EXTRA LOADED ELASTIC BASE*

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The three-dimensional Galin's type contact problem for a two-layered elastic base (a layer completely attached to a half-space from another material) is investigated when an extra loading (concentrated force) is applied outside the contact area. The contact zone is supposed to be unknown. The punch foot form is an elliptic paraboloid. The problem is reduced to an integral equation with respect to the unknown contact pressure distributed in the unknown contact zone. Galanov's method of nonlinear boundary integral equations is used to determine the contact pressure and the contact zone simultaneously. Calculations made for various values of elastic and geometric parameters allow estimating an extra force input to the dependence between the punch settlement and the force applied to the punch. The problem is important for the strength analysis of coated surfaces of various elastic solids subjected to contact and extra loadings. The solution can be also useful in the frame of the discrete contact theory for bodies with rough surfaces.

Keywords: *theory of elasticity, contact problems, two-layered elastic base, nonlinear boundary integral equations, Galanov's method.*

Introduction. Bodies with coverings represent a widespread class of materials, so their study has the great theoretical and practical significance. L. A. Galin was probably the first who has considered the contact problem for a half-space with an additional concentrated force applied outside the contact area [1]. A similar contact problem for a two-layered elastic base was investigated earlier [2] without additional force. The Galanov's method used below to take the additional concentrated force into account allows us to estimate the influence of the extra force onto the contact pressure as well as onto the force applied of the punch. This problem is of interest for the contact mechanics of the bodies with coverings.

Statement of problem. Consider the contact problem of the indentation of a punch into an elastic layer of the thickness h completely attached to an elastic half-space. The punch is acted on by a force P. The shape of the punch foot is described by the function $f(x, y)$. The elastic characteristics of the layer and half-space are G_1, ν_1 and G_2, ν_2, respectively. For simplicity, we assume that the punch has the form of an elliptic paraboloid, i. e.

$$f(x,y)=\frac{x^2}{2R_1}+\frac{y^2}{2R_2}.$$

The force P is applied to the punch so that the punch penetrates without rotation by a depth δ. Assume that an additional normal concentrated force Q is applied at the point $x = c$, $y = 0$.

By means of the methods of the operational calculus [2], one can derive the following integral equation with respect to the normal contact pressure $q(x, y)$:

$$\iint_{\Omega} q(\xi,\eta)K\left(\frac{R}{h}\right)d\xi d\eta = 2\pi h\theta_1\left(\delta - f(x,y)\right) - QK\left(\frac{R_1}{h}\right),\quad (x,y)\in\Omega, \tag{1}$$

$$K(t)=\int_0^{\infty} N(u)J_0(ut)\,du,\quad R=\sqrt{(x-\xi)^2+(y-\eta)^2},\quad R_1=\sqrt{(x-c)^2+y^2}, \tag{2}$$

$$N(u)=\frac{M+4u\exp(-2u)-L\exp(-4u)}{M-\left(1+4u^2+LM\right)\exp(-2u)+L\exp(-4u)},$$

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$$M = \frac{G_2\kappa_1 + G_1}{G_1 - G_2},\ L = \frac{G_1\kappa_2 - G_2\kappa_1}{G_1\kappa_2 + G_2},\ \theta_n = \frac{G_n}{1 - \nu_n},\ \kappa_n = 3 - 4\nu_n \quad (n = 1, 2).$$

Here, Ω is an unknown contact area.

To determine the connection between the force P and the settlement δ one can use the integral condition of equilibrium of the punch

$$\iint_{\Omega} q(x, y)\,dxdy = P.$$

The Galanov's method. To solve the contact problem and equation (1) we use the method of nonlinear boundary integral equations based on the theorems discussed in [2, 3].

Let the contact area be a priori included into the rectangle

$$S = \{|x| \le a, |y| \le b\},\ b \ge a.$$

We introduce the following dimensionless notation:

$$\frac{x}{b} = x',\ \frac{y}{b} = y',\ \frac{R}{b} = R',\ \frac{R_1}{b} = R_1',\ \frac{\delta}{b} = \delta',\ \frac{h}{b} = \lambda,\ \frac{a}{b} = \varepsilon',\ \frac{c}{b} = c',$$

$$\frac{b}{2R_1} = A',\ \frac{b}{2R_2} = B',\ \frac{q(x, y)}{2\pi\theta_1} = q'(x', y'),\ \frac{P}{2\pi\theta_1 b^2} = P',\ \frac{Q}{2\pi\theta_1 b^2} = Q',\ \text{etc.}$$

We will omit the primes. The dimensionless parameter λ characterizes the relative thickness of the elastic layer.

To calculate the function $K(t)$ in formula (2) one should extract its principal term by using the integral

$$\int_0^{\infty} J_0(ut)\,du = \frac{1}{t}.$$

The elastic materials chosen for calculations and their elastic parameters (Young's modulus E and Poisson's ratio ν) are presented in Table 1. The shear modulus is given by formula $G = \dfrac{E}{2(1+\nu)}$.

Table 1

Elastic parameters

Modulus Material	$E\times10^{-4}$ (MPa)	ν
Carbon steel	20	0.28
Cold-drawn brass	9	0.35
Concrete	2	0.17

The two following cases were taken for the calculations: steel on brass (case A) and steel on concrete (case B). The values of the contact pressure $q_0 = q(0, 0)$ and of the force P applied to the punch are tabulated in Table 2 for δ = A = B = ε = 1.

Table 2

Values of pressure q_0 and force P

λ	Q	c	q_0	P	q_0	P
			Case A		Case B	
1	0.1	1.5	0.229	0.163	0.127	0.0322
0.5	0.1	1.5	0.191	0.137	0.0631	0.0167
1	0.05	1.5	0.236	0.183	0.159	0.0547
0.5	0.05	1.5	0.194	0.154	0.112	0.0343
1	0.1	1.2	0.226	0.156	0.111	0.0267
0.5	0.1	1.2	0.188	0.129	0.0145	0.0102
1	0.05	1.2	0.235	0.179	0.155	0.0542
0.5	0.05	1.2	0.193	0.149	0.104	0.0304

As one can see from Table 2, for case A the force *P* and the pressure q_0 are greater than those in case B. As the value of λ decreases, the force *P* and the pressure drop because the top steel layer becomes thinner. As the force *Q* increases and also as the value of *c* decreases (the force *Q* goes to the contact area), the force *P* and the pressure will drop because of the force *Q* input becomes significant.

Conclusion. The obtained results have a clear mechanical sense helping us to estimate the extra force influence onto the typical contact characteristics. The Galanov's method is quite effective for the three-dimensional elastic contact analysis in the presence of an additional concentrated force applied outside the contact area.

Reference

1. Argatov, I. I., Dmitriev, N. N. Foundations of the theory of elastic discrete contact. Saint Petersburg : Polytekhnika, 2003, 233 p. (in Russian).
2. Alexandrov, V. M., Kalker, J. J., Pozharskiy, D. A. Three-dimensional contact problem for a two-layered elastic base with an unknown contact area. Mechanics of Solids, 1999, no. 4, pp. 41–45.
3. Alexandrov, V. M., Pozharskiy, D. A. Three-dimensional contact problems. Dordrecht : Kluwer, 2001, 406 p.

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Библиографический список

1. Аргатов, А. А., Дмитриев, Н. Н. Основы теории упругого дискретного контакта / А. А. Аргатов, Н. Н. Дмитриев. — Санкт-Петербург : Политехника, 2003. — 233 с.
2. Alexandrov, V. M., Kalker, J. J., Pozharskiy, D. A. Three-dimensional contact problem for a two-layered elastic base with an unknown contact area. Mechanics of Solids, 1999, no. 4, pp. 41–45.
3. Alexandrov, V. M., Pozharskiy, D. A. Three-dimensional contact problems. Dordrecht : Kluwer, 2001, 406 p.

Трёхмерная контактная задача для двухслойного дополнительно нагруженного упругого основания*

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Исследуется трёхмерная контактная задача (типа задачи Галина) для двухслойного упругого основания (слой полностью сцеплен с полупространством из другого материала) при действии дополнительной нагрузки (сосредоточенной силы) вне области контакта. Предполагается, что зона контакта неизвестна. Форма основания штампа — эллиптический параболоид. Задача сводится к интегральному уравнению относительно неизвестного контактного давления, распределённого в неизвестной области контакта. Используется метод нелинейных граничных интегральных уравнений, предложенный Галановым и позволяющий одновременно определить контактные давления и область контакта. Расчёты, сделанные для различных значений упругих и геометрических параметров, позволяют оценить вклад дополнительной силы на зависимость между осадкой штампа и приложенной к штампу силой. Задача важна для анализа контактной прочности поверхностей упругих тел, имеющих покрытия. Найденное решение также полезно в теории дискретного контакта шероховатых тел.

Ключевые слова: *теория упругости, контактные задачи, двухслойное упругое основание, нелинейные граничные интегральные уравнения, метод Галанова.*

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